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Vector bundles on ample divisors

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Introduction.

Suppose that a scheme A lies as an ample divisor in another scheme M. Then, as we saw in [5] and [2], the structure of M is closely related to that of A. Keeping this principle in mind, we study in § 1 the behaviour of a vector bundle F on M in relation to that of F_A. In § 2 and § 3 we prove the following extendability criterion announced in [1]: *A vector bundle E can be extended to a vector bundle on M if $H^2(A, \mathcal{E}nd(E)[-tA])=0$ for any $t\geqq 1$, $H^p(A, E[tA])=0$ for any $0<p<\dim A$, $t\in\boldsymbol{Z}$ and if M is non-singular.* In § 4 and § 5, as an application, we show that the Grassmann variety $G_{n,r}$ parametrizing r-dimensional linear subspaces of an n-dimensional vector space cannot be an ample divisor in any manifold except the well known classical cases, namely the cases in which $r=1$, $r=n-1$ or $(n, r)=(4, 2)$.

Notation, Convention and Terminology.

In this paper we fix once for all an algebraically closed field k of any characteristic and assume that everything is defined over k. Basically we employ the same notation as in [2]. In particular, vector bundles are confused with the locally free sheaves of their sections. Here we show examples of symbols.

$E^\vee$: The dual vector bundle of a vector bundle E.

S^iE: The i-th symmetric product bundle of E.

$\mathcal{E}nd(E)$: $=\mathcal{H}om(E, E)=E^\vee\otimes E$.

$\mathcal{F}[E]$: $=\mathcal{F}\otimes_{\mathcal{O}}\mathcal{O}[E]$ where F is a coherent $\mathcal{O}$-module.

$[D]$: The line bundle associated with a Cartier divisor D.

$Bs\Lambda$: The intersection of all the members of a linear system Λ.

Note that a line bundle L is generated by its global sections if and only if $Bs|L|=\emptyset$.

ρ_Λ: The rational mapping induced by Λ.

L_T: The pull back of L to T.

§ 1. The hyperplane section principle of Lefschetz.

(1.0) Throughout this section A is an ample divisor on a scheme S. We fix $a \in H^0(S, [A])$ which defines A. For a coherent sheaf $\mathcal{F}$ on S, let $a_{\mathcal{F}}$ denote the natural homomorphism $\mathcal{F}[-A] \to \mathcal{F}$, and let $\mathcal{F}_A$ denote Coker$(a_{\mathcal{F}})$. This is consistent with the fact that $\mathcal{O}_A = \text{Coker}(a_{\mathcal{O}_S})$.

(1.1) PROPOSITION. *Let $\mathcal{F}$ be a coherent sheaf on S such that $a_{\mathcal{F}}$ is injective. Let p be a positive integer such that the natural mapping $H^{p-1}(S, \mathcal{F}[tA]) \to H^{p-1}(A, \mathcal{F}[tA]_A)$ is surjective for any $t>0$. Then $H^p(S, \mathcal{F})=0$.*

For a proof, see [**2**, (2.1)].

REMARK. $a_{\mathcal{F}}$ is injective if S is irreducible and reduced and if $\mathcal{F}$ is locally free.

(1.2) PROPOSITION. *Let $\mathcal{E}$ be a locally free sheaf on S and suppose that S is non-singular. Let p be an integer less than* $\dim S$ *such that $H^p(A, \mathcal{E}[-tA]_A) = 0$ for any $t \geqq 0$. Then $H^p(S, \mathcal{E})=0$.*

For a proof, see [**5**, Lemma I-B].

(1.3) Let L be a line bundle on S and let $G(S, L)$ denote the graded k-algebra $\bigoplus_{t \geq 0} H^0(S, tL)$. For any coherent sheaf $\mathcal{F}$ on S, $M(\mathcal{F}, L) = \bigoplus_{t \in Z} H^0(S, \mathcal{F}[tL])$ has a natural graded $G(S, L)$-module structure. Of course the grading is given by $M_t(\mathcal{F}, L) = H^0(S, \mathcal{F}[tL])$.

Let $\delta \in H^0(S, dL)$ for some $d>0$ and let D be the divisor of zeros of δ. By $M(\mathcal{F}, L)_D$ we denote the image of the natural mapping $M(\mathcal{F}, L) \to M(\mathcal{F}_D, L_D)$.

THEOREM. *Let G be a graded subalgebra of $G(S, L)$ containing δ. Suppose that $\delta_{\mathcal{F}}$ is injective and that $M_t(\mathcal{F}, L)=0$ for $t \ll 0$. Let Σ be a set of homogeneous elements of $M(\mathcal{F}, L)$ such that $M(\mathcal{F}, L)_D$ is generated by the image of Σ as a G-module. Then $M(\mathcal{F}, L)$ is generated by Σ as a G-module.*

PROOF. Let Z be the G-submodule of $M(\mathcal{F}, L)$ generated by Σ. Putting $Z_t = Z \cap M_t(\mathcal{F}, L)$, we show $Z_t = M_t(\mathcal{F}, L)$ by induction on t. This is obvious for $t \ll 0$ by assumption. Let us assume $Z_p = M_p(\mathcal{F}, L)$ for any $p<t$. The injectivity of $\delta_{\mathcal{F}}$ implies $\text{Ker}(M(\mathcal{F}, L) \to M(\mathcal{F}, L)_D) = \delta M(\mathcal{F}, L)$ via the long exact sequence of cohomology. On the other hand, $Z \to M(\mathcal{F}, L)_D$ is surjective by assumption. Hence we infer that $M_t(\mathcal{F}, L) = Z_t + \delta M_{t-d}(\mathcal{F}, L) = Z_t + \delta Z_{t-d} = Z_t$. Thus we prove the assertion.

(1.4) COROLLARY. *Let $\mathcal{F}$ be a coherent sheaf on S such that $a_{\mathcal{F}}$ is injective. Suppose that the natural mappings $H^0(S, \mathcal{F}) \to H^0(A, \mathcal{F}_A)$ and $H^0(S, [A]) \to H^0(A, [A]_A)$ are surjective. Suppose further that the natural mapping $H^0(A, \mathcal{F}[tA]_A) \otimes H^0(A, [A]_A) \to H^0(A, \mathcal{F}[(t+1)A]_A)$ is surjective for every $t \geqq 0$. Then, the natural mappings $H^0(S, \mathcal{F}[tA]) \to H^0(A, \mathcal{F}[tA]_A)$ and $H^0(S, \mathcal{F}[tA]) \otimes H^0(S, [A]) \to H^0(S, \mathcal{F}[(t+1)A])$ are surjective for any $t \geqq 0$.*

PROOF. Put $L=[A]$ and let G be the subalgebra of $G(S, L)$ generated by

$H^0(S, [A])$. Then, letting $\Sigma = H^0(S, \mathcal{F})$, a similar argument as in (1.3) proves this corollary.

