11. *Studies on Holonomic Quantum Fields. VII*

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In [1] we have constructed field operators $\varphi_F(x)$ and $\varphi^F(x)$, which satisfy a simple commutation relation with the auxiliary free fermi field $\psi(x)$ ((37) in [2]). This commutation relation implies a remarkable monodromy structure of the wave function $\langle\psi(x)\varphi_F(a_1)\cdots\varphi^F(a_\nu)\cdots\varphi_F(a_n)\rangle$. Namely, as a function of x it changes the sign when prolonged around each branch point a_μ $(\mu=1,\cdots,n)$.

The present note deals with the following three topics, extending the above mentioned results.

i) Construction of operators with arbitrary exponents of local monodromy.

ii) Construction of an $n(n-1)/2$ parameter family of wave functions with different global monodromy structures.

iii) Computation of global monodromy at degenerate fibers in the above family.

1. Let $\psi(u)$, $\psi^*(u)$ $(u>0)$ and $\psi^\dagger(u)\underset{\text{def}}{=}\psi^*(-u)$, $\psi^{*\dagger}(u)\underset{\text{def}}{=}\psi(-u)$ $(u>0)$ denote annihilation and creation operators, respectively. They satisfy the following.

$$(1)\qquad \begin{pmatrix}[\psi(u),\psi(u')]_+ & [\psi(u),\psi^*(u')]_+\\ [\psi^*(u),\psi(u')]_+ & [\psi^*(u),\psi^*(u')]_+\end{pmatrix} = \begin{pmatrix}\langle\psi(u),\psi(u')\rangle & \langle\psi(u),\psi^*(u')\rangle\\ \langle\psi^*(u),\psi(u')\rangle & \langle\psi^*(u),\psi^*(u')\rangle\end{pmatrix} = \begin{pmatrix}0 & 2\pi|u|\,\delta(u+u')\\ 2\pi|u|\,\delta(u+u') & 0\end{pmatrix},$$

$$(2)\qquad \begin{pmatrix}\langle\psi(u)\psi(u')\rangle & \langle\psi(u)\psi^*(u')\rangle\\ \langle\psi^*(u)\psi(u')\rangle & \langle\psi^*(u)\psi^*(u')\rangle\end{pmatrix} = \begin{pmatrix}0 & 2\pi u_+\delta(u+u')\\ 2\pi u_+\delta(u+u') & 0\end{pmatrix}.$$

We set

$$(3)\qquad \psi_l(x)=\int \underline{du}(0+iu)^l e^{-im(x-u+x+u^{-1})}\psi(u),$$

$$\psi_l^*(x)=\int \underline{du}(0+iu)^l e^{-im(x-u+x+u^{-1})}\psi^*(u),$$

where $l\in \boldsymbol{C}$ and $(0+iu)^l=e^{\pm\pi il/2}|u|^l$ if $u\gtrless 0$. We abbreviate $\psi_{\pm 1/2}(x)$ (resp. $\psi^*_{\pm 1/2}(x)$) to $\psi_\pm(x)$ (resp. $\psi^*_\pm(x)$) and set

$$\psi(x)={}^t(\psi_+(x),\psi_-(x)),\qquad \psi^*(x)={}^t(\psi^*_+(x),\psi^*_-(x)).$$

Following (12) in [3] we set

$$(4)\quad \rho_F(x;l)=\iint \underline{du}\,\underline{du'}[R(u,u';l)\psi(u)\psi^*(u')+R(u,u';-l)\psi^*(u)\psi(u')]\times e^{-im(x^-(u+u')+x^+(u^{-1}+u'^{-1}))}$$

where $R(u,u';l)=2\sin\pi l\left(\frac{u-i0}{u'-i0}\right)^{-l}\frac{i}{u+u'-i0}\sqrt{u-i0}\sqrt{u'-i0}$, and define operators $\varphi_F(x;l)$, $\varphi^F_{l'}(x;l)$ and $\varphi^{F*}_{l'}(x;l)$ by

$$(5)\quad \begin{aligned}\mathrm{Nr}\,(\varphi_F(x;l))&=\exp(\rho_F(x;l)/2),\\ \mathrm{Nr}\,(\varphi^F_{l'}(x;l))&=\psi_{l'}(x)\exp(\rho_F(x;l)/2),\\ \mathrm{Nr}\,(\varphi^{F*}_{l'}(x;l))&=\psi^*_{l'}(x)\exp(\rho_F(x;l)/2),\end{aligned}$$

respectively.

