Funktionentheorie. By C. Carathéodory. 2 vols. Basel, Birkhäuser, 1950.

The appearance of a book by a master of the stature of Carathéodory which is concerned for the most part with the classical aspects of a classical subject is an occasion of great interest. This is particularly so in the present case because it is reported that Carathéodory himself regarded his *Funktionentheorie* as his finest achievement. In apparaising this work it is well to recall that on the one hand, Carathéodory was a mathematician of very broad interests and on the other hand the theory of functions of a complex variable was an ever returning theme in his research.

It is therefore natural to anticipate that the present book would be written from a catholic point of view, that the treatment of general questions of convergence, continuity, and so on, would have a strong "real variable" flavor. This is indeed the case. Throughout the book there is constant reference to his *Reelle Funktionen*. One meets everywhere striking formulations of concepts which conventionally are phrased in other ways. For this reason the *Funktionentheorie* will be of considerable interest to the specialist who likes to compare notes.

Carathéodory's contributions to the theory of analytic functions are many and of lasting importance. We need call to mind only his early work on the Picard theorem which in turn led to the study of the coefficient problem for analytic functions with positive real part, the boundary behavior of conformal maps, the conformal mapping of variable regions, to mention but a few of his contributions. These interests are reflected in the second volume where the theory of bounded analytic functions, conformal mapping, the triangle functions and the Picard theorem are treated.

The author places great emphasis on the geometric point of view. The Weierstrassian aspects find less prominence than is often the case; instead the Riemannian aspect dominates. In fact, to lend point to this geometric tendency, some sixty pages of the beginning of the book are devoted to the geometry of the circle and non-euclidean geometry. In the introduction, Carathéodory says that he regards this chapter of geometry as the best entry to the theory of functions of a complex variable and he cites the role that these methods played in the achievements of Schwarz. In the treatment of this material synthetic and analytic methods are interwoven.

The first volume is divided into five parts. The first of these is concerned with the geometry of the circle, as we have just mentioned. The second part gives a brief resumé of material on convergence, continuity, connectedness, the topology of the plane, line integrals.

Emphasis is placed on the chordal metric and the unity it lends to the study of meromorphic functions.

The third part is concerned with the classical Cauchy theory and its immediate consequences. The maximum principle enters early and is used to establish the interiority of the mapping defined by an analytic function in a manner which is modeled on G. Kowalewski's treatment of continuously differentiable point transformations with nonvanishing Jacobian. The Poisson integral is then studied and the relation between harmonic and analytic functions is developed. In the definition of harmonicity, differentiability demands are reduced to a minimum. This part concludes with an introductory account of meromorphic functions, partial fractions, Liouville's theorem, and the fundamental theorem of algebra.

The author's concept of continuous convergence plays a central role in the fourth part: the generation of analytic functions by limiting processes. Normal families of meromorphic functions are considered and here the theorems of Vitali and Osgood find their place. A short chapter is devoted to power series. The Mittag-Leffler theorem and the calculus of residues terminate the section.

The elementary transcendental functions and the gamma function are the concern of the fifth part. The elementary functions are studied with reference to their classical expansions and the mappings they define. Mention should be made of the elegant treatment of the partial fraction expansions of the cotangent which was communicated orally to the author by Herglotz.

The second volume contains parts six and seven. Part six, *Foundations of geometric function theory*, treats the theory of bounded analytic functions, conformal mapping, and behavior of the mapping function on the boundary. Here one finds the Schwarz-Pick lemma with applications, the Julia lemma, Fatou's theorem (Carathéodory's 1912 proof), the Riemann mapping theorem, the Koebe distortion theorem. The general theory of prime ends is not considered but the mapping of Jordan regions is studied in detail.

The final part, which is introduced by a chapter on functions of several complex variables and differential equations, gives a detailed account of the triangle functions. The Picard theorems are then developed via Landau and Schottky. An account of Carathéodory's 1947 paper (Comment. Math. Helv. vol. 19, pp. 263–278) on the cluster values of meromorphic functions completes the book.

Carathéodory did not aim at writing a comprehensive treatise of modern geometric function theory. Many topics such as the general theories of Riemann surfaces and uniformization are frankly omitted.

On the other hand, it would appear that Carathéodory wished to revive certain classical aspects of the theory of analytic functions which generally do not receive much attention nowadays.

Here is a book which will be of permanent interest not only to the specialist but to all who are inclined to graze in function-theoretic pastures.

MAURICE HEINS

Lezioni de geometria moderna. Vol. 1. *Fondamenti di geometria sopra un corpo qualsiasi.* By B. Segre. Bologna, Zanichelli, 1948. 4+195 pp. 1200L.

This admirable little book comprises a course given by the author at the University of Bologna. It will be followed by two volumes devoted, respectively, to non-linear projective geometry and invariants of birational transformations.

Since an objective is to have the basis of (projective) geometry reflect the great generality achieved in recent years by abstract algebra, almost half of the 180 pages of text (twelve of the seventeen chapters) are exclusively algebraic. In a rapid but clear manner the reader is presented with the essentials of residue classes of integers, groups, rings, corpora and fields, homomorphisms, sub-rings and ideals, zeros and decomposability of polynomials, algebraic and transcendental extensions of fields, finite corpora, and Galois fields.

The word "corpus" (plural, corpora) requires an explanation. The author avoids the contradiction in terms current in English (and other languages) that refers to an algebraic structure which has all the properties of a field except that commutativity of multiplication is not assumed (and may even be denied) as a *noncommutative field.* He calls such a structure a corpus (corpo) and reserves "campo" for a field. The reviewer feels that this terminology might well be generally adopted.

The algebraic preliminaries disposed of, the remaining five chapters proceed at a still brisk but somewhat slower pace. In Chapter 13 a *(right) linear space over a corpus* γ is defined as a set S of "points" in which certain subsets (subspaces) are distinguished, and the following two properties subsist.

I. There is a one-to-one correspondence between the points ξ of S and ordered (right) homogeneous n-tuples $(x_1, x_2, \cdots, x_n)$ of elements of γ (not all zero); that is

$$\xi \sim (x_1, x_2, \cdots, x_n) = (x_1c, x_2c, \cdots, x_nc) \neq (0, 0, \cdots, 0), \qquad c \in \gamma.$$

II. A subspace S' of S consists of all points ξ of S representable by