(1.5) REMARK. Let $\mathcal{F}$ be as in (1.4). Then $\mathcal{F}$ is generated by its global sections.

Indeed, the second assertion implies that $\mathcal{F}[tA]$ is generated by its global sections if $\mathcal{F}[(t+1)A]$ is so. On the other hand, $\mathcal{F}[lA]$ is generated by its global sections for $l \gg 0$ since A is ample. Therefore $\mathcal{F}[tA]$ is generated by its global sections for any $t \geqq 0$.

(1.6) REMARK. Return to the situation in (1.3). Consider the case in which Σ is a finite set $\zeta_1, \cdots, \zeta_m$. Each ζ_j defines a G-homomorphism $G \to M(\mathcal{F}, L)$. Combining them together we get a surjective G-homomorphism $\Phi: G \oplus \cdots \oplus G \to M(\mathcal{F}, L)$. To give a fundamental system of relations among $\zeta_1, \cdots, \zeta_m$ is equivalent to give a generator system of Ker(Φ) as a G-module.

Since Σ generates $M(\mathcal{F}, L)_D$, we have similarly a surjective homomorphism $\Phi_D: G_D \oplus \cdots \oplus G_D \to M(\mathcal{F}, L)_D$. Then, we can show, similarly as in [**2**, (3.2)], that the natural homomorphism Ker$(\Phi) \to$ Ker(Φ_D) is surjective. In other words, any relation among $\zeta_1, \cdots, \zeta_m$ on D can be lifted to a relation on S.

However, such lifted relations do not always generate Ker(Φ). This is because Ker$(G \to G_D)$ might be greater than δG. So, suppose in addition that $0 \to \delta G \to G \to G_D \to 0$ is exact. Then, quite similarly as in [**2**, (3.2)], we can show that any lift of a generator system of Ker(Φ_D) to Ker(Φ) becomes a generator system of Ker(Φ) as a G-module.

We omit detailed arguments since we don't use these facts in the following sections.

(1.7) Before closing this section we present the following

PROPOSITION. *Suppose that there is a morphism $\pi: S \to A$ such that the restriction of π to A is the identity. Then $Pic(S) \to Pic(A)$ is not injective.*

PROOF. Put $L = [A]_A \in Pic(A)$. Then $\pi^* L_A = [A]_A$. On the other hand, $\pi^* L$ is not ample since dim $A <$ dim S, while A is ample. Hence $Pic(S) \to Pic(A)$ is not injective.

§ 2. Formal extendability of vector bundles.

(2.1) The purpose of this section is to prove the following

PROPOSITION. *Let A be an effective divisor on a scheme S such that $a_{\mathcal{O}_S}$ is injective. Let E be a vector bundle on A such that $H^2(A, \mathcal{E}nd(E)[tA]_A) = 0$ for any $t < 0$. Then E can be extended to a vector bundle on the formal completion $\hat{S}$ of S along A.*

(2.2) Let $\mathcal{I}$ be the defining ideal of A. Then $\mathcal{I} \cong \mathcal{O}_S[-A]$ since $a_{\mathcal{O}_S}$ is injective. Moreover, $\mathcal{I}^k/\mathcal{I}^{k+1}$ is canonically isomorphic to $\mathcal{O}_A[-kA]$ for any $k \geqq 0$.

(2.3) Let $\mathcal{F}$ be a coherent sheaf on S. For each open set U in S, let

$\mathcal{M}_n(\mathcal{F})(U)$ be the module of (n, n) matrixes each (i, j) component of which is an element of $\Gamma(U, \mathcal{F})$. Clearly this defines a coherent sheaf $\mathcal{M}_n(\mathcal{F})$ on $\mathcal{S}$.

Let $\mathcal{B}$ be a sheaf of $\mathcal{O}_S$-algebra. Then $\mathcal{M}_n(\mathcal{B})(U)$ has a natural (probably non-commutative) $\Gamma(U, \mathcal{O}_S)$-algebra structure. Let $\mathbf{1}_U$ denote the unit of it. Let $\mathcal{G}l_n(\mathcal{B})(U)$ be the set of $P\in\mathcal{M}_n(\mathcal{B})(U)$ such that $PX=YP=\mathbf{1}_U$ for some $X, Y\in\mathcal{M}_n(\mathcal{B})(U)$. Then this gives rise to a sheaf of (probably non-abelian) groups on S, which is denoted by $\mathcal{G}l_n(\mathcal{B})$.

(2.4) For each $k\geqq 0$, we have the following natural exact sequence of sheaves of groups: $\{1\}\to\mathcal{M}_n(\mathcal{J}^k/\mathcal{J}^{k+1})\to\mathcal{G}l_n(\mathcal{O}_S/\mathcal{J}^{k+1})\to\mathcal{G}l_n(\mathcal{O}_S/\mathcal{J}^k)\to\{1\}$. Here we map $X\in\mathcal{M}_n(\mathcal{J}^k/\mathcal{J}^{k+1})(U)$ to $\mathbf{1}_U+X\in\mathcal{G}l_n(\mathcal{O}_S/\mathcal{J}^{k+1})$.