The following local expansion formula is valid (cf. (29), (30) in [2]).

$$(6)\quad \begin{aligned}&\mathrm{Nr}\,(\psi(x)\varphi_F(a;l))\\ &=\sum_{j=0}^{\infty}\mathrm{Nr}\,(\varphi^F_{-l+1/2+j}(a;l))\cdot w_{-l+1/2+j}(-(x-a)^-+i0,(x-a)^+-i0)\\ &\quad+\sum_{j=0}^{\infty}\mathrm{Nr}\,(\varphi^F_{-l-1/2-j}(a;l))\cdot w^*_{l+1/2+j}(-(x-a)^-+i0,(x-a)^+-i0),\\ &\mathrm{Nr}\,(\psi^*(x)\varphi_F(a;l))\\ &=\sum_{j=0}^{\infty}\mathrm{Nr}\,(\varphi^{F*}_{l+1/2+j}(a;l))\cdot w_{l+1/2+j}(-(x-a)^-+i0,(x-a)^+-i0)\\ &\quad+\sum_{j=0}^{\infty}\mathrm{Nr}\,(\varphi^{F*}_{l-1/2-j}(a;l))\cdot w^*_{-l+1/2+j}(-(x-a)^-+i0,(x-a)^+-i0).\end{aligned}$$

$\mathrm{Nr}\,(\varphi_F(a;l)\psi(x))$ and $\mathrm{Nr}\,(\varphi_F(a;l)\psi^*(x))$ are given by the right hand sides of (6) with $\mp(x-a)^\mp\pm i0$ replaced by $\mp(x-a)^\mp\mp i0$. Similarly we have

$$(7)\quad \begin{aligned}&\mathrm{Nr}\,(\psi(x)\varphi^{F*}_{l'}(a;l))\\ &=\frac{i}{2\cos\pi l'}\mathrm{Nr}\,(\varphi_F(a;l))\cdot(w_{-l'}(-(x-a)^-+i0,(x-a)^+-i0)\\ &\qquad\qquad -w^*_{l'}(-(x-a)^-+i0,(x-a)^+-i0))\\ &\quad+\mathrm{Nr}\,(\psi(x)\varphi_F(a;l))\psi^*_{l'}(a),\\ &\mathrm{Nr}\,(\psi^*(x)\varphi^F_{l'}(a;l))\\ &=\frac{i}{2\cos\pi l'}\mathrm{Nr}\,(\varphi_F(a;l))\cdot(w_{-l'}(-(x-a)^-+i0,(x-a)^+-i0)\\ &\qquad\qquad -w^*_{l'}(-(x-a)^-+i0,(x-a)^+-i0))\\ &\quad+\mathrm{Nr}\,(\psi^*(x)\varphi_F(a;l))\psi_{l'}(a).\end{aligned}$$

$-\mathrm{Nr}\,(\varphi^{F*}_{l'}(a;l)\psi(x))$ and $-\mathrm{Nr}\,(\varphi^F_{l'}(a;l)\psi(x))$ are given by the right hand sides of (6) with $\mp(x-a)^\mp\pm i0$, $\mathrm{Nr}\,(\psi(x)\varphi_F(a;l))$ and $\mathrm{Nr}\,(\psi^*(x)\varphi_F(a;l))$ replaced by $\mp(x-a)^\mp\mp i0$, $\mathrm{Nr}\,(\varphi_F(a;l)\psi(x))$ and $\mathrm{Nr}\,(\varphi_F(a;l)\psi^*(x))$, respectively.

