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A Five-Valued Model of the *E-p-q*-Theses

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The first eleven-character single axioms for the classical equivalential calculus were discovered by Łukasiewicz, who proves in [3] that each of *EEpqEErqEpr*, *EEpqEEprErq* and *EEpqEErpEqr* will (with substitution and detachment) suffice.

Is there a yet shorter formula which will do the same job? To show that there is not, Łukasiewicz proceeds in [3] to establish that if a shorter single axiom existed there would be one among

EEEppqq	*EEpEpqq*	*EEppEqq*	*EpEEpqq*	*EpEpEqq*
EEEpqpq	*EEpEqpq*	*EEpqEpq*	*EpEEqpq*	*EpEqEpq*
EEEpqqp	*EEpEqqp*	*EEpqEqp*	*EpEEqqp*	*EpEqEqp.*

He then considers one two-valued, three three-valued and two four-valued matrices, showing that each of the fifteen formulas listed is a tautology of one of the six matrices though *EEpqEErqEpr* fails in all of them.

The author has not found in the literature the single matrix

$\mathcal{E} =$

E	1	2	3	4	5
*1	1	2	3	4	5
2	2	1	4	3	1
3	3	4	1	2	1
4	4	3	2	1	1
5	5	1	1	1	1,

whose tautologies include all fifteen formulas, and more. Indeed, the reader can verify that each submatrix of $\mathcal{E}$ generated by two elements validates *EEpqEErqEpr*, whence *all* two-letter *E*-theses are $\mathcal{E}$-tautologies. But $EE43EE53E45 = 2$, whereupon *every* complete set of equivalential axioms must (cf. [1]) include an axiom involving three or more distinct letters and so—since it is known from [2] that letters occur an even number of times each in *E*-theses—an axiom at least eleven characters long.

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REFERENCES

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[3] Łukasiewicz, Jan, "The equivalential calculus,", pp. 88–115 in *Polish Logic, 1920–1939*, ed., Storrs McCall, Clarendon Press, Oxford, 1967.

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