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Fourier analysis, by T. W. Körner. Cambridge University Press, Cambridge, 1988, xii + 591 pp., $95.00. ISBN 0-521-25120-6

When Arthur Dent, fortified by a prophylactic glass of beer, set off in ultimate pursuit of Life, the universe and everything, he was armed with only a towel. He was no braver than Tom Körner. This is truly an ambitious voyage through Fourier Analysis. Tom has real armour as a harmonic analyst of considerable personal accomplishment. Yet in both cases, the coping mechanism is the same—a kind of gentle English silliness which amuses, irritates, and finally enchants.

Let's go back to the beginning. The declared assignment is to provide a shop window for Fourier Analysis in a textbook which can be understood by a British undergraduate who possesses that knowledge which can be "supposed after two years of study." (A word of warning: the author teaches at the University of Cambridge where quite a lot is supposed to happen.) It follows that there are some bread and butter issues on which we must agree. What precise mathematical background is to be assumed, how do we organize the material so as to incorporate historical perspective, and which subject matter do we choose from the vast treasure house of Fourier Analysis?

The first practical decision on mathematical background concerns Lebesgue integration. Although Hardy wrote in 1922 that "No account of the theory of Fourier's series can possibly satisfy the imagination if it takes no account of the ideas of Lebesgue; the loss of elegance and of simplicity of statement is overwhelming" there still appears to be great reluctance to introduce these ideas early. Dr. Körner goes out of his way to avoid the Lebesgue integral (although he is obliged to define a null set in order to state Carleson's convergence theorem) and, it must be admitted, does so in a thoroughly sensible way. He concentrates wherever possible on continuous functions with finite integrals and even labels that class

$L^1 \cap C$ so the reader will find no future notational inconsistency. More importantly he eschews fussy results with detailed side conditions and goes for the broad generic statement. The worrisome thing is that our student reader might just end up with "Fear of Flying" Lebesgue-style. Lebesgue's integral appears in an appendix labelled "Weakening hypotheses," where Hardy is quoted as above (immediately after a traditional misprint called Reimann!) and where it is stated that "measure theory is subtle rather than clever and so requires hard work to master." Yet these comments come in a book which gives a careful discussion of nondifferentiable Brownian paths and sketches Burgess Davis' proof of Picard's theorem! Where is the message that probabilistic intuition and measure theory reinforce each other and that the Lebesgue integral is to be preferred because it is so beautifully *simple* to use? No matter—a reasonable retort might be that attention must not be deflected from Fourier Analysis itself. Witness the unconscionable amount of space which has been devoted in this review to the side issue of integration!

Let us agree to respect the author's choice and discuss some other aspects of mathematical background. The major assumption is a certain maturity with respect to rigorous analytic arguments. The author has a clear straightforward style but brooks no compromise on matters of accuracy of presentation by the standards of pure mathematics. He is also confident of his reader's desire to master the material rather than merely sample its flavour. That said, he has the good sense and good taste to vary the depth of treatment to suit the local discussion. He reminds us of Aristotle's advice that "it is the mark of the educated mind to use for each subject the degree of exactness which it admits," and reminds us both explicitly and by example.

Körner does not believe that history is bunk, but he notes that "a glance at the average history of mathematics shows that mathematicians are remarkably incompetent historians." This harsh judgement appears to contain the questionable implication that the average history of mathematics is written by a mathematician and perhaps he really has some specific examples in mind. In any event there is no "capital H" history here and the author's preferred choice as a secondary source is Herivel's biography "Joseph Fourier." Herivel explicitly divides his book in two parts: the first concerns the man and in the second he is "only concerned with Fourier the mathematician in so far as this is necessary for an understanding of Fourier the physicist." (Incidentally the gremlins struck again and the first formal reference to Herivel's book has the author as Herival.)

Our book then does not start with a historical summary nor does it end with one. It is not a chronological development of Fourier Analysis but rather a collection of some six extended essays on the subject; separate but of course interconnected. Stop a while! These remarks are potentially misleading. One of the great strengths of the book is the author's delight in the apposite historical anecdote, his skill in providing natural contextual background for the mathematical discussion. This is particularly important in a field such as Fourier Analysis which has been developed over several mathematical generations. For example how many students of mathematics are aware of Tchebychev's deep interest in Watt's design of mechanical linkages and the manner in which this informs the discussion of best uniform approximation? How many know that Airy's record as a tipster falls far behind that of Hot Horse Herbie?

We come to the choice of subject matter. Some general areas demand representation. We cannot conceive of a book on Fourier Analysis which does not discuss summability kernels and Fourier series, orthogonal expansions, or the Fourier transform. These topics are indeed the subject matter of three of the extended essays in Körner's book (Parts I, III, IV). The remaining core part (Part II) is entitled "Some Differential Equations," while the remaining two essays take up later work in the field. Roughly speaking Part V selects topics which can be seen as developing forth from what has already been discussed while Part VI chooses some material which shows Fourier Analysis moving in new directions. ('New' is of course a relative concept so that Dirichlet's use of L-series constitutes a new idea in this sense.)

Part II is a good idea. It is very much in the spirit of Fourier to discuss the time evolution of systems determined by second order ordinary differential equations. Liapounov stability, resonance, and the mysterious qualitative behaviour of nonlinear systems are all good stuff and highly appropriate in context. Nevertheless this turned out to be the least satisfactory part of the book for the present reviewer who found the treatment too pure. One is reminded in another part of the book of some sage advice of Heaviside. "The practice of eliminating the physics by reducing a problem to a purely mathematical exercise should be avoided as much as possible. The physics should be carried on right through, to give life and reality to the problem..."

