Functoriality and Grammatical Role
in Syllogisms

MARIE LA PALME REYES, JOHN MACNAMARA,
and GONZALO E. REYES

Abstract   We specify two problems in syllogistic: the lack of functoriality of predicates (although a thief is a person, a good thief may not be a good person) and the change of grammatical role of the middle term, from subject to predicate, in some syllogisms. The standard semantics, the class interpretation, by-passes these difficulties but, we argue, in a manner that is at odds with logical intuition. We propose a semantics that is category theoretic to handle these difficulties. With this semantics we specify when syllogisms are valid and we set limits to the class interpretation. To perform this task we show how to construct the categorical notion of an entity in a system of kinds. We devote two brief sections to an argument that our approach is very much in the spirit of Aristotle.

1 Introduction   The aim of this paper is to point out two major difficulties in Aristotle's syllogistic and to suggest a way of handling them; a way which we believe is in agreement with the spirit of Aristotle's overall system.

The first difficulty is a consequence of the sorting of predicables by the nouns to which they are attached. Consider for instance the following syllogism in Barbara:

$$\frac{\begin{array}{l} \text{Every M is L} \\ \text{Every S is M} \end{array}}{\text{Every S is L}}$$

In the first premise "L" stands for a predicable sorted by the noun "M", whereas in the conclusion, "L" stands for a predicable sorted by the noun "S". But according to Aristotle there may be a change in meaning of the predicable as it transfers from one noun to another. This being so, how can we be sure that the meaning of "L" has not undergone change from the first premise to the conclusion, with disastrous results for the validity of the syllogism? We can pose this question without defining the notion of "change of meaning" in this connection. For the moment we will use the expression informally, because the phenomenon to which we draw attention is quite clear.

To see the relevance of the point for syllogisms consider the following:

> Every person in the ward is a baby in the ward
> Every baby in the ward is big
> ─────────────────────────────────
> Every person in the ward is big

The inference fails because even though a baby is a person, a big baby is not a big person.

The importance of this point may not be generally appreciated. Everyone knows that certain adjectives, like "addled", can be applied only to "eggs" and "brains". But they may not appreciate how extensive the effect of sorting is. Compare the words "white" and "black" as applied to "human skin" and to "animal". White animal is exemplified by such creatures as white rabbits and white rats, which are of a quite different shade from white people. A black person is of a very different shade from a black cat. The dullness of a dull day and a dull knife have little to do with one another. Indeed much of what is classed as metaphor is in large part the result of sorting predicables by count nouns; which is not to say that sorting is all there is to metaphor.

The second difficulty relates to changes in grammatical role. Take again the first syllogism of this section. In the first premise "M" is a noun, the subject of the sentence, whereas in the second premise it is a predicable. But nouns and predicables have very different grammatical and semantical roles, as we shall argue in Section 2.6.

For the present we illustrate this problem with the following pseudo syllogism:

> All fire engines are red
> All reds are marxists
> ─────────────────────
> All fire engines are marxists

"Red" in the first premise is an adjective sorted by "fire engine" and as such it picks out a set of fire engines (the red ones). In the second premise, "reds" is the plural form of the CN "red" which refers to the kind *RED* (consisting of persons belonging to a certain political group). It follows that the putative inference, made plausible by an unmarked change in grammatical category associated with the change in grammatical role, is spurious.

The first difficulty has been pointed out and studied by several people, independently from syllogistic, under the heading of 'non-intersecting' or 'non-absolute' modifiers (cf. [6], [12], [13] and [16]). Unfortunately, these studies take for granted the so-called class interpretation, an interpretation that, as we shall argue, is plagued with several difficulties of its own when considered from the point of view of syllogistic.

On the other hand, it is fair to say that Peter Geach [9] and Mary Mulhern [18] are aware of the second difficulty. In fact, in his inaugural address at the University of Leeds Peter Geach [9] refers to the problem

> It is logically impossible for a term to shift about between subject and predicate position without undergoing a change in sense as well as a change of (grammatical) role.

While the claim may be too strong, it is addressed precisely to the problem that concerns us. Unfortunately, neither Geach nor Mulhern proposes any solution.

Our paper tries to provide a solution to these difficulties which, as we said, is in agreement at least with the spirit if not the letter of peripatetic syllogistic. To show agreement "to the letter" would require a philological point by point analysis of the

original texts, a task beyond our capabilities and concerns. We are simply confronted with serious difficulties in the theory of syllogism, difficulties that we have not seen discussed in depth by any of the commentators that we have consulted.

An interesting feature of our paper is that it makes essential use of category theory as will appear, thus establishing a connection between syllogistic and categorical logic.

Our work is divided into four sections and an appendix. The first contains a detailed criticism of the class or set-theoretic interpretation which is usually presented as *the* interpretation of the Aristotle's syllogistic. It also contains a prolegomenon to our interpretation, the semantics of kinds. In this section we introduce (informally) kinds as interpretations of count nouns and predicates as interpretations of predicables. Furthermore, we introduce underlying maps of kinds as special relations between kinds. In this context we formulate the fundamental notion of a predicable being functorial, which is the key to our solution of the first problem. We also indicate the lines of our solution of the second problem (related to change in grammatical role) by showing the connection between the interpretation of a CN (like "dog") and that of the predicable derived from it ("to be a dog"). To do this, we need a notion of entity for a system of kinds, a notion suggested by Bill Lawvere. The aim of this section is to present informally the main ideas that are presented rigorously in the next two sections. The second section is a mathematical formulation of the theory of kinds. The third section presents the interpretations of both "normal" and "deviant" syllogisms and studies their validity. It also specifies the conditions under which the class interpretation in which terms are interpreted as subsets of entities is justified. We end our paper with a section on Aristotle and his modern commentators and on problems related to translating his work into modern English. To help the reader, we have added an appendix with a list of all syllogisms of the first three figures.

2 The class interpretation and alternatives

2.1 The class interpretation   Syllogisms are generally considered unproblematic on the grounds that an adequate semantics for them exists: the class interpretation. The idea of the class interpretation is to assign to the count nouns (CNs) and predicables of a syllogism non-empty subsets of a supposed universal kind, say *ENTITY*. The members of this supposed kind are sometimes spoken of as bare particulars. Thus "baby" is interpreted as the subset of entities that have the property of being a baby, "person" as the subset of entities that have the property of being a person and the relation between babies and persons is just one of inclusion. Similarly "big" is interpreted as the subset of entities that have the property of being big. None of the difficulties pointed out in the introduction arise in this interpretation. To start with, there are no problems relating to the sorting of predicables, since predicables are not sorted or rather, what amounts to the same, are sorted by the unique sort "entity". The problem of switching between a CN and a predicable does not arise in this interpretation either, since both CNs and predicables are interpreted as subsets of a set of entities and hence it makes sense to require equality between these subsets. This class interpretation has been widely popularized with the help of Euler or Venn diagrams and is usually presented as *the* interpretation of Aristotelian syllogistic.

Nevertheless, there are several things wrong with the class interpretation. We first notice that it does not allow the possibility that a predicable may change its meaning due to a change of sorting: a big baby is an entity which has the property of

being a baby and the property of being big as well as the property of being a person. Therefore, the entity in question has the property of being a person and of being big. But as we have seen, there is a change of meaning: a big baby is not a big person. Following this lead we can find patently invalid syllogisms that turn out to be valid under the class interpretation. The following example is adapted from Keenan and Faltz [12]:

> Some persons are presidents
> All presidents are required to be at least 35 years old
> ───
> Some persons are required to be at least 35 years old

The trouble here is precisely that the meaning of the predicable "to be required to be at least 35 years old" changes with the change in sorting from "president" to "person".

Another problem with the class interpretation is that it does violence to grammar, since it does not distinguish between CNs and predicables, interpreting both as subsets of a set of entities. However in every language that has the grammatical category CN (nearly all languages do), a CN is required for the use of certain quantifiers. A predicable without a CN will not do, see Bach [1]. The quantifiers in question include: "every", "there is a", "many", "another", "one", "two", etc. Another way of arguing the same point is to note that we are unable to grasp conceptually and operate with the notion of a universal kind *ENTITY*. If we try to count the entities in a room we do not know whether to count a woman as one or to count her limbs separately or the cells in her body, or the molecules in the cells. And even if we hit upon some strategy about what to count, say the cells, should we count the woman herself as an entity separate from the cells? The point has been made by Geach (cf. [8], p. 63).

A further trouble for the class interpretation, discussed already by medieval logicians, concerns relations among kinds (or interpretations of CNs), a difficulty which we illustrate with a modern example inspired by Gupta [10]. The class interpretation recognizes only one basic relation between kinds: set-theoretical inclusion. For instance, the class interpretation of "baby", namely the set of entities that have the property of being babies is included in the set of entities that have the property of being persons, which is the interpretation of "person". But what about the relation between the interpretations of "passenger" and "person"? Is it just set-theoretical inclusion? A person travelling three times during a year with an airline will be counted as three passengers so that, in general, the number of passengers will be greater than the number of persons who travelled with the airline. This means that the relation between the interpretation of "passenger" and "person" cannot be the set-theoretical inclusion. In the class interpretation it is only the persons who travelled rather than the passengers themselves who constitute the interpretation of "passenger". The same applies to the way we count diners in a restaurant, patients in a hospital and countless other examples.

