$[\alpha],\beta$ is the proposition, "α implies β," expressed in *Principia* as $\alpha x \supset_x \beta x$. It is perhaps worth while to observe that [ ] is a propositional function of two variables, not in the sense of Quine, but in the sense of Schönfinkel, since, if α is a propositional function of one variable, $[\alpha]$ is a propositional function of one variable.

There is no slur on the invaluable pioneer work of Whitehead and Russell when it is said that their system is unsatisfactory from the viewpoints of formal definiteness and of mathematical elegance. The work of Quine is in both respects an important improvement over the system of *Principia*, and, although open to criticism in certain directions, is probably not too highly praised by Whitehead when he calls it, "A landmark in the history of the subject".

ALONZO CHURCH

AMERICAN MATHEMATICS BEFORE 1900

A History of Mathematics in America before 1900. By David Eugene Smith and Jekuthiel Ginsburg. (The Carus Monographs, No. 5.) Mathematical Association of America, 1934. x+209 pp.

The Committee on the Carus Monographs had a happy inspiration when it was led to induce Professor Smith to prepare this history. He was in every way qualified for the task—through his unique knowledge of the subject, through his attractive literary style, and through the excellence of his judgment in dealing with a great mass of material and in presenting its essence in well-balanced and compact form. All of these qualities are very much in evidence in the little volume under review. Only one who has had considerable experience in such matters can truly appreciate the great amount of research which went into the preparation of the manuscript. In this research Professor Smith had the valuable assistance of Professor Ginsburg of Yeshiva College, the editor-in-chief of Scripta Mathematica.

For the purposes of the history "America" was roughly considered as the territory north of the Caribbean Sea and the Rio Grande River. In 1938 fifty years of activity of the American Mathematical Society will be celebrated, and a number of scholars will doubtless cooperate in presenting a historical picture of each of the fields of American mathematics during that period. Such a survey, and the complementary work under review, will thus give an up-to-date panorama of outstanding mathematical activities of the past. The importance of these activities after 1875 for the extraordinary development in the twentieth century will be assessed, and Professor Smith's delineation of milestones of earlier progress will be recalled.

In the sixteenth and seventeenth centuries the mathematical needs of the early American settlers were few, and even at Harvard and William and Mary Colleges, nothing noteworthy was done. Astronomical observations were made to a certain extent, and almanacs prepared; astrologers were by no means unknown. "The century that saw the work of Galileo, Kepler, . . . ,* Napier,

* The name "Gilbert" occurred here in the original sentence (p. 13). The reviewer is unequal to guessing to whom it was intended to refer.

Fermat, Descartes, Pascal, Huygens, Newton and Leibniz, in countries from which the settlers had come, saw among the intelligentsia no apparent appreciation of the discoveries of scholars of this class."

The eighteenth century was the first in America when any special interest in mathematics was shown, and to this our authors devote 50 pages. The contributions from colleges, private instruction, textbooks, astronomy, navigation, and geodesy, learned societies and scientific periodicals, are all considered. Among outstanding individuals whose mathematical work, or influence on mathematics, is discussed, are Isaac Greenwood, John Winthrop, David Rittenhouse, Benjamin Franklin, and Thomas Jefferson.

The third chapter (pp. 65–101) gives us a general survey of the nineteenth century, including a further account of the mathematics of colleges and universities, and of scientific societies and periodicals. In the first complete paragraph on page 87 there is a call for revision throughout. It is not recognized that the periodicals Analyst or Mathematical Museum (5 nos., 150 pp., Philadelphia, 1808–11), and The Analyst (1 no., New York, 1814), were both edited by Robert Adrain. In this connection a change is also necessary in the first line on page 92. On page 88 the title of Marrat's The Scientific Journal is given incorrectly [see Mathematical Gazette, 1929, p. 393]. To introduce Gill's Mathematical Miscellany with "Among the problem-solving periodicals," is surely misleading. The powerful French influence in the early part of the century is not forgotten and prominent names during 1800–1875 are considered. Among these are Robert Adrain, Nathaniel Bowditch, Charles Gill,* and Ferdinand Rudolph Hassler. It is not clear why Benjamin Peirce is not listed here rather than later.

The fourth and final chapter (pp. 102–200) is devoted to the period 1875–1900. This began with the founding of the Johns Hopkins University, the appointment of its extraordinarily inspiring first professor of mathematics James Joseph Sylvester, and the founding of the American Journal of Mathematics. The New York (later American) Mathematical Society was founded in 1888 and soon the publication of its Bulletin was started. The great European (especially German) influence on American mathematics is indicated by the listing (without any pretense to completeness) of American mathematicians who got doctor's degrees from European universities before 1900. This list might have been increased by at least these four other names: T. H. Gronwall (Upsala) 1898; T. E. Hart (Heidelberg) 1866; H. D. Thompson (Göttingen) 1892; M. F. Winston (Göttingen) 1896. The name "Foche, Anne B. (Göttingen)," on the list, would not be recognized by those unaware that Anne Bosworth, who got the degree, later became the wife of Dean Foche.