(2.5) Let $\{U_\alpha\}$ be a sufficiently fine affine open covering of S such that E is free on each $V_\alpha=A\cap U_\alpha$. Let $\mathfrak{e}_{\alpha,1}, \cdots, \mathfrak{e}_{\alpha,r}\in\Gamma(V_\alpha, E)$ be a free base of E on V_α, where $r=\operatorname{rank} E$. Then $\mathfrak{e}_{\alpha,i}=\sum_{j=1}^{r}(g_{\alpha\beta})_{i,j}\mathfrak{e}_{\beta,j}$ for some $g_{\alpha\beta}\in\Gamma(V_{\alpha\beta}, \mathcal{M}_n(\mathcal{O}_A))$ on $V_{\alpha\beta}=V_\alpha\cap V_\beta$. Clearly $g_{\alpha\beta}g_{\beta\alpha}=\mathbf{1}$. Hence $g_{\alpha\beta}\in\Gamma(V_{\alpha\beta}, \mathcal{G}l_n(\mathcal{O}_A))$. Moreover, $g_{\alpha\beta}g_{\beta\gamma}g_{\gamma\alpha}=\mathbf{1}$ on each $V_{\alpha\beta\gamma}=V_\alpha\cap V_\beta\cap V_\gamma$. So $\{g_{\alpha\beta}\}$ is a 1-cocycle defined on the covering $\{V_\alpha\}$ with coefficients in $\mathcal{G}l_n(\mathcal{O}_A)$.

(2.6) We want to find a 1-cocycle $\{\hat{g}_{\alpha\beta}\}$ defined on $\{V_\alpha\}$ with coefficients in $\mathcal{G}l_n(\mathcal{O}_{\hat{S}})=\mathcal{G}l_n(\text{proj}\cdot\lim_{k\to\infty}(\mathcal{O}_S/\mathcal{J}^k))$ such that $\hat{g}_{\alpha\beta}=g_{\alpha\beta}$ modulo $\mathcal{J}$. If this is done, then $\{\hat{g}_{\alpha\beta}\}$ defines a vector bundle $\hat{E}$ on $\hat{S}$ with $\hat{E}_A=E$.

(2.7) To find $\{\hat{g}_{\alpha\beta}\}$ is equivalent to find a compatible system of 1-cocycles $g\{^{(k)}_{\alpha\beta}\}$, $k=0, 1, 2, \cdots$, where $g^{(k)}_{\alpha\beta}$ is with coefficients in $\mathcal{G}l_n(\mathcal{O}_S/\mathcal{J}^{k+1})$. Clearly we must set $g^{(0)}_{\alpha\beta}=g_{\alpha\beta}$ in order to have $\hat{g}_{\alpha\beta}=g_{\alpha\beta}$ modulo $\mathcal{J}$.

(2.8) Assuming that we have already constructed $\{g^{(k-1)}_{\alpha\beta}\}$ with $g^{(k-1)}_{\alpha\beta}\equiv g_{\alpha\beta}\bmod\mathcal{J}$, we want to get a 1-cocycle $\{g^{(k)}_{\alpha\beta}\}$ with coefficients in $\mathcal{G}l_n(\mathcal{O}_S/\mathcal{J}^{k+1})$ such that $g^{(k)}_{\alpha\beta}=g^{(k-1)}_{\alpha\beta}\bmod\mathcal{J}^k$.

(2.9) Since $V_{\alpha\beta}$ is affine, we can find $g'_{\alpha\beta}\in\Gamma(V_{\alpha\beta}, \mathcal{G}l_n(\mathcal{O}_S/\mathcal{J}^{k+1}))$ such that $g'_{\alpha\beta}=g^{(k-1)}_{\alpha\beta}\bmod\mathcal{J}^k$. We may assume $g'_{\alpha\beta}g'_{\beta\alpha}=g'_{\beta\alpha}g'_{\alpha\beta}=\mathbf{1}$ fo each α, β.

(2.10) Clearly $g'_{\alpha\beta}g'_{\beta\gamma}g'_{\gamma\alpha}=\mathbf{1}\bmod\mathcal{J}^k$. Hence $g'_{\alpha\beta}g'_{\beta\gamma}g'_{\gamma\alpha}=\mathbf{1}+x_{\beta\gamma\alpha}$ for some $x_{\beta\gamma\alpha}\in\Gamma(V_{\alpha\beta\gamma}, \mathcal{M}_n(\mathcal{J}^k/\mathcal{J}^{k+1}))$. By (2.9) we infer $x_{\beta\gamma\alpha}=0$ if any two of α, β, γ are the same index.

(2.11) We have $\mathbf{1}+x_{\beta\gamma\alpha}=g'_{\alpha\beta}g'_{\beta\gamma}g'_{\gamma\alpha}=g'_{\alpha\gamma}(g'_{\gamma\alpha}g'_{\alpha\beta}g'_{\beta\gamma})g'_{\gamma\alpha}=g'_{\alpha\gamma}(\mathbf{1}+x_{\alpha\beta\gamma})g'_{\gamma\alpha}=\mathbf{1}+g_{\alpha\gamma}x_{\alpha\beta\gamma}g_{\gamma\alpha}$. Therefore $x_{\beta\gamma\alpha}=g_{\alpha\gamma}x_{\alpha\beta\gamma}g_{\gamma\alpha}$. Note that $\mathcal{J}\cdot x_{\alpha\beta\gamma}=0\bmod\mathcal{J}^{k+1}$ and the right hand side is well defined as a section of $\mathcal{M}_n(\mathcal{J}^k/\mathcal{J}^{k+1})$.

(2.12) On each $V_{\alpha\beta\gamma\delta}$ we have $g_{\delta\gamma}x_{\alpha\beta\gamma}g_{\gamma\delta}=x_{\gamma\alpha\delta}+x_{\alpha\beta\delta}+x_{\beta\gamma\delta}$. This follows from: $\mathbf{1}+g_{\delta\gamma}x_{\alpha\beta\gamma}g_{\gamma\delta}=g'_{\delta\gamma}(\mathbf{1}+x_{\alpha\beta\gamma})g'_{\gamma\delta}=g'_{\delta\gamma}g'_{\gamma\alpha}g'_{\alpha\beta}g'_{\beta\gamma}g'_{\gamma\delta}=g'_{\delta\gamma}g'_{\gamma\alpha}g'_{\alpha\delta}g'_{\delta\alpha}g'_{\alpha\beta}g'_{\beta\delta}g'_{\delta\beta}g'_{\beta\gamma}g'_{\gamma\delta}$ $=(\mathbf{1}+x_{\gamma\alpha\delta})(\mathbf{1}+x_{\alpha\beta\delta})(\mathbf{1}+x_{\beta\gamma\delta})=(\mathbf{1}+x_{\gamma\alpha\delta}+x_{\alpha\beta\delta}+x_{\beta\gamma\delta})$.