2.2 Count nouns and kinds   Since CNs are at the heart of our concerns we begin with a remark on their interpretation. Readers wishing for a fuller treatment of the logic of kinds may consult [15] and [19].

The CN "dog" refers to the kind *DOG*. The kind is a couple consisting of the set D of dogs and a function ϵ_D. The set D embraces all the dogs that ever were, are, and ever will be. That it should do so is borne out by the fact that we use the word "dog" in exactly the same sense to speak about dogs long since dead and dogs as yet

unborn as well as dogs that are living at the present time. This means that the word "dog" does not change its reference as old dogs die and young ones are born. We refer to this property of CNs as "modal constancy". We remark that we consider only CNs and we will not consider, in this paper, syllogisms which have in their premises mass nouns (like "water") or abstract nouns (like "beauty" or "justice"). Syllogisms with proper names will also be excluded from our considerations.

The function ϵ_D assigns to each member d of D the set of situations, factual and counterfactual, in which d appears or, as we prefer to say, of which d is a constituent. Situations are naturally pre-ordered under the relation $\leq$ of "having at least as much information as". We refer to this property of sets of situations as their being "downward closed". Since the information that d is a constituent of U is inherited by any situation V having at least as much information as U, it follows that $\epsilon_D(d)$ is a downward closed subset of situations (for the pre-order $\leq$).

Remark 2.1   Although we have spoken about situations, they do not play a central role in our work and they were chosen mainly for reasons of exposition. Indeed, Reyes in [19] defined kinds relative to an arbitrary topos defined over a basic topos, of which the presheaf topos generated by a preorder of "situations" is a particular determination. This determination, however, simplifies computations. For instance, the interpretation of the syllogisms uses only the familiar Kripke forcing (rather than the more general and more formidable Beth-Kripke-Joyal forcing). On the other hand, the more general notion of kind may be useful in handling problems for which sets may be too rigid. We emphasize that the central notion in our semantics is reference and kinds are essential in our theory of reference as Reyes [19] argues. Thus our motivation is different from that of Barwise and Perry [3] whose central concern is the pragmatic relation between character and content (or reference) in the sense of Kaplan [11]. Consequently, situations play a basic role in their semantics.

2.3 Relations among kinds   We have seen that the relation among such kinds as *PASSENGER* and *PERSON* is not set-theoretical inclusion, since a single person may be counted as several passengers. How then can we conceptualize the relation between these kinds, expressed by "is" in the colloquial phrase "a passenger is a person"? Our answer is to posit a map

$$u : PASSENGER \longrightarrow PERSON$$

which associates with a passenger that person that the colloquial expression claims the passenger is. On u we impose the condition that if a passenger p is a constituent of a situation, then $u(p)$ is also a constituent of the same situation. We will say that $u(p)$ is the person underlying p and that u is an underlying map. These maps enable us to express the identity of a baby with the man he later becomes by saying that the person underlying the baby is the same person as that underlying the man. This sidesteps what would otherwise be an impossibility: to say of a baby that he is the same baby as a man or, for that matter, that he is the same man as a certain man. Thus, to interpret "is" as set-theoretical inclusion (the standard approach) is a source of cheap paradoxes.

Kinds and underlying maps are not an arbitrary collection of certain sets and functions, but they constitute a category. Thus given the connection (i.e. the underlying map) between *POODLE* and *DOG* and that between *DOG* and *ANIMAL*, there

is an obvious connection between *POODLE* and *ANIMAL*: the composition of the corresponding underlying maps. Thus, kinds and their underlying maps constitute a category $\mathcal{K}$ in a natural way. (Notice that the identity map is underlying).

We shall see, by means of an example, that CNs themselves (in a given "universe of discourse") constitute a preordered set and thus a category, $\mathcal{N}$, the *nominal category*. Its role is to act as a "blueprint" to organize the CNs in such a way as to indicate the connection between the kinds interpreting these CNs.

Assume that, like Aristotle, we are organizing and developping a given subject, say Zoology. In Zoology we have CNs ("mammal", "whale", "fish", "animal", etc.) and predicables ("having a heart", "breathing air", "being a mammal") which combine with CNs to make significant sentences. Furthermore, we have relations between CNs described by sentences of the type "a dog is an animal", "a whale is a mammal", which we assume as postulates. These postulates impose constraints on the interpretation of these CNs, constraints which are partly conventional and partly empirical and are subject to revision. Recall that for a long time people accepted that a whale was a fish.

The CNs and their relations constitute a category $\mathcal{N}$ under the following definition of objects and morphisms: an *object* of $\mathcal{N}$ is a CN $\underline{A}$. A *morphism* $\underline{u} : \underline{A} \longrightarrow \underline{B}$ is the postulate "an $\underline{A}$ is a $\underline{B}$".

To check that $\mathcal{N}$ is indeed a category, we need our postulates to constitute a *deductive system* in the sense that

1. The set of postulates contains all sentences of the type "a dog is a dog" (identity morphisms).

2. The set of postulates is closed under Cut (or Modus Ponens). Thus if both "a whale is a mammal" and "a mammal is an animal" are postulates, then so is "a whale is an animal" (composition of composable morphisms).

Needless to say, any set of postulates generates a deductive system in the obvious way.

As we mentioned, CNs and their relations may be interpreted as kinds and connections between kinds. Such an interpretation turns out to be precisely a functor between the nominal category $\mathcal{N}$ and the category of kinds $\mathcal{K}$, the *interpretation functor*.

Remark 2.2   It was Bill Lawvere who suggested to consider the nominal category $\mathcal{N}$. To appreciate the need for this category let us look at the following syllogism in Disamis:

> Some women are odd
> All women are even natural numbers
> ————————————————
> Some even natural numbers are odd

Even a pythagorean who accepts the premises would certainly reject the conclusion and we can ask what went wrong. The trouble is that the interpretation of the predicable "odd" sorted by "woman" is quite unrelated to that of "odd" sorted by "natural number". In fact, they cannot be compared at all. So even if an interpretation happens to assign "woman" and "even natural number" to the same kind, we would still lack linguistic/conceptual tools to compare the corresponding interpretations of the predicable in question. This missing link is provided precisely by $\mathcal{N}$. If two CNs $\underline{A}$

and $\underline{B}$ are connected by a morphism of $\mathcal{N}$, i.e. by a postulate such as "An $\underline{A}$ is a $\underline{B}$", then the interpretation of this postulate provides us at the level of kinds with a morphism that interprets the connection between the two CNs, and hence with means to compare the corresponding interpretations of the predicable. This possibility of comparing the interpretations of CNs is at the basis of the notion of functoriality of predicates required for the validity of syllogisms.

A further question arises: accepting passengers as distinct from persons, how can we avoid what Gupta [10] calls a "bloated ontology" in which besides persons there are passengers and wives and teachers and so on crowded into airplanes? Our answer is that for certain purposes such as the number of seats required, what matters is the number of entities underlying passengers, persons, men, wives and so on. One entity, one seat.

This presupposes the construction of a notion of entity for the relevant system of kinds; a notion that will consider, say, a certain wife, a certain passenger and a certain person as just one entity. This entity will occupy one seat. From the standpoint of ticket prices, however, an agent needs to know whether an entity is a child or an adult. This brings out the important point that airline personnel must for some purposes hold onto the individual kinds that constitute the system while collapsing across those kinds for other purposes. This sets the entities that our theory recognizes in sharp contrast to the unstructured notion of entity envisaged by the class interpretation. Sometimes when one has collapsed across the kinds of the system one finds a convenient natural language expression, such as "person", to cover the resulting entities, but there are more complicated cases when there is no readily available word. We give a construction of a kind *ENTITY* relative to a system of kinds in Section 3.3.

Remark 2.3 It will appear that the construction of the entity, which is the basic construction to solve the problem of grammatical change, is a particular case of the colimit of a functor: the interpretation functor. Functors will also appear in the interpretation of predicables, giving rise to a natural generalization of the notion of "intersective" or "absolute" predicable, as we will see in Section 3.4. These examples further illustrate the essential use of category theory in our paper.

2.4 Predicables and predicates We will use "predicable" for that grammatical category of expressions consisting of adjectives, verbs or adjectival and verb phrases. Predicables include expressions such as "to be a person" which are derived from CNs. We interpret predicables as certain families of predicates, where a predicate (of a given kind) is a map from the kind into the set of downward closed sets of situations. For example the predicate *SICK* of the kind *BOY* is the map which associates with a given boy b the set of all situations in which b is sick. (This set is clearly downward closed). We think that each predicable has a domain of application, much in line with the intuition of Sommers (cf. [21], p. 297). For instance "sick" may be applied to persons, dogs, plants and so on, but not to electrons or natural numbers; similarly "red" may not be applied to atoms or ideas but to books, balls and even daffodils. This is the reason for interpreting a predicable as a family of predicates, a family indexed by the system of kinds that belong to its domain of application. On the other hand, we insist that the domain should be a system rather than a class of kinds, which is how our approach differs from that of Sommers. This is essential to define the notion of functoriality of an interpreted predicable in the next section.