Consequently we have the following commutation relations in the region where x and a are mutually spacelike.

$$(8)\quad \varphi_F(a;l)\psi(x)=\begin{cases}\psi(x)\varphi_F(a;l) & \text{if } x^+>a^+ \text{ and } x^-<a^-,\\ e^{2\pi il}\psi(x)\varphi_F(a;l) & \text{if } x^+<a^+ \text{ and } x^->a^-.\end{cases}$$

$$(9)\qquad \varphi_F(a\,;\,l)\psi^*(x)=\begin{cases}\psi^*(x)\varphi_F(a\,;\,l) & \text{if } x^+>a^+ \text{ and } x^-<a^-,\\ e^{-2\pi i l}\psi^*(x)\varphi_F(a\,;\,l) & \text{if } x^+<a^+ \text{ and } x^->a^-.\end{cases}$$

If $l'\equiv l+1/2 \pmod{\boldsymbol{Z}}$, we have

$$(10)\qquad \varphi_{l'}^{F*}(a\,;\,l)\psi(x)=\begin{cases}-\psi(x)\varphi_{l'}^{F*}(a\,;\,l) & \text{if } x^+>a^+ \text{ and } x^-<a^-,\\ -e^{2\pi i l}\psi(x)\varphi_{l'}^{F*}(a\,;\,l) & \text{if } x^+<a^+ \text{ and } x^->a^-.\end{cases}$$

If $-l'\equiv l+1/2 \pmod{\boldsymbol{Z}}$, we have

$$(11)\qquad \varphi_{l'}^{F}(a\,;\,l)\psi^*(x)=\begin{cases}-\psi^*(x)\varphi_{l'}^{F}(a\,;\,l) & \text{if } x^+>a^+ \text{ and } x^-<a^-,\\ -e^{-2\pi i l}\psi^*(x)\varphi_{l'}^{F}(a\,;\,l) & \text{if } x^+<a^+ \text{ and } x^->a^-.\end{cases}$$

The asymptotic fields $\phi_\pm(u)$ for $\varphi_{l'}^F(x\,;\,l)$ are defined by

$$(12)\qquad \varepsilon(u)\phi_\pm(u)=\lim_{t\to\pm\infty}\frac{i}{2}\int_{x^0=t}dx^1[e^{im(x^-u+x^+u^{-1})}(\partial/\partial x^0)\varphi_{l'}^F(x\,;\,l)$$
$$-\varphi_{l'}^F(x\,;\,l)(\partial/\partial x^0)e^{im(x^-u+x^+u^{-1})}].$$

We find

$$(13)\qquad \mathrm{Nr}\,(\phi_\pm(u))=(iu)^{l'}\psi(u)\exp\int_0^\infty \underline{du'}$$
$$\times\theta(\pm(|u|-u'))\{(e^{2\pi i l}-1)N^*(u')+(e^{-2\pi i l}-1)N(u')\},$$

$$(14)\qquad \phi_\pm(u)=\begin{cases}(iu)^{l'}\left\{\exp 2\pi i l\int_0^\infty \underline{du'}\theta(\pm(|u|-u'))(N^*(u')-N(u'))\right\}\psi(u) & \text{if } u>0,\\ (iu)^{l'}\psi(u)\left\{\exp 2\pi i l\int_0^\infty \underline{du'}\theta(\pm(|u|-u'))(N^*(u')-N(u'))\right\} & \text{if } u<0.\end{cases}$$

Here we have set $N(u)=\psi^\dagger(u)\psi(u)$, $N^*(u)=\psi^{*\dagger}(u)\psi^*(u)$.