(2.13) Put $\beta=\gamma$ in the above situation. Then $0=x_{\beta\alpha\delta}+x_{\alpha\beta\delta}$ by (2.10).

(2.14) We define $\varphi_{\beta\gamma\alpha}\in\Gamma(V_{\alpha\beta\gamma}, \mathcal{E}nd\,(E)(\mathcal{J}^k/\mathcal{J}^{k+1}))=\Gamma(V_{\alpha\beta\gamma}, \mathcal{H}om\,(E, E(\mathcal{J}^k/\mathcal{J}^{k+1})))$ by $\varphi_{\beta\gamma\alpha}(\mathfrak{e}_{\alpha,i})=\sum_{j=1}^{r}(x_{\beta\gamma\alpha})_{i,j}\mathfrak{e}_{\alpha,j}$. Then $\varphi_{\beta\gamma\alpha}(\mathfrak{e}_{\gamma,i})=\varphi_{\beta\gamma\alpha}\left(\sum_j(g_{\gamma\alpha})_{i,j}\mathfrak{e}_{\alpha,j}\right)$

$=\sum_{j,k}(g_{\gamma\alpha})_{i,j}(x_{\beta\gamma\alpha})_{j,k}e_{\alpha,k}=\sum_{j,k,l}(g_{\gamma\alpha})_{i,j}(x_{\beta\gamma\alpha})_{j,k}(g_{\alpha\gamma})_{k,l}e_{\gamma,l}=\sum_{l}(g_{\gamma\alpha}x_{\beta\gamma\alpha}g_{\alpha\gamma})_{i,l}e_{\gamma,l}$ $=\sum_{l}(x_{\alpha\beta\gamma})_{i,l}e_{\gamma,l}=\varphi_{\alpha\beta\gamma}(e_{\gamma,i})$. Hence $\varphi_{\beta\gamma\alpha}=\varphi_{\alpha\beta\gamma}$. Combining this with (2.13) we infer that $\{\varphi_{\alpha\beta\gamma}\}$ is a 2-cochain.

(2.15) In view of (2.12) we calculate: $\varphi_{\alpha\beta\gamma}(e_{\delta,i})=\varphi_{\alpha\beta\gamma}\left(\sum_j(g_{\delta\gamma})_{i,j}e_{\gamma,j}\right)$ $=\sum_{j,k}(g_{\delta\gamma})_{i,j}(x_{\alpha\beta\gamma})_{j,k}e_{\gamma,k}=\sum_{j,k,l}(g_{\delta\gamma})_{i,j}(x_{\alpha\beta\gamma})_{j,k}(g_{\gamma\delta})_{k,l}e_{\delta,l}=\sum_{l}(g_{\delta\gamma}x_{\alpha\beta\gamma}g_{\gamma\delta})_{i,l}e_{\delta,l}$ $=\varphi_{\gamma\alpha\delta}(e_{\delta,i})+\varphi_{\alpha\beta\delta}(e_{\delta,i})+\varphi_{\beta\gamma\delta}(e_{\delta,i})$. Thus we have $\varphi_{\alpha\beta\gamma}=\varphi_{\gamma\alpha\delta}+\varphi_{\alpha\beta\delta}+\varphi_{\beta\gamma\delta}$. This means that $\{\varphi_{\alpha\beta\gamma}\}$ is a 2-cocycle.

(2.16) $\{V_\alpha\}$ is an affine covering of A and $H^2(A,\mathcal{E}nd(E)(\mathcal{J}^k/\mathcal{J}^{k+1}))$ $=H^2(A,\mathcal{E}nd(E)[-kA]_A)=0$ for any $k>0$. Hence by Čech theory $\{\varphi_{\alpha\beta\gamma}\}$ must be a 2-coboundary. Namely we have a 1-cochain $\{\varphi_{\alpha\beta}\}$ such that $\varphi_{\alpha\beta\gamma}=\varphi_{\beta\gamma}-\varphi_{\alpha\gamma}$ $+\varphi_{\alpha\beta}$ for any α,β,γ.

(2.17) Let $\varphi_{\alpha\beta}(e_{\beta,i})=\sum_{j=1}^{r}(y_{\alpha\beta})_{i,j}e_{\beta,j}$ for $y_{\alpha\beta}\in\Gamma(V_{\alpha\beta},\mathcal{M}_n(\mathcal{J}^k/\mathcal{J}^{k+1}))$. Then $\varphi_{\alpha\beta\gamma}=\varphi_{\beta\gamma}-\varphi_{\alpha\gamma}+\varphi_{\alpha\beta}$ implies $x_{\alpha\beta\gamma}=y_{\beta\gamma}-y_{\alpha\gamma}+g_{\gamma\beta}y_{\alpha\beta}g_{\beta\gamma}$. This follows from a calculation as in (2.14). Similarly we obtain $y_{\beta\alpha}+g_{\alpha\beta}y_{\alpha\beta}g_{\beta\alpha}=0$ from $\varphi_{\beta\alpha}+\varphi_{\alpha\beta}=0$.

(2.18) We put $g^{(k)}_{\alpha\beta}=g'_{\alpha\beta}(\mathbf{1}-y_{\alpha\beta})$. Then, using (2.17), we see $g^{(k)}_{\gamma\alpha}g^{(k)}_{\alpha\beta}g^{(k)}_{\beta\gamma}$ $=g'_{\gamma\alpha}(\mathbf{1}-y_{\gamma\alpha})g'_{\alpha\beta}(\mathbf{1}-y_{\alpha\beta})g'_{\beta\gamma}(\mathbf{1}-y_{\beta\gamma})=g'_{\gamma\alpha}g'_{\alpha\beta}g'_{\beta\gamma}-g_{\gamma\alpha}y_{\gamma\alpha}g_{\alpha\beta}g_{\beta\gamma}-g_{\gamma\beta}y_{\alpha\beta}g_{\beta\gamma}-y_{\beta\gamma}$ $=(\mathbf{1}+x_{\alpha\beta\gamma})+y_{\alpha\gamma}-g_{\gamma\beta}y_{\alpha\beta}g_{\beta\gamma}-y_{\beta\gamma}=\mathbf{1}$. Thus $\{g^{(k)}_{\alpha\beta}\}$ is a 1-cocycle having the property in (2.8).