We are particularly interested in predicates that have the following property: if the predicate holds of a member b of the kind at the situation U, then b appears at U. We call these ϵ-*predicates*. Most predicates are ϵ-predicates. Take $SICK$ once again: if a boy is sick at U, then he must appear at U. A situation, then, is required to decide whether the predicate SICK applies to the boy. On the other hand, the predicate *TO BE IDENTICAL TO ITSELF* of the kind *PERSON* may be predicated of Socrates at any situation, regardless whether he appears in the situation or not. In other words, a situation is not needed to ground the truth of "Socrates is identical to himself". Therefore, this is an example of a predicate which is not an ϵ-predicate. We will only consider ϵ-predicates in this paper.

2.5 *Functoriality*   Some predicables "behave well" semantically when transferred from one kind to another which underlies it. For instance, 'male' does so when we go from *BABY* to *PERSON* in the precise sense that, at a given situation in which a baby b appears, b is a male precisely when the person underlying the baby is a male. We express this property of the (interpreted) predicable by saying that "male" is *functorial* relative to the system of kinds consisting of *BABY*, *PERSON* and the underlying map *BABY* $\longrightarrow$ *PERSON*.

The predicable "to appear" or "to be a constituent" is functorial relative to any system, by the very definition of morphism in the category of kinds. On the other hand, the predicable "big" does not behave well under this transfer, as we saw already. We say of "big" in this connection that it *is not functorial* or that *it fails to be functorial*. A further example of an interpreted predicable that fails to be functorial is "good". In fact, it was pointed out in antiquity that while every thief is a person, a good thief is not usually a good person. Another predicable that fails to be functorial is "required to be at least 35 years old". Even if every president is required to be at least 35 years old and some persons are presidents, no person is required to be at least 35 years old.

Using a term borrowed from physics, we may express functoriality of a predicable relative to a system by saying that the predicable "keeps phase" when we go from one kind to another along the underlying maps of the system. Similarly, failure of functoriality may be expressed by saying that the predicable "does not keep phase" along the underlying maps. The notion of "functoriality" can be generalized to any predicable and any system of kinds.

Our solution to the first problem is to require for validity of syllogisms that the interpreted predicables involved should be functorial (relative to their domains). Notice that this notion is relative and depends essentially on the system of kinds considered and hence on the chosen nominal category. This nominal category, we emphasize, depends on our concerns and is far from being unique. Had we chosen a subcategory of the given nominal category, we would have (in general) increased the number of functorial interpreted predicables for the new system of kinds. As a simple example, consider the nominal category consisting of the CNs "person", "rat" and "animal" and the obvious maps between them. Under the natural interpretation, the interpretation of "white" is not functorial for this system, but it becomes functorial when the nominal category is cut down to the subcategory whose CNs are "rat" and "animal". On the other hand, those predicables whose interpretations were functorial ("male", for instance) with respect to the original system, have the same property when restricted to a new system of kinds defined by any subcategory. In passing we remark that whether or not an interpreted predicable is functorial relative to a system

of kinds strikes us, from the practical point of view, as similar to the frame problem familiar to workers in the field of artificial intelligence. Its solution in connection with syllogisms and other forms of inference would seem to depend on a type of detailed knowledge that is difficult to program in a computer in sufficient abundance without running into serious problems of retrieval and control.

In Section 3.4 we prove a theorem that for a predicable interpreted as an ϵ-predicate of a system of kinds, to be functorial is equivalent to being extensional. The second notion is easier to work with, although the former seems more intuitive. For this reason we make considerable use of the notion of extensionality in the next section, a notion that here we present informally. First we need the notion of *coincidence* between two kinds A and B of a system of kinds. We say, disregarding mathematical precision for the moment, that a in A and b in B are coincident at a situation U if a and b correspond to a single entity for the system. We say that a predicable γ whose domain of application includes both A and B is *extensional* if at a situation U in which a coincides with b , U being in $\gamma(a)$ implies that U is in $\gamma(b)$. More concretely, *SICK* as applied to *PASSENGER* and *PERSON* is extensional in that a sick passenger is necessarily a sick person. The reader will see immediately how closely this notion of extensionality is related to the notion of a predicate being functorial. Although the notion of functoriality is a guiding thread in this work, it turns out that the notion of extensionality is technically easier to handle, even though less intuitive. However that these notions coincide is shown in Section 3.4. We remark that the syllogism from Section 2.1 is not valid since the predicable "required to be at least 35 years old" is not extensional. All predicables coming from CNs are extensional relative to any system (see Section 4).

2.6 Change in grammatical role   In all syllogisms, one of the terms changes from subject position into predicate position or vice-versa. (When we say "all syllogisms" we restrict ourselves to categorical or assertorical syllogisms of the first three figures, as opposed to modal or hypothetical ones.) In the first figure it is the middle term that so changes grammatical role; in the second it is the major term; in the third it is the minor term. The trouble is that the interpretation of a CN is a kind, whereas that of a predicable sorted by a CN is a predicate of a kind and it makes no sense to say that a kind is a predicate. But clearly, there must be some connection between the two. How can we describe this connection in such a way as to distinguish between permissible changes of grammatical role and changes that may lead to fallacies? Remember the pseudo-syllogism from Section 1 (the last syllogism of that section).

There is often some relation, perhaps metaphorical, between the meanings of adjectives and their derived nominals and between the meanings of verbs (e.g. "to travel") and their derived nominals ("travel", "travelling"), but it is obviously unable to support a semantics based on reference as our semantics of kinds.

Our answer to the question of the connection between the interpretation of the CN and that of the predicable derived from it is roughly the following: the interpretation of the predicable is a family of predicates of kinds such that each predicate of the family (of a given kind) associates with a member of that kind the set of situations in which that member coincides with a member of the kind which interprets the CN.

At this moment, we have all the elements required to formulate our semantics mathematically and discuss conditions of validity of syllogisms, when "changes of phase" and changes of grammatical role are taken into account.

3 Mathematical formulation of the theory of kinds

3.1 Situations Let $\mathcal{P} = <P, \leq>$ be a non-empty pre-ordered set, namely, $\leq$ satisfies $U \leq U$ and if $W \leq V$ and $V \leq U$ then $W \leq U$. We think of P as a set of "possible situations", and of $\leq$ as the relation of "having more information than", namely, $V \leq U$ whenever V has more information than U.

A set D is *downward closed* if whenever $U \in D$ and $V \leq U$ then $V \in D$. We define

$$\Gamma(\Omega) = \{D \subseteq P : D \text{ is downward closed}\}$$

(Both Γ and Ω and hence $\Gamma(\Omega)$ have definite meanings in topos theory. In this particular case however, $\Gamma(\Omega)$ turns out to be the set of downward closed subsets of P.) Clearly $\Gamma(\Omega)$ is a distributive lattice with respect to the set-theoretical operations of union ($\cup$) and intersection ($\cap$) and has a smallest ($\varnothing$) as well as a largest (P) element. In fact, more is true of this lattice.

Proposition 3.1 $(\Gamma(\Omega), \cup, \cap, \varnothing, P)$ *is a Heyting algebra.*

Proof: It is clear that $\Gamma(\Omega)$ is closed under arbitrary unions and intersections. We may define the implication ($\Rightarrow$) as follows:

$$D \Rightarrow D' = \cup\{D'' \in \Gamma(\Omega) : D \cap D'' \subseteq D'\}.$$

It is easy to check that

$$D'' \subseteq D \Rightarrow D' \text{ iff } D \cap D'' \subseteq D'.$$

This proves that we really have defined the Heyting implication.

In what follows, we will abbreviate $(\Gamma(\Omega), \cap, \cup, \varnothing, P)$ to $\Gamma(\Omega)$.

3.2 Kinds and predicates In terms of $\mathcal{P}$ we define a *kind* as a couple (A, ϵ_A) where A is a set and $\epsilon_A : A \longrightarrow \Gamma(\Omega)$ a map. We think of kinds as interpretations of count nouns (CNs) such as "person", "dog", "animal". The interpretation of "dog", for instance, is the set *DOG* of all dogs that ever were, are or will be together with the map $\epsilon_{DOG} : DOG \longrightarrow \Gamma(\Omega)$. We think of ϵ_{DOG} as the map that associates with a particular dog, say Freddie, the set of situations of which Freddie is a constituent. Notice that if $U \in \epsilon_{DOG}(Freddie)$ and $V \leq U$, then the information that Freddie is a constituent is preserved from U to V, namely, $V \in \epsilon_{DOG}(Freddie)$ and so $\epsilon_{DOG}(Freddie) \in \Gamma(\Omega)$. Notice also that "Freddie" is a proper name and is interpreted as a member of the kind *DOG*.

To simplify the notation we will often replace the couple (A, ϵ_A) by A.

Kinds constitute a category $\mathcal{K}$ under the following definition of morphism: a *morphism* $f : (A, \epsilon_A) \longrightarrow (B, \epsilon_B)$ is a function $f : A \longrightarrow B$ such that $\epsilon_A(a) \subseteq \epsilon_B(f(a))$ for all $a \in A$. In fact, the category of kinds constitutes a category of fuzzy sets but we will not go into that subject. Nevertheless, we remark that our interpretation is not one of fuzzy membership but of degree of constituency or more intuitively of extent of existence.

