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Zonal Polynomials

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Purdue University

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Shanti S. Gupta, Series Editor

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PREFACE

This monograph presents a self-contained development of zonal polynomials in the framework of standard multivariate analysis. Except for standard tools of multivariate normal distribution theory and linear algebra, no extensive mathematics is assumed in this work. This contrasts with earlier treatments of the theory of zonal polynomials. It is hoped that the present approach makes zonal polynomials and the theory of noncentral distributions in multivariate analysis accessible to much wider audience.

This work has been based on my doctoral dissertation submitted to Stanford University in 1982. I am indebted to many people in completing my thesis and in preparing this monograph. Professor T.W. Anderson as my thesis advisor had a distinctive influence on me. Professor Ingram Olkin and Professor Persi Diaconis read the thesis and suggested many improvements. My friends Fred Huffer and Satish Iyengar read an earlier version of this work and gave very careful and useful comments. In preparing this monograph I benefited from thorough reviews by three referees. Professor Shanti Gupta, the editor of this monograph series, has been encouraging throughout the preparation of this monograph. This work was supported in part by Office of Naval Research Contract N00014-75-C-0442 and U.S. Army Research Office Contract DAAG29-82-K-0156.

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