(2.19) In such a way we can construct $\{g^{(k)}_{\alpha\beta}\}_{k=0,1,2,\dots}$ inductively and we obtain $\hat{g}_{\alpha\beta}=\text{proj lim}\, g^{(k)}_{\alpha\beta}$. $g^{(k)}_{\alpha\beta}g^{(k)}_{\beta\alpha}=\mathbf{1}$ implies $\hat{g}_{\alpha\beta}\hat{g}_{\beta\alpha}=\mathbf{1}$. Hence $\hat{g}_{\alpha\beta}\in\Gamma(V_{\alpha\beta},$ $\mathcal{G}l_n(\mathcal{O}_{\hat{S}}))$ and $\{\hat{g}_{\alpha\beta}\}$ is a 1-cochain. Similarly we infer that $\{\hat{g}_{\alpha\beta}\}$ is a 1-cocycle since each $\{g^{(k)}_{\alpha\beta}\}$ is so. Thus we get a 1-cocycle $\{\hat{g}_{\alpha\beta}\}$ as in (2.6). This proves the proposition (2.1).

§ 3. Global extendability of vector bundles.

(3.1) The purpose of this section is to prove the following

PROPOSITION. *Let A be an ample divisor on a non-singular variety M with* $\dim M\geqq 3$. *Let $\hat{E}$ be a vector bundle on the formal completion $\hat{M}$ of M along A. Suppose that $H^p(A, E[tA]_A)=0$ for any $t\in\mathbf{Z}$, $0<p<\dim A$, where E is the restriction of $\hat{E}$ to A. Then there is a vector bundle $\tilde{E}$ on M such that $\tilde{E}_{\hat{M}}=\hat{E}$.*

For a proof, we recall the following lemmas.

(3.2) LEMMA. *Let F be any vector bundle on M. Then the natural mapping $H^0(M, F)\to H^0(\hat{M}, F_{\hat{M}})$ is bijective.*

(3.3) LEMMA. *For any vector bundle V on $\hat{M}$, $\bigoplus_{t\in Z}\Gamma(\hat{M}, V[tA]_{\hat{M}})$ is finitely generated as a $G(M,[A])=\bigoplus_{t\geqq 0}\Gamma(M,[tA])$ module.*

(3.4) LEMMA. *For any locally free sheaf $\mathcal{F}$ on $\hat{M}$, $\mathcal{F}[tA]$ is generated by its global sections if $t\gg 0$.*

For a proof of the above facts, see Hartshorne [**3**, Chap. IV].

(3.5) DEFINITION. Let L be a line bundle on a scheme T and let V be a vector bundle on T. Then V is said to be *L-free* if V is a direct sum of line bundles of the form tL, $t\in \boldsymbol{Z}$.

(3.6) LEMMA. *For any vector bundle V on $\hat{M}$, there are a $[A]_{\hat{M}}$-free vector bundle W on $\hat{M}$ and a homomorphism $\Phi: W\to V$ such that $H^0(\Phi[tA])$: $H^0(\hat{M}, W[tA])\to H^0(\hat{M}, V[tA])$ is surjective for every $t\in \boldsymbol{Z}$.*

PROOF. We take homogeneous generators $\xi_1, \cdots, \xi_n$ of $\bigoplus_{t\in Z}H^0(\hat{M}, V[tA])$ by (3.3). Each $\xi_j\in\Gamma(\hat{M}, V[d_jA])$ defines a homomorphism $\varphi_j: [-d_jA]_{\hat{M}}\to V$. Put $W=\bigoplus_{j=1}^{n}[-d_jA]_{\hat{M}}$ and let Φ be the map $W\to V$ obtained by φ_j. This is easily seen to have the desired property.

(3.7) The above Φ is surjective. This follows from (3.4).

(3.8) Now we prove (3.1). By (3.6) we have a $[A]_{\hat{M}}$-free vector bundle F on $\hat{M}$ and a homomorphism $\Phi: F\to\hat{E}$ such that $H^0(\Phi[tA])$ is surjective for any $t\in \boldsymbol{Z}$. Φ is surjective by (3.7). So $Ker(\Phi)$ is locally free. Again by (3.6) and (3.7), we obtain a $[A]_{\hat{M}}$-free vector bundle F' and a surjective homomorphism $\Phi': F'\to Ker(\Phi)$. This induces a homomorphism $\Psi: F'\to F$ such that $\hat{E}=\mathrm{Coker}(\Psi)$.

Both F and F' are $[A]_{\hat{M}}$-free. Hence they can be extended to $[A]$-free vector bundles on M. We denote them by $\tilde{F}$ and $\tilde{F}'$. By (3.2), $\Psi\in\mathrm{Hom}(F', F)$ extends uniquely to $\tilde{\Psi}\in\mathrm{Hom}_M(\tilde{F}', \tilde{F})$. Let $\mathcal{E}'=\mathrm{Coker}(\tilde{\Psi})$. Then clearly $\mathcal{E}'_{\hat{M}}=\hat{E}$ and $\mathcal{E}'$ is locally free in a neighbourhood of A.

Since $H^0(A, \mathcal{E}'[-tA]_A)=0$ for $t\gg 0$, we have $q\in \boldsymbol{Z}$ such that $H^0(M, \mathcal{E}'[-tA])\cong H^0(M, \mathcal{E}'[-qA])$ for any $t\geqq q$. Let $\mathcal{N}$ be the subsheaf of $\mathcal{E}'$ generated by $\mathrm{Im}(\varphi)$, where φ moves in $\mathrm{Hom}_M([qA], \mathcal{E}')\cong H^0(M, \mathcal{E}'[-qA])$. Let $a\in H^0(M, [A])$ be a defining section of A as in (1.0). Then, $a_{\mathcal{N}}$ is surjective since $\mathcal{N}[-qA]$ is generated by its global sections and $\Gamma(\mathcal{N}[-(q+1)A])\cong\Gamma(\mathcal{N}[-qA])$. This implies $\mathcal{N}_A=0$ and $Supp(\mathcal{N})\cap A=\emptyset$. So $Supp(\mathcal{N})$ is a finite set since A is ample. Put $\tilde{\mathcal{E}}=\mathcal{E}'/\mathcal{N}$. It is easy to see $H^p(M, \mathcal{E}'[tA])\cong H^p(M, \tilde{\mathcal{E}}[tA])$ for any $t\in \boldsymbol{Z}$, $p>0$ and $H^0(M, \tilde{\mathcal{E}}[-tA])=0$ for $t\geqq q$.

