solutions for abstract parabolic equations by a different method. The author applies these results to semilinear parabolic equations and the Navier-Stokes equations. This provides another proof for regularity criteria in Chapter IV. The major part of Chapters IV and V is based on recent results of the author and Sohr. Each chapter has a section for comments on related results which is very helpful for the reader. Most of prerequisites are given in the book, with or without proof. Finally, the reviewer acknowledges that the book is well proofread, although there are a lot of complicated formulas with sub- and superscripts.

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Differential geometry and topology, by A. T. Fomenko; translated by D. A. Leites. Consultants Bureau, New York and London, 1987, xiii + 323 pp., $ 75.00. ISBN 0-306-10995-6

Most mathematicians soon acquire the habit of scribbling diagrams and drawing pictures when reading mathematics to help them follow the argument. This book, however, contains so many useful and attractive diagrams (234 figures in 323 pages) that scribbling is much less necessary than usual. It is a book for readers who like thinking in pictures better than following a proof through line by line. Explanations are stressed more than formal proofs, theorems which are not needed but are considered interesting are stated without proofs, and useful methods are illustrated by the presentation of important special cases rather than by the development of the most general theory. This makes the book easier to read for nonspecialists who want to gain a good understanding of the material and the ability to apply differential geometry and topology to problems in pure and applied mathematics. However, having no index, it is less satisfactory as a work of reference.

The heart of the book is the study of critical points and level sets of smooth functions on manifolds and their relation to the topology of manifolds. The author goes on to consider several rather different areas of differential geometry: the topology of three-dimensional manifolds, the geometry of Lie groups, Lie algebras and symmetric spaces, and lastly symplectic geometry leading into mechanics through Hamiltonian systems. Although most of the book concerns pure mathematics the reader is reminded of its relevance to mechanics and theoretical physics, and the final chapter looks at applied problems such as the motion of a solid body in an ideal fluid.

A smooth real-valued function with nondegenerate critical points (a "Morse function") on a compact manifold X can be used to give X the structure of a CW complex. A CW complex is a topological space with a decomposition as a disjoint union of subsets, called cells, each homeomorphic to an open unit ball in Euclidean space of some dimension. The topology of the CW complex is built up inductively: a cell is "glued" to the union of the cells of lower dimension by a continuous map from the boundary of the appropriate unit ball in Euclidean space. Any manifold can be made into a CW complex, but so can spaces which are not necessarily manifolds—for example level sets of smooth functions on manifolds. From the description of a space as a CW complex one can obtain information about topological invariants such as its homology groups. A simple example is the fact that the dimension of the kth real homology group is at most the number of k-dimensional cells. Much more subtle information than this can also be obtained.

Given a Morse function f on a compact manifold X, one can make X into a CW complex whose cells correspond to critical points of f by putting a Riemannian metric on X and considering the downward trajectories of the gradient flow of f on X. The cell corresponding to a critical point p is the set of all points in X whose downward trajectories converge to p (and the codimension of this cell is called the index of p). It is an important fact that on any compact connected manifold there exist "admissible" Morse functions, i.e., Morse functions with exactly one minimum and one maximum and whose value at any critical point is the index of that point. This implies several topological properties of compact manifolds not held by arbitrary compact CW complexes. For example if X is an oriented compact n-dimensional manifold there is a perfect pairing between its real homology groups of dimensions k and $n - k$ for any k ("Poincaré duality"). This can be seen by comparing a suitable Morse function f with the Morse function $-f$.

The existence of admissible Morse functions on a compact connected manifold X can also be used to show that if X is three-dimensional and oriented then X has a Heegaard splitting of some genus g. That is, X is the union of two compact submanifolds-with-boundaries X_1 and X_2 such that $\partial X_1 = \partial X_2 = X_1 \cap X_2$ is a two-dimensional submanifold of X diffeomorphic to a sphere with g handles M_g, and X_1 and X_2 are both diffeomorphic to the corresponding "solid sphere with g handles" H_g (often called a handlebody). If f is an admissible Morse function on X then its critical points all have indices $0, 1, 2$, or 3 and if $1 < \alpha < 2$ then

$$X_1 = f^{-1}([0, \alpha]), \qquad X_2 = f^{-1}([\alpha, 3])$$

defines a Heegaard splitting of X.