2. Let $a_1,\cdots,a_n$ denote mutually spacelike points in X^{Min} and let $l_1,\cdots,l_n$ denote complex numbers. When we investigate the product $\varphi_F(a_1\,;\,l_1)\cdots\varphi_F(a_n\,;\,l_n)$ it is natural to consider it in the Clifford algebra over the extended orthogonal space parametrized by a symmetric matrix Λ as in § 3 of [4]. Let $\psi^{(\nu)}(u)$, $\psi^{*(\nu)}(u)$, $\psi^{(\nu)}(x)$ and $\psi^{*(\nu)}(x)$ denote ν-th copies of $\psi(u)$, $\psi^*(u)$, $\psi(x)$ and $\psi^*(x)$, respectively. The orthogonal structure $\langle\ ,\ \rangle_\Lambda$ and the expectation value $\langle\ \ \rangle_\Lambda$ are given by

$$(15)\qquad \begin{pmatrix}\langle\psi^{(\mu)}(u),\psi^{(\nu)}(u')\rangle_\Lambda & \langle\psi^{(\mu)}(u),\psi^{*(\nu)}(u')\rangle_\Lambda\\ \langle\psi^{*(\mu)}(u),\psi^{(\nu)}(u')\rangle_\Lambda & \langle\psi^{*(\mu)}(u),\psi^{*(\nu)}(u')\rangle_\Lambda\end{pmatrix}$$
$$=\begin{pmatrix}0 & \lambda_{\mu\nu}2\pi\,|u|\,\delta(u+u')\\ \lambda_{\mu\nu}2\pi\,|u|\,\delta(u+u') & 0\end{pmatrix},$$
$$\begin{pmatrix}\langle\psi^{(\mu)}(u)\psi^{(\nu)}(u')\rangle_\Lambda & \langle\psi^{(\mu)}(u)\psi^{*(\nu)}(u')\rangle_\Lambda\\ \langle\psi^{*(\mu)}(u)\psi^{(\nu)}(u')\rangle_\Lambda & \langle\psi^{*(\mu)}(u)\psi^{*(\nu)}(u')\rangle_\Lambda\end{pmatrix}$$
$$=\begin{pmatrix}0 & \lambda_{\mu\nu}2\pi u_+\delta(u+u')\\ \lambda_{\mu\nu}2\pi u_+\delta(u+u') & 0\end{pmatrix}.$$

We define $\varphi_F^{(\nu)}(x\,;\,l)$, $\varphi_{l'}^{F(\nu)}(x\,;\,l)$ and $\varphi_{l'}^{F*(\nu)}(x\,;\,l)$ by (5) with $\psi(u)$ and $\psi^*(u)$ replaced by $\psi^{(\nu)}(u)$ and $\psi^{*(\nu)}(u)$. We abbreviate $\varphi_F(a_\nu\,;\,l_\nu)$, $\varphi_{l'_\nu}^F(a_\nu\,;\,l_\nu)$ and $\varphi_{l'_\nu}^{F*}(a_\nu\,;\,l_\nu)$ to $\varphi_F^{(\nu)}, \varphi^{F(\nu)}$ and $\varphi^{F*(\nu)}$, respectively. We shall specify l'_ν $(\nu=1,\cdots,n)$ later.

We denote by E_ν the $n\times n$ matrix $(\delta_{\mu\nu}\delta_{\mu'\nu})_{\mu,\mu'=1,\cdots,n}$ and set $M_\nu=1+(e^{-2\pi i l_\nu}-1)E_\nu\Lambda$ and $M_\nu^*=1+(e^{-2\pi i l_\nu}-1)E_\nu\Lambda$. (9) and (15) yield

$$\varphi_F^{(\nu)}\psi^{(\mu)}(x)=\sum_{\kappa=1}^n \psi^{(\kappa)}(x)\varphi_F^{(\nu)}m(x)_{\kappa\mu}, \tag{16}$$
$$\varphi_F^{(\nu)}\psi^{*(\mu)}(x)=\sum_{\kappa=1}^n \psi^{*(\kappa)}(x)\varphi_F^{(\nu)}m^*(x)_{\kappa\mu},$$

where

$$(m(x)_{\kappa\mu})_{\kappa,\mu=1,\cdots,n}=\begin{cases}1 & \text{if } x^+>a_\nu^+ \text{ and } x^-<a_\nu^-,\\ M_\nu & \text{if } x^+<a_\nu^+ \text{ and } x^->a_\nu^-,\end{cases}$$

and

$$(m^*(x)_{\kappa\mu})_{\kappa,\mu=1,\cdots,n}=\begin{cases}1 & \text{if } x^+>a_\nu^+ \text{ and } x^-<a_\nu^-,\\ M_\nu^* & \text{if } x^+<a_\nu^+ \text{ and } x^->a_\nu^-.\end{cases}$$