$H^0(M, \tilde{F}[tA])\cong H^0(\hat{M}, F[tA])\to H^0(\hat{M}, \hat{E}[tA])$ is surjective for any t by construction of F. This is factored to $H^0(M, \tilde{F}[tA])\to H^0(M, \mathcal{E}'[tA])\to H^0(M, \tilde{\mathcal{E}}[tA])\to H^0(\hat{M}, \hat{E}[tA])$. So $H^0(M, \tilde{\mathcal{E}}[tA])\to H^0(\hat{M}, \hat{E}[tA])$ is surjective for any $t\in \boldsymbol{Z}$. On the other hand, $H^0(\hat{M}, \hat{E}[tA])\to H^0(A, E[tA]_A)$ is surjective since $H^1(A, E[tA]_A)=0$ for any $t\in \boldsymbol{Z}$. Therefore $H^0(M, \tilde{\mathcal{E}}[tA])\to H^0(A, E[tA]_A)$ is surjective for any t. Hence (1.1) applies to the effect that $H^1(M, \tilde{\mathcal{E}}[tA])=0$ for any t. (1.1) proves also that $H^p(M, \tilde{\mathcal{E}}[tA])=0$ for any $t\in \boldsymbol{Z}$, $0<p<\dim M$.

Thus we have $H^p(M, \tilde{\mathcal{E}}[-tA])=0$ for $p<\dim M$ and $t\gg 0$. Then $Ext^q_M(\tilde{\mathcal{E}}, \omega_M[tA])=0$ for $q>0$, $t\gg 0$ by Serre duality, where ω_M is the canonical sheaf of M. We claim that this implies $\mathcal{H}^q=\mathcal{E}xt^q_{\mathcal{O}_M}(\tilde{\mathcal{E}}, \omega_M)=0$ for $q>0$. To see this, consider the spectral sequence $E_2^{p,q}\to Ext_M^{p+q}(\tilde{\mathcal{E}}, \omega_M[tA])$ with $E_2^{p,q}$

$=H^p(M, \mathcal{H}^q[tA])$. $E_2^{p,q}=0$ for $p>0$ if $t\gg 0$. Therefore, if $t\gg 0$, $H^0(M, \mathcal{H}^q[tA])$ $=E_2^{0,q}=Ext_M^q(\tilde{\mathcal{E}}, \omega_M[tA])=0$ for $q>0$. This implies $\mathcal{H}^q=0$ for $q>0$ since A is ample.

Thus we obtain $\mathcal{E}xt_{\mathcal{O}_M}^q(\tilde{\mathcal{E}}, \omega_M)=0$ for $q>0$. This implies that $\tilde{\mathcal{E}}$ is locally free, since M is non-singular and ω_M is invertible. So $\tilde{\mathcal{E}}$ is a desired extension of $\hat{E}$.

(3.9) THEOREM. *Let A be an ample divisor on a non-singular variety M with* $\dim M\geqq 3$. *Let E be a vector bundle on A such that $H^2(A, \mathcal{E}nd(E)[-tA]_A)$ $=0$ for any $t>0$ and that $H^p(A, E[tA]_A)=0$ for any $t\in \boldsymbol{Z}$, $0<p<\dim A$. Then there exists a vector bundle $\tilde{E}$ on M such that $\tilde{E}_A=E$.*

For a proof, combine (2.1) and (3.1).

§ 4. Geometries on flag manifolds.

In this section, for the convenience of the reader, we review several results on Grassmann varieties which we use in the next section. Most of them are more or less known.

(4.1) We fix a vector space V over k with $\dim V=n$. Let $R=\{r_1, \cdots, r_h\}$ be a set of integers such that $n>r_1>r_2>\cdots>r_h>0$. Then we denote by $F_R(V)$, or by F_R as an abbreviated form, the flag manifold parametrizing the filtrations $0\subset V_1\subset\cdots\subset V_h\subset V$ of linear subspaces of V such that codim $V_j=r_j$ for $j=1, \cdots, h$. $F_{\{n-1, n-2, \cdots, 2, 1\}}$ is denoted by F. $F_{\{r\}}$ is a usual Grassmann variety $G_{n, n-r}$.

(4.2) For any subset S of R, we have a natural morphism $\Pi_{R/S}: F_R\to F_S$. It is easy to see that $\Pi_{R/S}$ makes F_R a fiber bundle over F_S with each fiber being isomorphic to a product of flag manifolds. It is also well known that $R^q(\Pi_{R/S})_*\mathcal{O}_{F_R}=0$ for $q>0$ and $(\Pi_{R/S})_*\mathcal{O}_{F_R}=\mathcal{O}_{F_S}$. Hence, for any vector bundle W on F_S, we have $H^p(F_S, W)\cong H^p(F_R, (\Pi_{R/S})^*W)$. We write W instead of $(\Pi_{R/S})^*W$ when there is no danger of confusion. In particular we have $H^p(F_R, W)\cong H^p(F, W)$.

(4.3) Let V_R denote the trivial vector bundle $V\times F_R$ over F_R. This has a natural filtration $0\subset E_{n-r_1}^*\subset E_{n-r_2}^*\subset\cdots\subset E_{n-r_h}^*\subset V_R$ of subbundles of V_R such that rank $E_j^*=j$. Putting $E_{r_j}=V_R/E_{n-r_j}^*$, we get a cofiltration $V_R\twoheadrightarrow E_{r_1}\twoheadrightarrow E_{r_2}$ $\twoheadrightarrow\cdots\twoheadrightarrow E_{r_h}\twoheadrightarrow 0$ of V_R with rank $E_j=j$. Obviously $(\Pi_{R/S})^*E_r=E_r$ for any $S\subset R$ and $r\in S$. Our notation is consistent with this fact.

(4.4) For any $i>j$ we have $E_{n-j}^*/E_{n-i}^*\cong\mathrm{Ker}\,(E_i\to E_j)$. This vector bundle is denoted by $E_{i/j}$. In particular, we set $E_{j/0}=E_j$ and $E_{n/j}=E_{n-j}^*$.