Other influences in the development of mathematics of the period were the founding of such periodicals as the The Analyst, Annals of Mathematics, American Mathematical Monthly, W. E. Story's Mathematical Review, and Martin's Mathematical Magazine and Mathematical Visitor. It is stated that of the Review "only three numbers appeared"; more accurately this should read "only two numbers appeared, and some pages of a third number were

* References to Gill may be augmented by one to S. Neumark, *Note on the life of Charles Gill*, Scripta Mathematica, vol. 2, pp. 139–142.

printed, but never distributed." The impression is given that only Volume 1, containing 12 numbers, "1882–1884," of the Mathematical Magazine was ever published; the facts are that Volume 1 was printed during the years 1884–1887, and Volume 2 in 12 numbers, No. 12 in 3 parts, during the years 1890–1913. American contributions to British, but not to German, periodicals are noted.

"Prominent names and special interests" of the period makes fascinating reading. Of Sylvester, Cayley, Hill, Newcomb, Gibbs, W. W. Johnson, E. H. Moore, Maschke, Cole, W. I. Stringham, and J. S. Hagen, for example, there are brief biographies. An obvious slip was made in referring to Sylvester* as "second senior wrangler" in 1837. The volume number of the publication containing a biographical memoir of Gibbs is lacking. Stringham's excellent book, *Uniplanar Algebra: being Part 1 of a Propædeutic to Higher Mathematical Analysis* (San Francisco, 1893) should not have been forgotten. If three volumes of Father Hagen's *Synopsis der Höheren Mathematik* are to be listed with dates, the fourth, 1930, should also be given; when the authors refer as they did to Father Hagen's index to Euler's writings I wonder if they were not really thinking of Eneström's bibliography, so much better and complete. Reference to Cayley in a history of American mathematics is not inappropriate since he not only lectured in Baltimore for a few months, but also published twenty-five memoirs in the American Journal of Mathematics.

In the next section on *American dissertations*, are listed doctoral dissertations accepted before 1901 at Chicago, Columbia, Harvard, The Johns Hopkins, and Yale universities. The "general trend of mathematical research in America for the period 1875–1900" is mainly found by study of the Jahrbuch über die Fortschritte der Mathematik. In the last decade of the period, it appeared, for example, that in each of the fields, algebra and geometry, nearly twice as much was done as in each of the fields "mechanics and mathematical

* Other statements about Sylvester are either misleading or incorrect. On page 75 it is stated that at the University of Virginia Sylvester "failed as a teacher and left at the end of a year." As a matter of fact, Sylvester arrived at the University in November, 1841, and resigned on February 24, 1842; his resignation had nothing whatever to do with his teaching. On page 126 is an equally inaccurate statement, namely: "After a few months of continued trouble he was compelled to leave"; he was not "compelled" to leave. (Anyone wishing to get the correct facts in this regard may refer to P. A. Bruce, *History of the University of Virginia*, 1819–1919, New York, vol. 3, 1921, pp. 75–77.) The statement is repeated (later on p. 126) that as a teacher "he was a failure"; and yet a third time (p. 128), though not quite so strongly. Sylvester's wonderfully inspiring professorial activities at The Johns Hopkins University, and the following quotation from Bruce's *History*, seem to be adequate refutation: "He has a good deal of hesitation, is not fluent, but is very enthusiastic, and commands the attention and interest of his class." Reference is made to Sylvester's "*Collected Works*" (p. 128); this should be *Collected Mathematical Papers*. Sylvester's poem *Spring's Début, a Town Idyll* was dated (inside, p. 28) January, 1880; hence the statement (p. 126) "*n.d.* but *c.* 1880" is hardly accurate.

physics," and "history, biography, philosophy, teaching"; and more than twice as much as in the field of "theory of functions." "Trends of important branches" are chiefly exhibited for each of 14 topics by a chronological listing of published papers. With the final three-page "retrospect," and eight-page index, the work concludes.

Six full-page portraits of John Winthrop, Benjamin Peirce, Sylvester, E. H. Moore, Bôcher, and Gibbs add materially to the interest of the volume. There are many references to sources where further information may be found. It is hard to see how so much information could have been better condensed into two hundred small pages. On the whole the work is exceedingly valuable and suggestive, and American mathematicians must be highly grateful to the authors for thus notably contributing to their enlightenment and edification.

The following corrigenda submitted to me by Mr. S. A. Joffe, have been checked, and are put on record for a new edition:

p. 9, ll. 8–9, *for* "all the early American colleges" *read* "every American college";
p. 37, l. 14, *for* "of" *read* "on";
p. 38, l. 12, *for* "Mans" *read* "Man's";
p. 39, l. 9, *for* "shows" *read* "show";
p. 86, in the 11-line quotation there are too many inaccuracies to list;
p. 94, ll. 4–5 from bottom, *for* "automaton professors there begin" *read* "automata professors there, begin";
p. 99, ll. 1–2 of first footnote, *omit* "24" and *substitute* "670" *for* "570";
p. 106, l. 1 of footnote, *for* "become" *read* "became";
p. 109, *omit* ll. 4–5, and "pers:" in l. 6, and *insert* in l. 8, "Norbert Herz (Vienna)";
p. 192, l. 6 from bottom, *for* "Sur la logarithme" *read* "Sur le logarithme".

R. C. ARCHIBALD