We define a *predicate* of a kind A to be a map $\varphi : A \longrightarrow \Gamma(\Omega)$. We think of predicates of A as the interpretation of predicables (adjectives, VP, etc.) such as

"white", "mortal", "run" and "find something" sorted by the CN whose interpretation is A. For instance, the predicate RUN of the kind $PERSON$ is the map

$$RUN : PERSON \longrightarrow \Gamma(\Omega)$$

which associates with a person, say John, the set of situations in which John runs. Once again it is easy to see that $RUN(JOHN)$ is a downward closed set, i.e., is a member of $\Gamma(\Omega)$.

We define an ϵ-*predicate* of a kind (A, ϵ_A) as a predicate φ of the kind (A, ϵ_A) such that $\varphi(a) \subseteq \epsilon_A(a)$ for all $a \in A$. We will come back to the notion of ϵ-predicate in Section 3.4.

ϵ-predicates constitute a category $Pred_\epsilon(K)$ with the following definition of objects and morphisms: an *object* is a couple (A, φ) where A is a kind (we will make the usual abuse of language and write "A" for "(A, ϵ_A)") and

$$\varphi : A \longrightarrow \Gamma(\Omega)$$

is an ϵ-predicate of A , i.e., such that $\varphi(a) \subseteq \epsilon_A(a)$ for all $a \in A$. A *morphism*

$$f : (A, \varphi) \longrightarrow (B, \psi)$$

is a map $f : A \longrightarrow B$ in $\mathcal{K}$ such that

$$\varphi(a) = \epsilon_A(a) \cap \psi(f(a)).$$

We have an obvious forgetful functor

$$\mathcal{F} : Pred_\epsilon(K) \longrightarrow K$$

sending the object (A, φ) into A and f into itself. (We remark that predicates also constitute a category, but we shall not use it here.)

3.3 Entities for a system of kinds   In this section we define a notion of entity for a system of kinds. We recall that a system of kinds is a functor

$$I : \mathcal{L} \longrightarrow \mathcal{K}$$

where $\mathcal{L}$ is a small category and $\mathcal{K}$ is the category of kinds. We define the entity for the system to be the colimit of the functor I .

We now give an explicit description of the colimit. Consider the set

$$E_0 = \{(a, i) : a \in I(i)\}$$

we define an equivalence relation on E_0 as follows:

$(a, i) \sim (b, j)$ iff there is a chain

in L and elements $a_k \in I(i_k)$ where $(0 \leq k \leq n)$ such that $a = a_0$, $b = a_n$, and $I(i_k \to i_{k-1})(a_k) = a_{k-1}$, $I(i_k \to i_{k+1})(a_k) = a_{k+1}$.

This is the smallest equivalence relation containing $((a, i), (b, j))$ whenever $\exists \alpha : i \longrightarrow j$ such that $I(\alpha)(a) = b$.

Now let $E = E_0/\sim$, the set of equivalent classes

$$\{[(a, i)] : a \in I(i)\}$$

We define the relation ϵ as follows:

$$\epsilon_E([(a, i)]) = \bigcup\{\epsilon_{I(j)}(b) : (a, i) \sim (b, j)\}.$$

It is easy to check that (E, ϵ_E) is the colimit of I.

Notice that we have a canonical map

$$\eta_i : I(i) \longrightarrow E$$

given by $\eta_i(a) = [(a, i)]$

We are now in the position to define a *coincidence relation*

$$\delta_I : E_0 \times E_0 \longrightarrow \Gamma(\Omega)$$

$U \in \delta_I((a, i), (b, j))$ iff there is a chain as before (see Section 3.3) in L and elements $a_k \in I(i_k)$ where $(0 \leq k \leq n)$ such that $a = a_0, b = a_n$, and $I(i_k \to i_{k-1})(a_k) = a_{k-1}$, $I(i_k \to i_{k+1})(a_k) = a_{k+1}$, and $U \in \epsilon_{I(i_k)}(a_k)$ for $(0 \leq k \leq n)$.

Example: let Joe be a man, l be a liar. So $U \in \delta_I((Joe, man), (l, liar))$ precisely when at U, Joe is l. Other examples will be given later. The following result is easily checked:

Proposition 3.2   *The coincidence relation δ_I has the following properties*
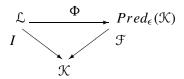
1. $\delta_I((a, i), (b, j)) = \delta_I((b, j), (a, i))$
2. $\delta_I((a, i), (b, j)) \cap \delta_I((b, j), (c, k)) \subseteq \delta_I((a, i), (c, k))$
3. $\delta_I((a, i), (b, j)) \subseteq \epsilon_{I(i)}(a) \cap \epsilon_{I(j)}(b)$.

3.4 Predicates of a system of kinds   We define an ϵ-predicate of a system $I :$ $\mathcal{L} \longrightarrow \mathcal{K}$ of kinds to be a family $(\varphi_i)_{i \in |\mathcal{L}|}$ such that each φ_i is an ϵ-predicate of the kind $I(i)$.

We say that an ϵ-predicate $(\varphi_i)_i$ of I is *functorial* iff there is a functor

$$\Phi : \mathcal{L} \longrightarrow Pred_\epsilon(\mathcal{K})$$

such that the diagram

commutes, and for each $i \in |\mathcal{L}|$, $\Phi(i) = \varphi_i$. $Pred_\epsilon(\mathcal{K})$ is the category of ϵ-predicates and F the forgetful functor. Equivalently, $(\varphi_i)_i$ is natural if for every $\alpha : i \to j \in \mathcal{L}$, $\varphi_i = \epsilon_{I(i)} \cap (\varphi_j \circ I(\alpha))$, i.e., the diagram

commutes "up to existence".

  We say that an ϵ-predicate $(\varphi_i)_i$ for I is *extensional* iff

$$\varphi_i(a) \cap \delta_I((a, i), (b, j)) \subseteq \varphi_j(b)$$

for all $i, j \in |\mathcal{L}|$, $a \in I(i)$, $b \in I(j)$.

Proposition 3.3   *An ϵ-predicate for a system I of kinds is functorial iff it is extensional.*

Proof: ($\Longleftarrow$) Let $\alpha : i \to j \in ;$. Given $a \in I(i)$, we have to check that

$$\varphi_i(a) = \epsilon_{I(i)}(a) \cap \varphi_j(I(\alpha)(a)).$$

Let $b = I(\alpha)(a)$. If $U \in \varphi_i(a)$, then $U \in \epsilon_{I(i)}(a)$ ($(\varphi_i)_i$ is a ϵ-predicate). Since $I(\alpha)$ is a morphism of $\mathcal{K}$, $U \in \epsilon_{I(j)}(b)$ and this shows that $U \in \delta_I((a, i), (b, j))$. By extensionality of $(\varphi_i)_i$ we conclude that $U \in \varphi_j(b)$. A similar argument shows that

$$\epsilon_{I(i)}(a) \cap \varphi_j(I(\alpha)(a)) \subseteq \varphi_i(a).$$

($\Longrightarrow$) Let $(\varphi_i)_i$ be a functorial ϵ-predicate. We have to show that

$$\varphi_i(a) \cap \delta_I((a, i), (b, j)) \subseteq \varphi_j(b).$$

Let $U \in \varphi_i(a)$, $U \in \delta_I((a, i), (b, j))$. We proceed by induction on the length of the chain defining δ_I (see Section 3.3). Let us do just the case n=1 to see what is involved: there is a diagram

$$
\begin{array}{ccc}
 & i_1 & \\
 \swarrow & & \searrow \\
i & & j
\end{array}
$$

in $\mathcal{L}$ and an element $a_1 \in I(i_1)$ which is sent into $a \in I(i)$ by $I(i_1 \to i)$ and into $b \in I(j)$ by $I(i_1 \to j)$ and such that

$$U \in \epsilon_{I(i)}(a) \cap \epsilon_{I(i_1)}(a_1) \cap \epsilon_{I(j)}(b).$$

By functoriality of $(\varphi_i)_i$,

$$\varphi_{i_1}(a_1) = \epsilon_{I(i)}(a_1) \cap \varphi_i(a)$$

and this implies that $U \in \varphi_{i_1}(a_1)$. Using functoriality once again

$$\varphi_{i_1}(a_1) = \epsilon_{I(j)}(b) \cap \varphi_j(b)$$

and this implies that $U \in \varphi_j(b)$.

Remark 3.4   Following a suggestion of the referee, we shall compare predicables whose interpretations are functorial with those studied by Keenan and Faltz [12] in the context of the class interpretation under the heading of "intersecting" or "absolute". We will see that the second notion is a particular case of the first. If we leave situations aside and assume that the category of kinds constitutes a Boolean algebra B of subsets of a given set U (thought of as a universal kind of things or objects), as is usually done in the class interpretation, then all underlying maps are simply set-theoretical inclusions. If furthermore we identify this Boolean algebra with the nominal category,

viewing each kind as a name of itself, the interpretation f (in our sense) of a predicable may be identified with a family of couples of sets $(X, f(X))$ belonging to B of which the second is a subset of the first. Each couple may be thought of as a subkind of the universal kind together with a predicate of that kind. Under these conditions, f is functorial iff for every X, $f(X) = X \wedge f(U)$ as may be easily checked. But this last condition is precisely the notion of f being intersecting in the sense of Keenan and Faltz.