If $l'_\nu\equiv l_\nu+1/2 \pmod{\boldsymbol{Z}}$, we have

$$\varphi^{F^*(\nu)}\psi^{(\mu)}(x)=-\sum_{\kappa=1}^n \psi^{(\kappa)}(x)\varphi^{F^*(\nu)}m(x)_{\kappa\mu}. \tag{17}$$

If $-l'_\nu\equiv l_\nu+1/2 \pmod{\boldsymbol{Z}}$, we have

$$\varphi^{F(\nu)}\psi^{*(\mu)}(x)=-\sum_{\kappa=1}^n \psi^{*(\kappa)}(x)\varphi^{F(\nu)}m^*(x)_{\kappa\mu}. \tag{18}$$

3. Now we assume that $a_1,\cdots,a_n$ are real points in X^{Euc} such that $a_1<\cdots<a_n$. Fixing a reference point a_0 with positive imaginary part, we denote by γ_ν a closed path from a_0 encircling a_ν clockwise. For a multi-valued real analytic function w defined on $X^{\mathrm{Euc}}-\{a_1,\cdots,a_n\}$, we denote by $\gamma_\nu w$ the analytic continuation along the path γ_ν.

Let $W^{l'_1,\cdots,l'_n}_{a_1,\cdots,a_n}(\Lambda)$ denote the space consisting of $2n$-tuples of multi-valued real analytic functions $w(\Lambda)=(w^{(1)}(\Lambda),\cdots,w^{(n)}(\Lambda))$ with $w^{(\nu)}(\Lambda)={}^t(w_+^{(\nu)}(\Lambda),w_-^{(\nu)}(\Lambda))$ defined on $X^{\mathrm{Euc}}-\{a_1,\cdots,a_n\}$ satisfying the following properties.

i) $(\gamma_\nu w^{(1)}(\Lambda),\cdots,\gamma_\nu w^{(n)}(\Lambda))=(w^{(1)}(\Lambda),\cdots,w^{(n)}(\Lambda))M_\nu$ with $M_\nu=1-(e^{2\pi i l'_\nu}+1)E_\nu\Lambda$.

ii) $(m-\Gamma)w^{(\nu)}(\Lambda)=0$.

iii) $w^{(\mu)}(\Lambda)$ admits the following expansion at each a_ν

$$\begin{aligned}w^{(\mu)}(\Lambda)=&\sum_{j=0}^\infty \lambda_{\mu\nu}c_j^{(\nu)}(w(\Lambda))w_{-l'_\nu+j}[a_\nu]\\ &+\sum_{j=0}^\infty \lambda_{\mu\nu}c_j^{*(\nu)}(w(\Lambda))w^*_{l'_\nu+j}[a_\nu]\\ &+\text{regular function.}\end{aligned} \tag{19}$$

iv) $|w_\pm^{(\mu)}(\Lambda)(z,\bar z)|=O(e^{-2m|z|})$ as $|z|\to\infty$ uniformly in any finite sector.

We set

$$w_{F,\nu}(\Lambda)=(w_{F,\nu}^{(1)}(\Lambda),\cdots,w_{F,\nu}^{(n)}(\Lambda)),\ w_{F,\nu}^{(\mu)}(\Lambda)=\langle\psi^{(\mu)}(x)\varphi_F^{(1)}\cdots\varphi^{F^*(\nu)}\cdots\varphi_F^{(n)}\rangle_\Lambda \tag{20}$$
$$(l'_\nu=l_\nu\pm1/2,\ \nu=1,\cdots,n),$$
$$w_{F,\nu}^*(\Lambda)=(w_{F,\nu}^{*(1)}(\Lambda),\cdots,w_{F,\nu}^{*(n)}(\Lambda)),\ w_{F,\nu}^{*(\mu)}(\Lambda)=\langle\psi^{*(\mu)}(x)\varphi_F^{(1)}\cdots\varphi^{F(\nu)}\cdots\varphi_F^{(n)}\rangle_\Lambda$$
$$(-l'_\nu=l_\nu\pm1/2,\ \nu=1,\cdots,n).$$