(4.5) Taking the dual of a cofiltration of V, we get a filtration of $V^\vee$. This induces an isomorphism $D: F_R(V)\to F_{R^*}(V^\vee)$, where R^* is the set $\{n-r_h, \cdots, n-r_1\}$. Note that $D^*(E_{n-r}(V^\vee))=(E_{n-r}^*(V))^\vee$ and $D^*(E_r^*(V^\vee))$

$=(E_r(V))^\vee$ for any $r\in R$.

(4.6) THEOREM. *Let* $R=\{r_1, \cdots, r_h\}$ *and* $S=R-\{r_j\}$ *and set* $r_0=n$ *and* $r_{h+1}=0$. *Then,* $(R^q\Pi_{R/S})_*(\mathcal{O}_{F_R}[E_{r_j/r_{j+1}}])=0$ *for* $q>0$ *and* $(\Pi_{R/S})_*(\mathcal{O}_{F_R}[E_{r_j/r_{j+1}}])\cong\mathcal{O}_{F_S}[E_{r_{j-1}/r_{j+1}}]$.

PROOF. First we consider the case in which $r_j=r_{j+1}+1$. Then $F_R\cong\boldsymbol{P}_{F_S}(E_{r_{j-1}/r_{j+1}})$ and $E_{r_j/r_{j+1}}$ is the tautological line bundle $\mathcal{O}(1)$. So the assertion is well known in this case. In the general case in which $r_j>r_{j+1}+1$, put $r=r_{j+1}+1$ and let $T=R\cup\{r\}$. Using the Leray spectral sequence $E_2^{p,q}\Rightarrow R^{p+q}(\Pi_{T/S})_*\mathcal{F}$ for $\mathcal{F}=\mathcal{O}_{F_T}[E_r/_{r_{j+1}}]$ such that $E_2^{p,q}=R^p(\Pi_{R/S})_*(R^q(\Pi_{T/R})_*\mathcal{F})$, we reduce the problem to the above special case.

(4.7) COROLLARY. *For any vector bundle* W *on* S, $H^p(F_S, E_{r_{j-1}/r_{j+1}}\otimes W)\cong H^p(F_R, E_{r_j/r_{j+1}}\otimes W)$.

(4.8) We denote $\det E_r$ by H_r. It is well known that H_r is very ample on $F_{\{r\}}$ and $\rho_{|H_r|}$ is the Plücker embedding of the Grassmann variety $F_{\{r\}}$. Clearly F_R is a submanifold of $F_{\{r_1\}}\times\cdots\times F_{\{r_h\}}$ and $H_{r_1}+\cdots+H_{r_h}$ is very ample on F_R. It is also well known that $\{H_r\}_{r\in R}$ gives an integral base of $Pic(F_R)$.

(4.9) Let K_R denote the canonical bundle of F_R. Then $K_R=-\sum_{j=1}^{h}(r_{j-1}-r_{j+1})H_{r_j}$, where $r_0=n$ and $r_{h+1}=0$.

To see this, note that $K_{\{r\}}=-nH_r$ since the tangent bundle of $F_{\{r\}}$ is canonically isomorphic to $Hom(E_{n-r}^*, E_r)$. We put $K_R=\sum_{j=1}^{h}\mu_jH_{r_j}$ since $Pic(F_R)$ is generated by $\{H_r\}_{r\in R}$. Let X be a fiber of $\Pi_{R/R-\{r_j\}}$. Then $X\cong F_{\{r_j-r_{j+1}\}}(V_j)$ for a vector space V_j with $\dim V_j=r_{j-1}-r_{j+1}$, $H_{r_j}|_X=H_{r_j-r_{j+1}}(V_j)$, $H_{r_i}|_X=0$ if $i\neq j$ and $K_R|_X=K_X=-(r_{j-1}-r_{j+1})H_{r_j-r_{j+1}}(V_j)$. This implies $\mu_j=-(r_{j-1}-r_{j+1})$.

(4.10) THEOREM. *For* $L=\sum_{j=1}^{n-1}\mu_jH_j\in Pic(F)$, *the following conditions are equivalent to each other.*

a) $\mu_j\geqq 0$ *for any* j.

b) $Bs|L|=\emptyset$. *Namely* L *is generated by its global sections.*

c) $|L|\neq\emptyset$.

PROOF. a) implies b) since H_j is very ample on $F_{\{j\}}$. Obviously b) implies c). Recall that $\Pi_j: F\to F_{\{n-1,\dots,j+1,j-1,\dots,1\}}$ makes F a $\boldsymbol{P}^1$-bundle and $H_i\{X_j\}=\delta_{ij}$ for any fiber X_j of Π_j. So c) implies $0\leqq LX_j=\mu_j$.

(4.11) COROLLARY. $H^0(F_{\{r\}}, E_r[-tH_r])=0$ *if* $t>0$.

PROOF. $H^0(F_{\{r\}}, E_r[-tH_r])\cong H^0(F_{\{r,1\}}, -tH_r+H_1)$ by (4.7). So (4.10) applies.

(4.12) REMARK. Suppose that $L\in Pic(F)$ satisfies the conditions a), b) and c) in (4.10). Let R be the set of j such that $\mu_j>0$. Then L comes from a very ample line bundle on F_R. Hence $\kappa(L)=\dim\rho_{|L|}(F)=\dim F_R$.

(4.13) THEOREM. *Suppose that* $L\in Pic(F)$ *satisfies the conditions in* (4.10). *Then* $H^p(F, -L)=0$ *unless* $p=\kappa(L)$.

For a proof, see Kempf [4, p. 328, Theorem 2]. If char $k=0$, this is a special case of Kodaira-Ramanujam's vanishing theorem.

(4.14) COROLLARY. *Let $L\in Pic(F_R)$ and suppose that $L-K_S$ is ample on F_S for some $S\supset R$. Then $H^p(F_R, L)=0$ for $p>0$.*

PROOF. Putting $q=\dim F_S-p$, we have $h^p(F_R, L)=h^p(F_S, L)=h^q(F_S, K_S-L)$ $=h^q(F, -[L-K_S])$. So (4.13) applies.

(4.15) COROLLARY. *$H^p(F_{\{r\}}, tH_r)=0$ for any $t\in\boldsymbol{Z}$, $0<p<\dim F_{\{r\}}$.*

PROOF. For $t<0$, (4.13) applies. For $t>-n$, (4.14) applies since $K_{\{r\}}=-nH_r$ by (4.9).