4 Interpretation of syllogisms

4.1 Interpretation of CNs and predicables   We must now specify the notion of validity for a syllogism. This notion is a particular case of the notion of validity in a topos (the Beth-Kripke-Joyal semantics) which is the natural generalization of the corresponding notion in sets (Tarski semantics). We formulate validity in terms of the forcing relation $\Vdash$ (which corresponds to the satisfaction relation of Tarski semantics). Instead of a complete definition we shall give only those clauses which are relevant to syllogisms.

There are two notions of validity depending on the way we consider syllogisms, either as axioms following Łukasiewicz [17] or as rules of inference following Corcoran [5]. The first takes a syllogism to be valid if whenever the premises are forced at a particular situation U then the conclusion is forced at U. The second takes a syllogism to be valid if whenever the premises are forced at every situation U so is the conclusion. Of course validity as axioms implies validity as rules of inference. So we will prove the validity of syllogisms as axioms.

We start with the interpretation of CNs or rather of the nominal category $\mathcal{N}$ introduced in Section 2.3.

An *interpretation* of $\mathcal{N}$ is a functor

$$\|...\| \; : \; \mathcal{N} \longrightarrow \mathcal{K} \;\; .$$

Thus $\|...\|$ is a family of kinds indexed by CNs and respecting the postulates in the sense that we have a morphism

$$\|\underline{A}\| \longrightarrow \|\underline{B}\|$$

in $\mathcal{K}$, whenever "an $\underline{A}$ is a $\underline{B}$" is a postulate.

At this point we can apply the theory developed in Section 3.3 and obtain the kind ENTITY for the system given by the interpretation as the colimit of $\|...\|$ and the coincidence relation $\delta_{\|...\|}$. To simplify the notation we shall write "$\delta(a, b)$" rather than "$\delta_{\|...\|}((a, A), (b, B))$" whenever A and B are clear from the context.

An *interpretation* of a predicable φ relative to an interpretation $\|...\|$ of $\mathcal{N}$ is a function $\|\varphi\|$ which associates with a CN $\underline{A}$ a couple $(\|\underline{A}\|, \varphi_{\underline{A}})$ where $\varphi_{\underline{A}}$ is an ϵ-predicate of $\|\underline{A}\|$, i.e.,

$$\varphi_{\underline{A}} : \|\underline{A}\| \longrightarrow \Gamma(\Omega) \; .$$

Thus $\|\varphi\|(\underline{A})$ is an object of $Pred_\epsilon(\mathcal{K})$.

Some predicables derive from CNs. For instance, "to be a dog" derives from "dog" and we need to specify the semantic connection between the interpretation of the CN and that of the predicable.

Let $\delta_{\|...\|}$ be the coincidence relation defined in Section 3.3. If $\underline{A}$ is a CN in $\mathcal{N}$, we interpret the predicable "to be an $\underline{A}$" as the predicate $(A_{\underline{B}})_{\underline{B} \in |\mathcal{N}|}$ defined by the formula

$$A_{\underline{B}}(b) = \bigcup \{\delta_{\|...\|}((a, \underline{A}), (b, \underline{B})) : a \in \|\underline{A}\|\} \,.$$

Using the properties of $\delta_{\|...\|}$ (see Section 3.3) we can easily check that

Proposition 4.1   *The interpretation $(A_{\underline{B}})_{\underline{B} \in |\mathcal{N}|}$ of the predicable "to be an $\underline{A}$" derived from the CN $\underline{A}$ is an extensional ϵ-predicate.*

Remark 4.2   Notice that for predicables derived from CNs, the order of predication makes no difference at the level of interpretation. Thus a quadruped mammal is the same as a mammalian quadruped. This is not so for other predicables as we saw in Section 2.

4.2 Normal syllogisms   We shall first interpret syllogisms in which only CNs occur in subject position. Although this restriction seems quite "normal", Aristotle also considered syllogisms in which genuine predicables occur in subject position. The interpretation of these "deviant" syllogisms will be given in Section 4.3.

Let us now interpret "All $\underline{A}$ are φ". Letting $(A, \epsilon_A) = \|\underline{A}\|$, we define $\Vdash$ as follows:

$U \Vdash$ All $\underline{A}$ are φ iff $\forall V \leq U \ \forall a \in A \ (V \in \epsilon_A(a) \Rightarrow V \in \varphi_{\underline{A}}(a))$.

We read "$U \Vdash$" as "U forces". We remind the reader that $\epsilon_A(a)$ is the set of situations both factual and counterfactual of which a is a constituent.

Similarly

$U \Vdash$ some $\underline{A}$ are φ iff $\exists a \in A \ (U \in \epsilon_A(a) \wedge U \in \varphi_{\underline{A}}(a))$

$U \Vdash$ all $\underline{A}$ are not φ iff $\forall V \leq U \ \forall a \in A \ (V \in \epsilon_A(a) \Rightarrow \forall W \leq V \ W \notin \varphi_{\underline{A}}(a))$

$U \Vdash$ some $\underline{A}$ are not φ iff $\exists a \in A \ (U \in \epsilon_A(a) \wedge \forall V \leq U \ V \notin \varphi_{\underline{A}}(a))$

$U \Vdash$ no $\underline{A}$ are not φ iff $\forall V \leq U \ \forall a \in A \ (V \in \epsilon_A(a) \Rightarrow \forall W \leq V \ \exists W' \leq W \ W' \in \varphi_{\underline{A}}(a))$.

The sentence "no $\underline{A}$ are φ" is interpreted exactly as the sentence "all $\underline{A}$ are not φ". We show that the Aristotelian syllogisms are valid constructively, namely, in what is normally regarded as a non-Aristotelian logic!

Let us look at the following syllogism in Barbara:

$$\frac{\begin{array}{c} \text{All } \underline{A} \text{ are } \underline{\beta} \\ \text{All } \underline{B} \text{ are } \underline{\psi} \end{array}}{\text{All } \underline{A} \text{ are } \underline{\psi}}$$

where β is the predicable "is a $\underline{B}$". To check validity we need an interpretation $\|...\|$ of $\mathcal{N}$, together with an interpretation $\|\psi\|$ of the predicable ψ (relative to $\|...\|$). Let $(A, \epsilon_A) = \|\underline{A}\|$ and $(\psi_{\underline{A}})_{\underline{A}} = \|\underline{\psi}\|$. Assume now that U is such that

1. $U \Vdash$ All $\underline{A}$ are $\underline{\beta}$
2. $U \Vdash$ All $\underline{B}$ are $\underline{\psi}$

Take $a \in A$ such that $U \in \epsilon_A(a)$. By 1, $U \in \beta_{\underline{A}}(a)$ and hence there is a $b \in B$ such that $U \in \delta(a, b)$. Since $U \in \epsilon_B(b)$, we obtain, by 2, $U \in \psi_{\underline{B}}$. Here we get stuck since we have specified no a priori connections between $\psi_{\underline{B}}$ and $\psi_{\underline{A}}$.

This is precisely the first problem discussed in the introduction. Should we conclude that syllogisms in Barbara are not valid and that, for instance, from

All Greeks are men
All men are mortal

we cannot conclude

All Greeks are mortal

because there are no connections between "mortal" applied to "men" and "mortal" applied to "Greek"? But then, how explain the success of syllogisms? We believe that their success depends on the choice of predicables, or rather the interpretations of predicables.

We say that the interpretation $\|\underline{\varphi}\|$ of a predicable $\underline{\varphi}$ (relative to $\|...\|$) is *functorial* if there is a functor Φ making commutative the diagram

$$L \xrightarrow{\quad\Phi\quad} Pred_\epsilon(K)$$
$$\|...\| \searrow \qquad \swarrow F$$
$$K$$

and such that $\Phi(\underline{A}) = (\|\underline{A}, \varphi_{\underline{A}})$, $\Phi(\underline{u} : \underline{A} \longrightarrow \underline{B}) = \|\underline{u}\|$. In other words, $\|\underline{\varphi}\|$ is functorial if the family $(\varphi_{\underline{A}})_{\underline{A}}$ may be "extended" to a functor Φ .

It follows from Section 3.4 that $\|\underline{\varphi}\|$ is functorial iff $\|\underline{\varphi}\|$ is extensional and we shall use this last characterization, since it is easier to apply.

Return to the syllogism in Barbara and assume that $\|\underline{\psi}\|$ is extensional (or, equivalently, functorial). We can now complete the proof of its validity as follows: by extensionality of $\|\underline{\psi}\|$,

$$\psi_{\underline{B}}(b) \cap \delta(a, b) \subseteq \psi_{\underline{A}}(a)$$

and therefore U , which belongs to the LHS (left hand side), belongs also to the RHS (right hand side), i.e., $U \in \psi_{\underline{A}}(a)$. We have thus shown

3. $U \Vdash$ All $\underline{A}$ are $\underline{\psi}$.

The previous interpretation assumed that only CNs appear in subject position, whereas genuine predicables or predicables derived from CNs may appear in predicate position.

