VI. Euler's recursion formula for the sum of the divisors of n. VII. Mathematical induction. Some non-routine applications, such as the following one: "If the polygon P is convex and contained in the polygon Q, the perimeter of P is shorter than the perimeter of Q." VIII. Maxima and minima, with, again, several ingenious, non-routine applications. IX. Minimum principles from optics and mechanics. X. The isoperimetric problem. XI. Miscellaneous problems. Sample: if the intersection of a solid sphere with a solid cylinder whose axis passes through the center is removed from the sphere, find the volume of the remainder in terms of the radius of the sphere and the height of the cylindrical hole.

The second volume is quite different from the first. The problems occupy less space (less than a third of the volume) and play a less important role. Their character is also different. Sample: "Check Heron's formula [for the area of a triangle in terms of the sides] as many ways as you can." Chapter XII discusses the "fundamental inductive pattern" and some of its variations. (If A implies B, then the discovery that B is true makes A more credible.) There is an interesting discussion of the author's teaching methods, centered around the formula for the area of the lateral surface of the frustum of a right circular cone. Chapter XIII continues in the same vein. By way of an example there is a long discussion of judicial proof with many details of a murder case. The best part of the volume for the mathematician is about a dozen pages (in the examples and comments on Chapter XIII) that discuss some of Pólya's own work with historical and psychological side lights. Chapters XIV and XV concern probability, mostly from a non-mathematical point of view. The main emphasis of the last chapter, Chapter XVI, is on the applications of the preceding discussion to pedagogic methods.

Neither volume has an index; there is, instead, a very detailed analytical table of contents. The physical appearance of the volumes is excellent. The style throughout is informal and charming.

PAUL R. HALMOS

P.S. The sequence 11, 31, 41, 61, 71, 101, 131, $\cdots$ consists of the primes ending in 1.

Elements of algebra. By Howard Levi. New York, Chelsea, 1954. 160 pp. $3.25.

This book is addressed to beginning students of mathematics; it is, roughly speaking, a text on freshman algebra. It is a text on freshman algebra in the sense that it talks about the removal of parentheses and the solution of quadratic equations (among other things). The level of the book, however, is so unusually high, mathematically as well as pedagogically, that it merits the attention of professional mathematicians (as well as of professional pedagogues) interested in the wider dissemination of their subject among cultured people. The book has its faults. The terminology, the exposition, the statements of the theorems, and the choice of subjects included and excluded are not always perfect. Despite its faults, however, the book is a closer approximation to the right way to teach mathematics to beginners than anything else now in existence.

The quickest way to summarize the contents of *Elements of algebra* is to say that it is *Grundlagen der Analysis* for freshmen. (The reference is, of course, to Landau's well-known derivation of the real number system from the Peano axioms.) Levi (unlike Landau) starts by describing some of the terminology of elementary set theory; he goes on to introduce the symbols $\cup$ and $\cap$ and to give a precise definition of the concept of function. The second chapter introduces what are unfortunately called *cardinal numbers*. (Perhaps *natural numbers* would have been more in accord with custom and less likely to confuse the student who subsequently finds out that infinite cardinals exist also.) The definition is along the lines of von Neumann's theory of ordinals. The number 0 is the empty set, the number 1 is the singleton $\{0\}$, the number 2 is the pair $\{0, 1\}$, etc. Based on this definition the elementary arithmetic operations are defined and their properties are derived.

Chapters III, IV, and V are a digression from the Landau program. In Chapter III *expressions* are defined (they are "well-formed formulas" involving cardinal numbers, variables, addition, multiplication, and parentheses), and a concept of *equality* is introduced for expressions (two expressions are equal if they represent the same function of numbers). Chapter IV defines a *monomial* as an expression not involving addition and a *polynomial* as a sum of monomials. The theorem that every expression is equal to some polynomial is offered as the meaningful substitute for the mysterious process of "simplifying" expressions by "removing parentheses." Chapter V concerns the unusual concept of a *number system*; this is defined to be a set with two commutative and associative binary operations satisfying one distributive law.

In Chapters VI, VII, and VIII the integers and the rational numbers are constructed by the usual method of ordered pairs. Chapter IX discusses equations; it includes the factor theorem and the solution of quadratic equations. Chapters X and XI construct the real number system (by means of infinite decimals) and prove that it has the upper-bound-property. An appendix makes brief mention of a few related matters (such as infinite cardinals and the definition of a group).

The book contains many striking phrases designed to eliminate some common sources of confusion. Here are two examples. (1) "It should be stated emphatically that there is nothing undesirable about parentheses. Our efforts at removing them are directed toward devising a test to determine if two given expressions are equal." (2) "The reader is urged to distinguish between his everyday use of the words [*greater* and *less*] and the mathematical one; otherwise he will not be impressed by their similarities nor take seriously their differences."

There are also examples of bad didactic technique. They are all of the same type: some difficult concepts are not sufficiently motivated. Thus the empty set is introduced without so much as a by-your-leave and the possibility of a set being a member of itself is casually (and needlessly) referred to. The construction of the integers via ordered pairs is introduced by the following sentence: "The reader who finds our definitions arbitrary and somewhat bizarre should be informed that they are the product of a long evolution of which our exposition gives only the final stage." (The patronizing tone of this sentence recurs frequently.) The fact that the author is thinking of (7, 11) as $7-11$ is kept a secret from the reader.

Mathematically, the author's treatment of statements involving variables ("if x is even, then $x+1$ is odd") is probably unobjectionable but certainly peculiar. The assertion that "we shall only accept those statements that are definitions and those statements that can be proved to be logical consequences of the definitions" is somewhat startling, to say the least.

Enough has been said to communicate the flavor of the work. The book can be useful to a beginner as an outline of territory whose detail maps are not available to him. As such, the reviewer recommends it, but he recommends also that an experienced guide be taken along on the tour.

PAUL R. HALMOS

Theory of games and statistical decisions. By D. Blackwell and M. A. Girshick. New York, Wiley, 1954. 12+355 pp. $7.50.

Statistical decision theory originated in Wald's 1939 paper (Ann. Math. Statist. vol. 10, pp. 299–326), whose interest is now almost purely historical. It was designed to embrace all problems of statistical inference which are the raison d'être of statistics; with inessential modifications it still does. In its simpler form the statistician has to