The results in § 2 imply that the analytic continuations to X^{Euc} of $w_{F,\nu}(\Lambda)$ and $w_{F,\nu}^*(\Lambda)$ belong to $W^{l'_1,\cdots,l'_n}_{a_1,\cdots,a_n}(\Lambda)$.

Now we define a generalization of τ function by $\tau_{F,\Lambda}(a_1,\cdots,a_n)=\langle\varphi_F^{(1)}\cdots\varphi_F^{(n)}\rangle_\Lambda$.

Set $L=(\delta_{\mu\nu}l_\nu)_{\mu,\nu=1,\cdots,n}$. For each choice (21)—(24) of l'_ν, the coefficient matrices $C_j(w)=(c_j^{(\mu)}(w_\nu))_{\mu,\nu=1,\cdots,n}$ and $C_j^*(w)=(c_j^{*(\mu)}(w_\nu))_{\mu,\nu=1,\cdots,n}$ of the

local expansion (19) are related to $\tau_{F,\Lambda}$ as follows.

(21)
$$l'_\nu = l_\nu + 1/2 \qquad (\nu=1,\cdots,n)$$
$$C_0(w_F(\Lambda)) = \frac{1}{2i\sin\pi L}\tau_{F,\Lambda},$$
$$(C_0(w_F(\Lambda))^{-1}C_1(w_F(\Lambda)))_{\mu\mu} = \frac{\partial}{\partial(-ma_\mu^-)}\log\tau_{F,\Lambda},$$

(22)
$$l'_\nu = l_\nu - 1/2 \qquad (\nu=1,\cdots,n)$$
$$C_0^*(w_F(\Lambda)) = \frac{1}{2i\sin\pi L}\tau_{F,\Lambda},$$
$$(C_0^*(w_F(\Lambda))^{-1}C_1(w_F(\Lambda)))_{\mu\mu} = \frac{\partial}{\partial(ma_\mu^+)}\log\tau_{F,\Lambda},$$

(23)
$$l'_\nu = -l_\nu + 1/2 \qquad (\nu=1,\cdots,n)$$
$$C_0(w_F^*(\Lambda)) = \frac{-1}{2i\sin\pi L}\tau_{F,\Lambda},$$
$$(C_0(w_F^*(\Lambda))^{-1}C_1(w_F^*(\Lambda)))_{\mu\mu} = \frac{\partial}{\partial(-ma_\mu^-)}\log\tau_{F,\Lambda},$$

(24)
$$l'_\nu = -l_\nu - 1/2 \qquad (\nu=1,\cdots,n)$$
$$C_0^*(w_F^*(\Lambda)) = \frac{-1}{2i\sin\pi L}\tau_{F,\Lambda},$$
$$(C_0^*(w_F^*(\Lambda))^{-1}C_1^*(w_F^*(\Lambda)))_{\mu\mu} = \frac{\partial}{\partial(ma_\mu^+)}\log\tau_{F,\Lambda}.$$

4. For a degenerate matrix Λ°, $w_{F,\nu}^{(\mu)}(\Lambda^\circ)$ $(\mu=1,\cdots,n)$ are not linearly independent. Namely, if $p={}^t(p^{(1)},\cdots,p^{(n)})$ belongs to Ker Λ°, we have $\langle\sum_{\mu=1}^n p^{(\mu)}\psi^{(\mu)}(u), \psi^{*(\nu)}(u')\rangle_{\Lambda^\circ}=0$ and $\langle(\sum_{\mu=1}^n p^{(\mu)}\psi^{(\mu)}(u))\psi^{*(\nu)}(u')\rangle_{\Lambda^\circ}=0$ $(\nu=1,\cdots,n)$. Hence $\sum_{\mu=1}^n p^{(\mu)}w_{F,\nu}^{(\mu)}(\Lambda^\circ)$ vanishes identically.