Under this natural assumption, we may, proceeding as before and recalling from the proposition of Section 4.1 that the interpretation of predicables derived from CNs are extensional, prove the following

Theorem 4.3   *If the interpretation of all genuine predicables in a syllogism not in D/F (Darapti and Felapton) are extensional, then the syllogism is valid.*

Proof: We give just a couple of examples, beginning with the syllogism in Baroco, a syllogism of the second figure:

All $\underline{A}$ are $\underline{\varphi}$
Some $\underline{C}$ are not $\underline{\varphi}$
Some $\underline{C}$ are not $\underline{\alpha}$

where $\underline{\alpha}$ is the predicable "is an $\underline{A}$". Let U be a situation such that

1. $U \Vdash$ All $\underline{A}$ are $\underline{\varphi}$
2. $U \Vdash$ Some $\underline{C}$ are not $\underline{\varphi}$

We show that $U \Vdash$ Some $\underline{C}$ are not $\underline{\alpha}$. So we must show that

$$\exists c \in C \ (U \in \epsilon_C(c) \land \forall V \leq U \ V \notin \alpha_C(c)).$$

By 2, for some $c \in C$ we have that $U \in \epsilon_C(c)$ and $\forall V \leq U \ V \notin \varphi_C(c)$. Take $V \leq U$, we claim that $V \notin \alpha_C(c)$. Indeed, otherwise $V \in \alpha_C(c)$, and hence by definition of $\alpha_C(c)$, $\exists a \in A \ V \in \delta(a, c)$. By property 3 of δ, this implies that $V \in \epsilon_A(a)$ and hence by 1, that $V \in \varphi_A(a)$. But $V \in \delta(a, c) \cap \varphi_A(a)$ implies that $V \in \varphi_C(c)$ by extensionality of $\|\varphi\|$ and this is a contradiction.

We check finally the validity of the syllogism in Datisi, a syllogism of the third figure.

$$\text{All } \underline{A} \text{ are } \varphi$$
$$\underline{\text{Some } \underline{A} \text{ are } \gamma}$$
$$\text{Some } \underline{C} \text{ are } \varphi$$

where γ is the predicable "to be a $\underline{C}$". Let U be a situation such that

1. $U \Vdash$ All $\underline{A}$ are φ
2. $U \Vdash$ Some $\underline{A}$ are γ

We must check that $U \Vdash$ Some $\underline{C}$ are φ . By 2, we have that for some $a \in A$ $U \in \epsilon_A(a)$, and $U \in \gamma_A(a)$. By definition of $\gamma_A(a)$, $U \in \delta(a, c)$ for some $c \in C$. By 1, $U \in \varphi_A(a)$. We then obtain that $U \in \delta(a, c) \cap \varphi_A(a)$ and by the proposition in Section 3.4 we can conclude that $U \in \varphi_C(c)$.

The following consequence of this theorem is worth noticing:

Corollary 4.4 *If all terms of a syllogism not in D/F are CNs or derived from CNs, then the syllogism is valid.*

The reader may well ask what is so special about syllogisms in D/F to deserve a special treatment. The answer is that in each we conclude an existential statement from universal premises and this is certainly not permissible, unless we make existential assumptions about kinds. In the class interpretation, validity of these syllogisms amounts to the requirement that interpretations of CNs and predicables should be non-empty subsets of the set of entities. In our theory,the corresponding requirement is rather strong: the interpretation A of the CNs $\underline{A}$ should satisfy a condition of "ubiquity".

Let us say that a kind (A, ϵ_A) is *ubiquitous* iff $\forall U \ \exists a \in A \ U \in \epsilon_A(a)$. With this notion we can prove the following:

Theorem 4.5 *If the interpetations of all CNs are ubiquitous and the interpretation of all genuine predicables are extensional, then the syllogism is valid.*

Proof: We check just the syllogism in Darapti

$$\text{All } \underline{A} \text{ are } \beta$$
$$\underline{\text{All } \underline{A} \text{ are } \varphi}$$
$$\text{Some } \underline{B} \text{ are } \varphi$$

where β is the predicable derived from the CN $\underline{B}$. Let U be a situation such that

1. $U \Vdash$ All $\underline{A}$ are $\underline{\beta}$

2. $U \Vdash$ All $\underline{A}$ are φ .

Since $\|\underline{A}\|$ is ubiquitous, there is some $a \in A$ such that $U \in \epsilon_A(a)$. By 1, since $U \in \beta_{\underline{A}}(a)$, there is some $b \in B$ such that $U \in \delta(a, b)$. By 2, $U \in \varphi_{\underline{A}}(a)$. By the extensionality of $\|\varphi\|$,

$$\varphi_{\underline{A}}(a) \cap \delta(a, b) \subseteq \varphi_{\underline{B}}(b)$$

and hence $U \in \varphi_{\underline{B}}(b)$. I.e., we have shown $U \Vdash$ Some $\underline{B}$ are φ .

4.3 Deviant syllogisms   There are examples due to Aristotle himself of genuine predicables appearing in subject position. We notice first that the grammar of the syllogism is then not quite correct since quantifiers apply only to CNs. Thus these syllogisms require special interpretation. We suggest the following interpretation:

$U \Vdash$ All $\underline{\varphi}$ are $\underline{\psi}$ iff
for every CN $\underline{A} \in \mathcal{N}$, $\forall V \leq U \ \forall a \in A \ (V \in \epsilon_A(a) \ \wedge V \in \varphi_{\underline{A}}(a) \Rightarrow V \in \psi_{\underline{A}}(a))$.
(Here and in the following clauses, we let (A, ϵ_A) be the interpretation of the $\overline{CN} \ \underline{A}$).

$U \Vdash$ Some $\underline{\varphi}$ are $\underline{\psi}$ iff
for some CN $\underline{A}$, $\exists a \in \overline{A} \ (U \in \epsilon_A(a) \ \wedge U \in \varphi_{\underline{A}}(a) \ \wedge U \in \psi_{\underline{A}}(a))$.

$U \Vdash$ All $\underline{\varphi}$ are not $\underline{\psi}$ iff
for every CN $\underline{A}$, $\forall V \leq \overline{U} \ \forall a \in A (V \in \epsilon_A(a) \wedge V \in \varphi_{\underline{A}}(a) \Rightarrow \forall W \leq V \ W \notin \psi_{\underline{A}}(a))$.

$U \Vdash$ Some $\underline{\varphi}$ are not $\underline{\psi}$ iff
for some CN $\underline{A}$, $\exists a \in A \ (\overline{U} \in \epsilon_A(a) \ \wedge U \in \varphi_{\underline{A}}(a) \ \wedge \forall V \leq U \ V \notin \psi_{\underline{A}}(a))$.

$U \Vdash$ No $\underline{\varphi}$ are $\underline{\psi}$ is interpreted as $U \Vdash$ All $\overline{\underline{\varphi}}$ are not $\underline{\psi}$

$U \Vdash$ All $\underline{\varphi}$ are not not $\underline{\psi}$ iff
for every CN $\underline{A}$, $\forall a \in A$ and $\forall V \leq U \ [V \in \varphi_{\underline{A}}(a) \Rightarrow \forall W \leq V \ \exists W' \leq W \ \ W' \Vdash \psi_{\underline{A}}(a)]$

With this interpretation we can prove the following

Theorem 4.6   *If all genuine predicables of a syllogism not in D/F are extensional, then the syllogism is valid.*

Once again syllogisms in D/F present problems related to both kinds and interpretation of predicables. Let us say that the interpretation of a predicable $\|\varphi\|$ is *ubiquitous* if there is a CN $\underline{A}$ such that $\forall U \ \exists a \in A \ \ U \in \varphi_{\underline{A}}(a)$, where $\|\underline{A}\| = (A, \epsilon_A)$.

Theorem 4.7   *If the interpretation of CNs are ubiquitous and the interpretation of genuine predicables are ubiquitous and extensional then the syllogism is valid.*

Proof: We check just validity of the following syllogism in Darapti:

$$\frac{\begin{array}{l} \text{All } \underline{\varphi} \text{ are } \underline{\psi} \\ \text{All } \underline{\varphi} \text{ are } \underline{\theta} \end{array}}{\text{Some } \underline{\theta} \text{ are } \underline{\psi}}$$

Let U be a situation such that

1. $U \Vdash$ All $\underline{\varphi}$ are $\underline{\psi}$
2. $U \Vdash$ All $\underline{\varphi}$ are $\underline{\theta}$

Since $\|\underline{\varphi}\|$ is ubiquitous, there is a CN $\underline{A}$ such that for some $a \in A = \|\underline{A}\|$, $U \in \varphi_{\underline{A}}(a)$. By 1, $U \in \psi_{\underline{A}}(a)$ and by 2, $U \in \theta_{\underline{A}}(a)$. This shows that $U \Vdash$ Some $\underline{\theta}$ are $\underline{\psi}$. Notice that we may have "mixed" syllogisms of the type:

$$\frac{\begin{array}{l} \text{All } \underline{A} \text{ are } \underline{\psi} \\ \text{All } \underline{A} \text{ are } \underline{\theta} \end{array}}{\text{Some } \underline{\theta} \text{ are } \underline{\psi}}$$

Remark 4.8 (1) If all terms of a syllogism are genuine predicables then we can omit "extensional" in the theorem. (2) The reader may have noticed a lack of symmetry between the treatment of the interpretation of (genuine) predicables in subject position and that of a CN. In the first case, we quantified over all kinds in the range of $\|...\|$, whereas in the second only the interpretation of the CN was involved.