(4.16) LEMMA. *$H^p(F_{\{r\}}, E_{n-r}^*[tH_r])=0$ for $p>0$, $t>-r$.*

PROOF. Using (4.7) we infer $H^p(F_{\{r\}}, E_{n-r}^*[tH_r])\cong H^p(F_{\{r+1,r\}}, E_{r+1/r}[tH_r])=$ $H^p(F_{\{r+1,r\}}, H_{r+1}+(t-1)H_r)$. Hence (4.14) applies since $K_{\{r+1,r\}}=-(n-r)H_{r+1}$ $-(r+1)H_r$ by (4.9).

(4.17) THEOREM. *Suppose that $r\geqq 2$ and $n-r\geqq 2$, namely, $F_{\{r\}}$ is not a projective space. Then $H^p(F_{\{r\}}, E_r[tH_r])=0$ for any $t\in\boldsymbol{Z}$, $0<p<\dim F_{\{r\}}$.*

PROOF. $H^p(F_{\{r\}}, E_r[tH_r])\cong H^p(F_{\{r,1\}}, H_1+tH_r)$ by (4.7). So (4.14) applies if $t>1-n$, since $K_{\{r,1\}}=-rH_1-(n-1)H_r$. On the other hand, we have $h^p(F_{\{r\}}, E_r[tH_r])$ $=h^q(F_{\{r\}}, E_r^\vee[-(n+t)H_r])$ for $q=\dim F_{\{r\}}-p$. In view of (4.5), we infer $h^q(F_{\{r\}},$ $E_r^\vee[-(n+t)H_r])=h^q(F_{\{n-r\}}, E_r^*[-(n+t)H_{n-r}])$. Therefore (4.16) proves the assertion if $t<-r$.

(4.18) THEOREM. *Suppose that $n-r\geqq 3$. Then $H^2(F_{\{r\}}, \mathcal{E}nd(E_r)\otimes[-tH_r])=0$ for any $t>0$.*

PROOF. On F, E_r has a co-filtration $E_r\to E_{r-1}\to\cdots\to E_1$ such that $\mathrm{Ker}(E_i\to E_{i-1})\cong E_{i/i-1}$ (see (4.3) and (4.4)). This induces a filtration of $E_r^\vee$. Combining them, we obtain a double filtration of $\mathcal{E}nd(E_r)\cong E_r^\vee\otimes E_r$. Hence we easily see that it suffices to show the following

(4.19) CLAIM. *$H^2(F, E_{j/j-1}^\vee\otimes E_{i/i-1}\otimes[-tH_r])=0$ for any $t>0$, $1\leqq i\leqq r$, $1\leqq j\leqq r$.*

PROOF. We put $L=E_{j/j-1}^\vee\otimes E_{i/i-1}\otimes[-tH_r]=H_i-H_{i-1}-H_j+H_{j-1}-tH_r$. Consider the projection $\Pi: F_{\{r,r-1,\dots,1\}}\to F_{\{r-1,\dots,1\}}$ and let X be a fiber of it. Then $X\cong\boldsymbol{P}^{n-r}$ and $\deg(L_X)=-t+\delta_{ir}-\delta_{jr}$. Hence $R^p\Pi_*L=0$ for $p\leqq 2$ unless $t=1$, $i=r$ and $j<r$. Using the Leray spectral sequence we infer $H^2(F, L)=0$ in that case. If $t=1$ and $r=i>j$, we infer $H^2(F, L)=H^2(F, -H_{r-1}-H_j+H_{j-1})=0$ by a similar argument using the projection $F_{\{r-1,\dots,1\}}\to F_{\{r-2,\dots,1\}}$.

(4.20) THEOREM. *The natural mapping $H^0(F_{\{r\}}, E_r[tH_r])\otimes H^0(F_{\{r\}}, H_r)\to$ $H^0(F_{\{r\}}, E_r[(t+1)H_r])$ is surjective for $t\geqq 0$.*

PROOF. Note that $H^0(F_{\{r\}}, E_r[tH_r])=H^0(F_{\{r,1\}}, H_1+tH_r)$ by (4.7). So this is a special case of [4, p. 327, Theorem 1, (3)].

§ 5. Grassmann varieties as ample divisors.

(5.1) Let $G_{n,r}$ denote the Grassmann variety $F_{[r]}(k^n)$ as in § 4. If $r=1$ or $n-r=1$, then $G_{n,r}$ is a projective space. It is well known that $G_{4,2}$ becomes a hyperquadric by the Plücker embedding. In these cases $G_{n,r}$ is an ample divisor on a manifold. However, for other Grassmann varieties, we have the following

(5.2) THEOREM. *$G_{n,r}$ cannot be an ample divisor in any manifold M unless $(n, r)=(n, 1)$, $(n, n-1)$ or $(4, 2)$.*

PROOF. We assume $G=G_{n,r}$ to be an ample divisor on a non-singular variety M. Assuming $r\neq 1$, $n-r\neq 1$ and $(n, r)\neq(4, 2)$, we will derive a contradiction. Since $G_{n,r}\cong G_{n,n-r}$, we may assume that $n-r\geqq 3$ and $r\geqq 2$. Put $L=[G]\in Pic(M)$. Note that $L_G=mH_r$ for some $m>0$. By (4.18) and (4.17) we can apply (3.9) in order to extend the vector bundle E_r to a vector bundle E on M. By (1.2), (4.11) and (4.17) we obtain $H^p(M, E[-L])=0$ for $p=0, 1$. Hence $H^0(M, E)\rightarrow H^0(G, E_r)$ is bijective. Similarly by (1.2) and (4.15) we infer that $H^0(M, L)\rightarrow H^0(G, L)$ is surjective. Moreover, the natural mapping $H^0(G, E[tL])\otimes H^0(G, L)\rightarrow H^0(G, E[(t+1)L])$ is surjective for $t\geqq 0$ by (4.20). Hence (1.4) applies and we see that E is generated by its global sections by (1.5). Since $h^0(M, E)=h^0(G, E_r)=n$, we get a morphism $\Pi: M\rightarrow G_{n,r}=G$ such that $\Pi^*E_r=E$. It is easy to see that the restriction of Π to G is the identity. So $Pic(M)\rightarrow Pic(G)$ cannot be injective by (1.7). This contradicts [**2**, (2.5)] since $H^1(G, -tL)=0$ for any $t>0$ by (4.15).

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