Using the Heegaard splitting one finds that any compact connected oriented three-manifold can be obtained from two copies of a handlebody H_g glued together by some diffeomorphism of its boundary M_g. When $g \leq 1$ the different three-manifolds obtained in this way are easy to describe. However in general the question of when two diffeomorphisms $F\colon M_g \to M_g$ and $\tilde{F}\colon M_{\tilde{g}} \to M_{\tilde{g}}$ give rise to the same three-manifold is an extremely difficult one. It was proved in 1980 that when $g = 2$ there is an algorithm for deciding when a diffeomorphism $F\colon M_g \to M_g$ gives rise to the standard three-sphere [**1**] but this no longer works when $g \geq 3$.

A rather different area of differential geometry considered in some detail in this book is the theory of Hamiltonian systems, which arose from the study of systems of differential equations in classical mechanics. To say that a differential system is Hamiltonian means that on the "phase space" M of the system there is a smooth real-valued function H and a symplectic form w (i.e., a closed nondegenerate 2-form) such that if locally we choose coordinates

$$(p_1, \ldots, p_n, q_1, \ldots, q_n)$$

on M such that

$$w = \sum_{1 \leq i \leq n} dp_i \wedge dq_i$$

then the system is given locally by the equations

$$\partial p_i / \partial t = \partial H / \partial q_i, \qquad \partial q_i / \partial t = -\partial H / \partial p_i.$$

Then the Hamiltonian function H is an integral of the system, in the sense that it is constant along trajectories. The system is said to be completely integrable in the sense of Liouville [**2**] if there are smooth functions $H_1 = H$, $H_2, \ldots, H_n$ on M whose gradients are linearly independent almost everywhere, and $\{H_j, H_k\} = 0$ where

$$\{H_j, H_k\} = \sum_{1 \leq i \leq n} (\partial H_j / \partial q_i)(\partial H_k / \partial p_i) - (\partial H_j / \partial p_i)(\partial H_k / \partial q_i)$$

in local coordinates. This implies that $H_1, \ldots, H_n$ are integrals of the system, and that for generic $a = (a_1, \ldots, a_n)$ the submanifold

$$M^a = \{x \in M\colon \; H_i(x) = a_i, \; 1 \leq i \leq n\}$$

is of the form $\mathbf{R}^n / \Lambda$ where Λ is a lattice, with coordinates $\varphi_1, \ldots, \varphi_n$ on $\mathbf{R}^n$ evolving *linearly* in time. Thus the set of integrals $H_1, \ldots, H_n$ (called a "full commutative set of functions on M") together with the "angle variables" $\varphi_1, \ldots, \varphi_n$ lead to a complete solution of the system.

Of course in general finding all the integrals of a Hamiltonian system is extremely hard. However it is possible to solve several important Hamiltonian systems by relating them to the theory of Lie groups, Lie algebras and symmetric spaces. A Lie group G acts on the dual g^* of its Lie algebra and every orbit in g^* has a canonical symplectic structure. It is conjectured in general and proved in many special cases that there are smooth functions $f_1, \ldots, f_k$ on g^* which restrict to a full commutative set of functions on generic orbits in g^*. Thus the Hamiltonian systems on the orbits defined by any of the functions f_i can be solved completely. In the last chapter of his book Fomenko shows how several differential systems arising from classical mechanics (such as the motion of a solid body in an ideal fluid) can be solved in this way by virtue of their equivalence to Hamiltonian systems on orbits in duals of Lie algebras.

Unfortunately this book suffers from having been poorly translated from Russian. Indeed a basic knowledge of the Russian language makes reading easier. In most cases the poor translation is merely a source of amusement or at worst irritation. However some errors are more serious, such as when the phrase "*must not be*" is used where "*is not necessarily*" is meant. These errors are likely to cause unnecessary confusion to those at whom the book is aimed, that is, graduate students and nonspecialists learning the subject for the first time.

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Pseudodifferential operators and spectral theory, by M. A. Shubin. Translated by Stig I. Andersson. Springer-Verlag, Berlin, Heidelberg, New York, 1987, x + 278 pp., $55.00. ISBN 3-540-13621-5

The publishers are to be commended for making this text by M. A. Shubin accessible to the mathematicians who do not read Russian. It contains a fairly short, yet highly readable account of pseudodifferential, and Fourier integral, operator theory, with extensive applications to the spectral theory of linear elliptic equations. Let me say right away that any mathematician tempted to give a first course in the subject of ΨDO and/or FIO should give serious consideration to Chapter I, plus Appendix I, of Shubin's book as a possible text. They present most of what is needed for a basic understanding of the theory in a style that is simple yet precise. They are independent of the other chapters, to which the reader might want to go for significant applications and extensions.

There is nothing novel in the application of pseudodifferential operators to elliptic problems. Elliptic problems are one of the main sources from which the