However, we can prove the following:

Proposition 4.9 $U \Vdash All \underline{B} \ are \ \underline{\varphi} \ iff \ U \Vdash All \ \underline{\beta} \ are \ \underline{\varphi}$, *provided that* $\|\underline{\varphi}\|$ *is extensional and* $\underline{\beta}$ *is the predicable corresponding to the CN* $\underline{B}$.

Proof: The proof that RHS implies LHS is obvious. Let us prove that LHS implies RHS. Let A be a kind, $a \in A$, $V \le U$ such that $V \in \beta_{\underline{A}}(a)$. We claim that $V \in \varphi_{\underline{A}}(a)$. Since $V \in \beta_{\underline{A}}(a)$, $\exists b \in B$ such that $V \in \delta(a, \overline{b})$. So by hypothesis $V \in \varphi_{\underline{B}}(b)$. By the extensionality of $\|\underline{\varphi}\|$, $V \in \varphi_{\underline{A}}(a)$.

4.4 The class interpretation revisited Our work should not be considered as refuting the class interpretation, but rather as determining limits for its validity, as we show in this section. In fact, the class interpretation may be obtained as a particular case of our semantics when kinds and predicates are suitably restricted.

Let us say that a kind (A, ϵ_A) is *situationless* if $\forall a \in A \ \epsilon_A(a) = P$ where P is the set of all situations. We notice that predicates of situationless kinds are automatically ϵ-predicates.

Assume now that we have a subject and a corresponding interpretation

$$\|...\| \ : \ \mathcal{N} \longrightarrow \mathcal{K}$$

such that for all $\underline{A}$ in $\mathcal{N}$, $\|\underline{A}\|$ is both situationless and ubiquitous. Assume, furthermore, that only predicables derived from CNs (of the subject) are considered. It then follows from the proposition of Section 4.1 that the interpretation of such predicables are functorial ϵ-predicates.

Proposition 4.10 *There is a bijection between functorial ϵ-predicates of $\|...\|$ and ϵ-predicates of the kind $E = colimit \ \|...\|$ of entities for $\|...\|$.*

Proof: We notice just that a predicate $(\varphi_{\underline{A}})_{\underline{A} \in \mathcal{N}}$ is functorial iff $\forall \underline{u} : \underline{A} \longrightarrow \underline{B}$ the diagram

commutes (since the kinds are situationless). The proposition follows from the universal property of the colimit of the system $\|...\|$.

Using our explicit construction of $colim \|...\| = E = E_0/\sim$, we may describe the 1-1 correspondance as follows: given a functorial predicate $(\varphi_{\underline{A}})_{\underline{A} \in \mathcal{N}}$, we define

$\varphi : E \longrightarrow \Gamma(\Omega)$ by $\varphi([(a, \underline{A})]) = \varphi_{\underline{A}}(a)$. This map is well defined because of the commutativity of the above diagram. Conversely, given $\varphi : E \longrightarrow \Gamma(\Omega))$, define $\varphi_{\underline{A}} = \varphi \circ \eta_{\underline{A}}$ and check that $(\varphi_{\underline{A}})_{\underline{A} \in \mathcal{N}}$ is functorial. We leave the verification that this correpondance is 1-1 to the reader.

   This result allows us to view $\|to\ be\ an\ \underline{A}\|$ as a predicate of the kind *ENTITY* for the interpretation $\|...\|$.

   Furthermore, since $\|\underline{A}\|$ is situationless, this predicate factors through

$$\{\varnothing, P\} \subseteq \Gamma(\Omega)$$

and this means that we may consider $\|to\ be\ an\ \underline{A}\|$ as a subset of E , indeed a non-empty subset of E , because of the ubiquity of $\|\underline{A}\|$.

   We thus obtain the following interpretation for the CNs and predicables (derived from CNs) as non-empty subsets of E:

$$\mid \underline{A} \mid = \|to\ be\ an\ \underline{A}\| \subseteq E$$

   It is an easy matter now to check that $U \Vdash$ All $\underline{A}$ are $\underline{\beta}$ iff $\mid \underline{A} \mid \subseteq \mid \underline{B} \mid$ where β is the predicable derived from the CN $\underline{B}$ and similarly for other clauses. In other words, we have obtained the class interpretation for terms which are "homogeneous with respect to their possible positions as subjects and predicates" as in Łukasiewicz [17] and other modern commentators (see Section 5).

5 Aristotle and his modern commentators

5.1 Aristotle   Aristotle was aware of several of the problems we raise. For example, he noticed that predicables are sorted by the nouns to which they are attached. From several texts we cite just one

> . . . good in the case of food is what is productive of pleasure and in the case of medicine what is productive of health. . . ([2], *Topics*, 107 a 5–6).

Aristotle was also sensitive to the logical problems that accompany the shift of grammatical role from predicable to subject. While he does not tackle the problem directly he touches upon it when discussing other problems. For instance

> The conversion of an appropriate name which is derived from an accident is an extremely precarious thing;... Names derived from definition and property and genus are bound to be convertible; e.g. if being a two-footed terrestrial animal belongs to something, then it will be true by conversion to say that it is a two-footed terrestrial animal ([2], *Topics*, 109 a 9–15).

With the notion of situation we try to take account of the aspect of contingency which is so central to Aristotle's philosophy. When Aritotle discusses the notion of accident, he recognizes that aspect

> . . . being seated may belong or not belong to some self-same thing. Likewise also whiteness; for there is nothing to prevent the same thing being at one time white at another not white ([2], *Topics*, 102 b 6–10).

At the end of his life, it seems that Aristotle was developing a modal logic based on the notions of time and change. We do not in this article consider modal syllogisms

but it would be "morally" wrong to debar any notion of contingency from our theory of his syllogistic since we believe contingency was one of his concerns.

Aristotle also considered predicables that we called ϵ- predicables. They are important since they relate to the aspect of contingency in his thought. For instance, we can from our perspective make perfectly good sense of the following passage, provided we take *SICK* to be an ϵ-predicate and interpret "existence" as "being constituent (of a situation)"

> For take "Socrates is sick" and "Socrates is not sick": if he exists it is clear that one or the other of them will be true or false, and equally if he does not; for if he does not exist "he is sick" is false but "he is not sick" true ([2], *Categories*, 13 b 30–35).

5.2 Some modern commentators   As we said in the introduction, we have not seen the difficulties in syllogistic that we have pointed out discussed at any length in the modern literature that we consulted. In his celebrated work on syllogistic, Łukasiewicz ([17] p. 5) notices that Aristotle, in his Prior Analytics, divides "things" into three classes: (1) those that cannot be predicated truly of anything at all like "Cleon" and "Callias", but others can be predicated of them; (2) those that are themselves predicated of others but nothing prior is predicated of them. No examples are given by Aristotle but it is clear that he meant what is most universal, like being; (3) those that are predicated of others and others are predicated of them. As an example, "person" is predicated of "man" in "a man is a person", and has "human being" predicated of it in "a person is a human being". According to Łukasiewicz, Aristotle would eliminate from his system those terms which, in his opinion, were not suited to be both subjects and predicates of true propositions. In other words, he would keep only terms of the third category. Łukasiewicz ([17] p. 7) goes on

> It is essential for the Aristotelian syllogistic that the same term may be used as a subject and as a predicate without any restriction. Syllogistic as conceived by Aristotle requires terms to be homogeneous with respect to their possible positions as subjects and predicates.

From the point of view of our interpretation, Łukasiewicz requires all terms to be CNs or to be derived from CNs and, in fact, we saw in Section 4.2 that such syllogisms are valid. On the other hand, we do not think that Aristotle accepted this limitation and later we will give some of the numerous examples of his use of terms which do not belong to the third category discussed by Łukasiewicz. Our interpretation has the merit of extending the validity of syllogisms to some terms which are not homogeneous in the sense of Łukasiewicz. For instance, in the following example a genuine adjective appears both in subject position and in predicable position:

> E.g. let A be altering, D changing, B enjoying... Now it is true to predicate both D of B and A of D; for the man who is enjoying himself is changing, and what is changing is altering ([2], *Posterior Analytics*, 87 b 7–11).

The work of Łukasiewicz on syllogistic has been criticized by Corcoran [5]. In particular, Corcoran disputes the view of Łukasiewicz that the Aristotelian syllogistic constitutes an axiomatic theory. The main thrust of Corcoran's work is to show that it is rather

> . . . an underlying logic which includes a natural deductive system and that it is not an axiomatic theory as had previously been thought ([5], p. 85)

The difficulties that we have raised in this paper are quite independent of the question raised by Corcoran and we need not take sides on this issue. Furthermore, his work offers us no help with our problems, since in relation to our second problem he just goes along with Łukasiewicz:

> In the first chapter of the first book of Prior Analytics (43 a 24-44) Aristotle also seems to exclude both adjectives and proper names from scientific languages. Łukasiewicz (p.7) seems correct in saying that both the latter were banned because neither can be used both in subject and in predicate positions... It must also be noted that our model makes no room for relatives (and neither does the Łukasiewicz interpretation). ([5], p. 100)

The only modern commentator among those that we have read who seems sensitive to the issues raised in this paper is Mary Mulhern. In her paper [18], she maintains that proper names, adjectives and relational expressions can appear in syllogistic premises, although their role in them is restricted. She gives examples even from the Prior Analytics of syllogisms containing proper names and adjectives. According to her, all that Aristotle's doctrine requires is the exclusion of proper names from the predicate place and the exclusion of disembodied accidents from the subject place. The latter condition is not stringent, since there are no such things as disembodied accidents. Names of accidental attributes, on the other hand, may take only the predicate place in sentences, never the subject place. She goes on

> When accidents appear to be treated as subjects, Aristotle holds, it is actually the object in which the accident is present which is the subject of predication.