We shall show below that n independent wave functions are recovered by a limiting procedure.

Let $\Lambda'=(\lambda'_{\mu\nu})_{\mu,\nu=1,\cdots,n}$ be an $n\times n$ symmetric matrix such that $\lambda'_{\nu\nu}=0$ $(\nu=1,\cdots,n)$, and set $\Lambda=\Lambda^\circ+\varepsilon\Lambda'$, with a small parameter ε. We choose Λ' so that the quadratic form $\sum_{\mu,\nu=1}^n \lambda'_{\mu\nu}p^{(\mu)}p^{(\nu)}$ is non-degenerate on Ker Λ°. In terms of a basis $p_j={}^t(p_j^{(1)},\cdots,p_j^{(n)})$ $(j=1,\cdots,r;$ $r=\dim\operatorname{Ker}\Lambda^\circ)$ of Ker Λ°, this is equivalent to $\det{}^tP\Lambda'P\neq 0$, where $P=(p_1,\cdots,p_r)$. Without loss of generality we assume the first $r\times r$ block P_1 of P to be non degenerate: ${}^tP=({}^tP_1,{}^tP_3)$, $\det P_1\neq 0$. We denote by $\begin{pmatrix}\Lambda_1^\circ & \Lambda_2^\circ\\ \Lambda_3^\circ & \Lambda_4^\circ\end{pmatrix}$ and $\begin{pmatrix}\Lambda_1' & \Lambda_2'\\ \Lambda_3' & \Lambda_4'\end{pmatrix}$ the corresponding partition into blocks of Λ° and Λ', respectively.

We set

(25)
$$w_{F,\nu}^{(j)}(\Lambda^\circ,\Lambda') = \lim_{\varepsilon\to 0}\frac{1}{\varepsilon}\sum_{\mu=1}^n p_j^{(\mu)}w_{F,\nu}^{(\mu)}(\Lambda) \qquad (j=1,\cdots,r).$$

Then $w_{F,\nu}^{(1)}(\Lambda^\circ,\Lambda'),\cdots,w_{F,\nu}^{(r)}(\Lambda^\circ,\Lambda')$, $w_{F,\nu}^{(r+1)}(\Lambda^\circ),\cdots,w_{F,\nu}^{(n)}(\Lambda^\circ)$ are linearly independent and satisfy the following monodromy properties.

$$(26)\quad (\gamma_\mu w_{F,\nu}^{(1)}(\Lambda^\circ, \Lambda'), \cdots, \gamma_\mu w_{F,\nu}^{(r)}(\Lambda^\circ, \Lambda'), \gamma_\mu w_{F,\nu}^{(r+1)}(\Lambda^\circ), \cdots, \gamma_\mu w_{F,\nu}^{(n)}(\Lambda^\circ)) = (w_{F,\nu}^{(1)}(\Lambda^\circ, \Lambda'), \cdots, w_{F,\nu}^{(n)}(\Lambda^\circ))M_\mu(\Lambda^\circ, \Lambda')$$

where

$$M_\mu(\Lambda^\circ, \Lambda') = 1 + (e^{2\pi i l_\mu} - 1)\begin{pmatrix} 0 & 0 \\ -P_3P_1^{-1} & 1 \end{pmatrix} E_\mu \begin{pmatrix} \Lambda_1'P_1 + \Lambda_2'P_3 & \Lambda_2^0 \\ \Lambda_3'P_1 + \Lambda_4'P_3 & \Lambda_4^0 \end{pmatrix}.$$

Remark. With suitable modifications the results in §§ 2—4 are also valid in our original case where $l_\nu' = 0$ ($\nu = 1, \cdots, n$).

References

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[3] ——: Ibid., 1–5 (1978).
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