Part III is a lovely treatment of orthogonal series and could well have come first. Indeed the author is at such pains to play down pointwise convergence of Fourier series in Part I that he makes an excellent case for delaying all discussion of infinite series! The real business of Part I is to show how finite trigonometric sums can be used to model physical phenomena. This is possible because of linear superposition of waves (an evocative passage by Helmholtz is quoted at length on p. 24) and because amplitudes corresponding to different frequencies can be picked out from the real sum h by computation of the limit, as $T \to \infty$, of

$$2T^{-1}\int_S^{S+T} h(t)\cos wt\,dt, 2T^{-1}\int_S^{S+T} h(t)\sin wt\,dt$$

for fixed S. (The author does not miss the opportunity to relate this to Kelvin's development of the harmonic analyzer.) Fourier series hang around in the background but are not of compelling interest at this stage of the game.

The treatment of the Fourier transform is admirably clear, careful, and simple. Moreover we discover its relevance to Kelvin's involvement with cable-laying entrepreneurs. We also become chastened after imagining that physical constraints would ensure a unique solution of the heat equation.

The author's reaction to this "great blast of heat from infinity" is revealing. "To the applied mathematician [this example] is simply an embarrassment reminding her of the defects of a model which allows an unbounded speed of propagation. To the numerical analyst it is just a mild warning that the heat equation may present problems which the wave equation does not. But the pure mathematician looks at it with the same simple pleasure with which a child looks at a rose which has just been produced from the mouth of a respectable uncle by a passing magician."

The choice of topics in Parts V, VI is freer so leaves more scope for argument. We might have expected to find spectra, the uncertainty principle, imaging and tomography. We *do* estimate the diameter of stars, speculate on the probity of Sir Cyril Burt (having developed the chi-squared test), and we contemplate public key encryption. Why argue? In every case Dr. Körner has something interesting to say.

We have covered the main strategic decisions which were open to the author. Tom Körner is fully aware that Fourier Analysis has been elaborated over two centuries and so has lost that overall immediacy of contact with the outside world which can act as a substitute for mathematical taste. Accordingly he has selected the main theorems and reproduced them in plain simple versions. He has used his personal interest to choose supplementary material and has been careful to present that in a physical and historical context. Above all he has insisted on a strong story line, and done that successfully. Let's return to the first paragraph and the question of style.

Our author is fond of the device of almost—but not quite—addressing his reader. "Continuing along this line of thought the reader will recall the following theorem. (If the reader has forgotten the proof she will find it in Chapter 53 (Lemma 53.2).)" These coy asides always use feminine pronouns; an idiosyncracy which can eventually become somewhat tiresome. (I hear a shrill voice cry "Precisely!".) He also enjoys being silly. The index has a self-reference for the Vladivostok telephone directory. Moreover, under Oxford, appears the entry "breakaway technical college somewhere in the fens (see Cambridge)." There are other little jokes sleeping there but the reader may discover these for herself.

Sometimes the dominie in Körner takes over and his reader is required to eat up all her porridge. "Before leaving this chapter the reader should convince herself that any two of Theorems 34.1, 34.2 and 34.4 can readily be deduced from the third" or "To understand this proof fully the reader must be clear in her own mind why we needed Lemma 44.5 . . . " His collar can be a little stiff. "We can measure in the contrast between the crude automatic mechanisms of the miller of 1770 and the sophisticated electronics that will back tomorrow's fighter pilot both the technical and the moral progress that mankind has made in the last 200 years." Generally, however, he retains his sense of fun like his hero Maxwell. Körner relates the fine tale of Maxwell's zoetropic leg-pull of Kelvin and quotes at length Maxwell's parody of Tennyson. Since much of Tennyson is close to self-parody, he might usefully have quoted Maxwell's parody of his fellow-countryman Burns. After declaring an intention to discuss Engineering

Mechanics by opening with the lines

"Gin a body meet a body
Flyin' through the air"

Maxwell played the tartan savage by claiming that

"Ilka problem has its method
By analytics high;
For me, I ken na ane o'them
But what the waur am I?"

Perhaps the greatest artifice in the book is the assumption (that we have so far taken at face value) that it is addressed to an undergraduate desirous of learning Fourier Analysis. That may have been the intent, but the result is an elegant and informative pastiche which should give pleasure to a very wide audience including even card-carrying harmonic analysts. Readers will enjoy the author's quiet sense of fun but should be aware that his didactic purpose is deadly serious. Recalling his reaction to the great blast of heat from infinity; it may well be that we shall never know the name of the rose, but I venture to suggest that the respectable uncle is called Dr. Körner.

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Representations of ∗-algebras, locally compact groups, and Banach ∗-algebraic bundles, by J. M. G. Fell and R. S. Doran. Academic Press, San Diego, 1988. vol. 1, *Basic representation theory of groups and algebras*, xviii + 746 pp., $99.00. ISBN 0-12-252721-6. vol. 2, *Banach ∗-algebraic bundles, induced representations, and the generalized Mackey analysis*, viii + 740 pp., $99.00. ISBN 0-12-252722-4

The theory of group representations has a long history in Mathematics and in Mathematical physics. It has its roots in two lines of mathematical thought. The first concerns the theory of Fourier series and the desire to extend these results, first to noncompact locally compact abelian groups, then to nonabelian compact groups, and finally to general locally compact groups. The second line centers around invariant theory, the Klein Erlanger program, and vector and tensor analysis. A historical account of the latter line may be found in **[13]**. In addition, a long motivational discussion concerning all these areas may be found in the first volume of the work being reviewed.

Let G be a locally compact group whose representation theory one wishes to study. Let λ be a Haar measure on G. The Banach space $L^1(G)$