We believe that our interpretation of "All φ are ψ" agrees with this passage.

5.3 Ancient texts and their modern translations

The formulation of our second difficulty in the theory of syllogism was in terms of CNs and predicables, grammatical categories of modern linguistics. As we emphasized, however, this distinction has a logical basis in so far as CNs provide means to specify and trace the identity of an individual, whereas predicables do not. It is in this sense that our problem is really a logical rather than a linguistic one. It so happens that modern linguistics goes along with logic in separating these parts of speech in the same way and our logical problem may be formulated in terms of grammatical categories. On the other hand, Mulhern [18] points out that the ancient Greeks did not distinguish parts of speech precisely as we do. Furthermore, by prefixing a definite article or by other means, they used adjectives as substantives. This feature of ancient Greek may have obscured the logical problem that we have discussed here. This is so much so that, according to Alexander, Aristotle preferred the forms "to be predicated of" and "to belong to" rather than the more common "to be" since these expressions distinguish more clearly between the subject and the predicate. In "All men are animal" both terms are in the nominative, whereas in the form preferred by Aristotle only the predicate is in the nominative. Alexander adds that to say as Aristotle does "Animal is predicated of all men" instead of the customary "All men are animal" was felt in ancient Greek to be as artificial as in modern languages (see Łukasiewicz [17]).

We may add that since articles were often implicitly understood and the order of the terms was not important, "men are animals" could also be expressed as "animals are men". Neither the order of the words, nor the cases, nor the articles, then could

differentiate between subject and predicate. Apart from the context, there were no means to express which term is predicated of the other. Aristotle writes

> If names and verbs are transposed they still signify the same thing, e.g. a man is white—white is a man ([2], *De Interpretatione*, 20b 1–3).

In the more artificial form preferred by Aristotle the predicate takes the nominative case while the subject takes the genitive or dative case, and thus there is a grammatical distinction between the two.

If we go back to to the second example from Section 5.2 we realize how difficult it is to reconcile ancient Greek, modern English and Aristotle's thought. That example is rendered into modern English as follows

> for the man who is enjoying himself is changing ([2], *Posterior Analytics*, 87b 10).

There is no term in the original Greek corresponding to "man" in the translation. Indeed the Greek text [20] reads as follows

$$o \ \gamma \alpha \rho \ \eta \delta o \mu \epsilon \nu o \varsigma \ \kappa \iota \nu \epsilon \iota \tau \alpha \iota$$

A literal translation would be rather like "the enjoyer is changing".

As we said before, it was common in ancient Greek to form a noun out of an adjective by prefixing an article. There are several examples where the English translation adds "man", "thing", "someone", "something" or a paraphrase to obtain a sentence which is grammatical. For instance, in the English translation, we find "the musical thing is white" ([2], *Posterior Analytics*, 83 a 11). The Greek text [20], on the other hand, reads

$$\tau o \ \mu o \upsilon \sigma \iota \kappa o \nu \ \lambda \epsilon \upsilon \kappa o \nu \ \epsilon \iota \nu \alpha \iota$$

A literal translation would be "the musical is white".

We make a last remark on the presentation of the syllogisms themselves. Nowadays, syllogisms are usually presented more or less as in Section 1. Aristotle never presents them in such a fashion and favors the presentation given in Section 5. When the syllogisms are given in terms of variables, the problem of transformation seems less acute, but this is of course not the case.

We conclude that Aristotle, contrary to what Łukasiewicz and other modern commentators claim, did not limit the application of his syllogisms to terms that were homogeneous with respect to the subject and predicate position. Łukasiewicz [17] has drawn a distinction between the peripatetic logic of terms and the later stoic logic of propositions and pointed out that it is only the latter that was incorporated in modern logic. In agreement with his emphasis on terms, Aristotle used all sorts of terms in his syllogisms and these terms underwent transformations from subject to predicate position and vice versa. In our work, we have tried to show conditions of validity of syllogisms when such transformations occur. We have also dealt with switches in the sorting of predicates. Our aim was to explain the nature of these transformations and switches at the semantic level, using the theory of kinds, thus renewing contact with a rich tradition unjustly neglected.

6 Appendix   We reproduce here the syllogisms that we have taken into account in this work. They constitute the first three figures. The variables X, Y and Z range over CNs and predicables.

Barbara   All X are Y
All Y are Z
_____
All X are Z

Celarent   All X are not Y
All Z are X
_____
All Z are not Y

Darii   All X are Y
Some Z are X
_____
Some Z are Y

Ferio   All X are not Y
Some Z are X
_____
Some Z are not Y

Cesare   All X are not Y
All Z are Y
_____
All Z are not X

Camestres   All X are Y
All Z are not Y
_____
All Z are not X

Festino   All X are not Y
Some Z are Y
_____
Some Z are not X

Baroco   All X are Y
Some Z are not Y
_____
Some Z are not X

Darapti   All X are Y
All X are Z
_____
Some Z are Y

Felapton   All X are not Y
All X are Z
_____
Some Z are not Y

Disamis   Some X are Y
            All X are Z
            Some Z are Y

Datisi   All X are Y
            Some X are Z
            Some Z are Y

Bocardo   Some X are not Y
            All X are Z
            Some Z are not Y

Ferison   All X are not Y
            Some X are Z
            Some Z are not Y

REFERENCES

[1]   Bach, E., "The semantics of syntactic categories: A cross-linguistic perspective," pp. 264–281 in *The logical Foundations of Cognition*, edited by J. Macnamara and G. E. Reyes, Oxford University Press, Oxford, 1994.

[2]   Barnes, J., (ed) *The Complete Works of Aristotle*, vol. 1, Princeton University Press, Princeton, 1984.

[3]   Barwise, J. and J. Perry, *Situations and Attitudes*, MIT Press, Cambridge, 1983.

[4]   Cooke, H. P. and H. Tredennick, *Aristotle*. vol. 1, William Heinemann, London, 1938.

[5]   Corcoran, J., "Aristotle's natural deduction system," pp. 85–131 in *Ancient Logic and its Modern Interpretations*, edited by J. Corcoran, Reidel, Dordrecht, 1974.

[6]   Creswell, M. "The semantics of degree," pp. 261–293 in *Montague Grammar* edited by B. Partee, The Academic Press, New York, 1976.

[7]   van Dalen, D., *Logic and Structure*, Springer-Verlag, Berlin, 1983.

[8]   Geach, P., *Reference and Generality*, Cornell University Press, Ithaca, 1962.

[9]   Geach, P., *Logic Matters*, University of California Press, Berkeley, 1972.

[10] Gupta, A., *The Logic of Common Nouns*, Yale University Press, New Haven, 1980.

[11] Kaplan, D., "Demonstratives; An essay on the semantics, logic, metaphysics and epistemology of demonstratives and other indexicals," pp. 221–250 in *Syntax and Semantics*, 9, edited by P. Cole, The Academic Press, New York, 1978.

[12] Keenan, E., and L.Faltz, *Boolean Semantics for Natural Language*, Synthesis Language Library, Reidel, Dordrecht, 1985.

[13] Klein, E., "The semantics for positive and comparative adjectives," *Linguistics and Philosophy*, 4 (1980), pp. 1–46.

[14] Kripke, S., *Naming and Necessity*, Basil Blackwell, Oxford, 1980.

[15] La Palme Reyes M., J. Macnamara, and G. Reyes, "Reference, kinds and predicates," pp. 90–143 in *The Logical Foundations of Cognition*, edited by J. Macnamara and G. Reyes, Oxford University Press, Oxford, 1994.

[16] Larson, R., "Scope and comparatives," *Linguistics and Philosophy*, 11 (1988), pp. 1–26.

[17] Łukasiewicz, J., *Aristotle's Syllogistic*, Clarendon Press, Oxford, 1957.

[18] Mulhern, M., "Corcoran on Aristotle's logical theory," pp. 133–158 in *Ancient Logic and its Modern Interpretations*, edited by J. Corcoran, Reidel, Dordrecht, 1974.

[19] Reyes, G. E., "A topos-theoretic approach to reference and modality," *Notre Dame Journal of Formal Logic*, 32 no.3 (1991), pp. 359-391.

[20] Ross, W., *Aristotelis Analytica Priora et Posteriora*, Scriptorum Classicorum Bibliotheca Oxoniensis, Oxford University Press, Oxford, 1964.

[21] Sommers, F., *The Logic of Natural Language*, Clarendon Press, Oxford, 1982.

Marie LaPalme Reyes
John Macnamara
Department of Psychology
McGill University
1205 Doctor Penfield Avenue
Montreal QC Canada H3A 1B1

Gonzalo E. Reyes
Department of Mathematics
University of Montreal
CP 6128 Succ "A"
Montreal QC Canada H3C 3J7
e-mail:reyes@mathcn.umontreal.ca