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TYPE, COTYPE AND LÉVY MEASURES IN BANACH SPACES

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A characterization of type p and cotype p separable Banach spaces is given in terms of integrability properties of Lévy measures. The following consequences are derived: (i) a separable Banach space is isomorphic to Hilbert space if and only if the set of Lévy measures on it coincides with the set of Borel measures which integrate the function $\min(1, \|x\|^2)$; and (ii) the classical Lévy–Khintchine representation of characteristic functions of infinitely divisible distributions holds in separable Banach spaces of cotype 2, in particular, in the separable L_p spaces for $p \in [1, 2]$.

1. Introduction. The classical Lévy–Khintchine formula states that φ is the characteristic function (ch.f.) of an infinitely divisible law in R^n if and only if

$$\varphi(y) = \exp\{i\langle y, x\rangle - \varphi_1(y) + \int K(x, y)\, d\mu(x)\} \tag{1.1}$$

where

$$K(x, y) = e^{i\langle y, x\rangle} - 1 - i\langle y, x\rangle/(1 + \|x\|^2)\,,$$

μ is a Lévy measure, i.e., a positive measure such that $\mu\{0\} = 0$ and $\int \min(1, \|x\|^2)\, d\mu(x) < \infty$, and φ_1 is the ch.f. of a centered Gaussian probability measure. By Theorem VI.4.10 in Parthasarathy (1967) this remains valid in separable Hilbert space. If in a Banach space $\int \min(1, \|x\|^2)\, d\mu(x) < \infty$, then $\int K(x, y)\, d\mu(x)$ still makes sense but (1.1) may not represent the ch.f. of a probability measure (Araujo (1975a)). It is therefore necessary to introduce the notation of a Lévy measure in the Banach space context in a different way.

Given a finite Borel measure μ on a separable Banach space B, define the Poisson probability measure with associated measure μ, Pois μ, as

$$\text{Pois}\, \mu = e^{-|\mu|} \textstyle\sum_{n=0}^{\infty} \mu^n/n!$$

where $|\mu| = \mu(B)$ and μ^n is the n-fold convolution of μ.

In view of the results of Tortrat ((1967), (1969)) on representation of infinitely divisible laws, the following definition seems to be adequate.

1.1. DEFINITION. A σ-finite Borel measure μ on a separable Banach space B is a Lévy measure if $\mu\{0\} = 0$ and there exist finite measures $\mu_n \uparrow \mu$ (setwise) and $x_n \in B$ such that $\{\sigma_{x_n} * \text{Pois}\, \mu_n\}$ converges weakly to a probability measure ν. We write $\nu = s\text{Pois}\, \mu$ (shifted Poisson probability measure with Lévy measure μ).

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REMARK. If μ is a symmetric Lévy measure then x_n may be taken to be $x_n = 0$. In this case the limit of {Pois μ_n} is denoted by Pois μ.

By Parthasarathy ((1967), VI.4.7 and VI.4.8) a σ-finite measure μ on a Hilbert space such that $\mu\{0\} = 0$ is a Lévy measure in the sense of this definition if and only if $\int \min(1, \|x\|^2)\, d\mu(x) < \infty$. Araujo ((1975a), (1977)) proved that $\int \min(1, \|x\|)\, d\mu(x) < \infty$ is sufficient for μ to be a Lévy measure in any Banach space and that $\int \min(1, \|x\| g(\|x\|))\, d\mu(x) < \infty$ with $g: \mathbb{R} \to \mathbb{R}$, $g(u) > 0$ for all u and $\lim_{u\to 0} g(u) = 0$, is not; moreover, $\int \min(1, \|x\|^2)\, d\mu(x) < \infty$ is not necessary for μ to be a Lévy measure either (Araujo (1975a)).

In this note we continue to study integrability properties of Lévy measures in separable Banach spaces. In fact we characterize type and cotype p spaces by means of integrability properties of the Lévy measures on them. In particular we obtain that a Banach space is isomorphic to a Hilbert space if and only if the Lévy measures are precisely the measures integrating the function $\min(1, \|x\|^2)$. As another byproduct we obtain the classical Lévy–Khintchine representation of the characteristic function of an infinitely divisible distribution in cotype 2 Banach spaces, and a sufficient condition for infinite divisibility of probability measures on type 2 spaces.

For definitions and properties of type p and cotype p Banach spaces we refer to Hoffmann-Jørgensen and Pisier (1976), Maurey and Pisier (1976), Pisier (1975) and references there. Here we recall a few facts. A Banach space is of type p-Rademacher, $p \in [1, 2]$ (cotype p-Rademacher, $p \in (0, \infty)$) if $E\|\sum_{i=1}^{\infty} \varepsilon_i x_i\|^p \leqq K_p \sum_{i=1}^{\infty} \|x_i\|^p$ ($\geqq$) for every sequence $\{x_i\} \subset B$, with $\{\varepsilon_i\}$ a *Rademacher sequence*, i.e., a sequence of independent identically distributed random variables such that $P\{\varepsilon_i = 1\} = P\{\varepsilon_i = -1\} = \frac{1}{2}$. If $\{X_i\}_{i=1}^{n}$ are independent centered random variables taking values in a type p-Rademacher (cotype p-Rademacher space) then $E\|\sum_{i=1}^{n} X_i\|^p \leqq K_p \sum_{i=1}^{n} E\|X_i\|^p$ ($\geqq$). B is type p-Rademacher (cotype p-Rademacher) if and only if $\sum_{i=1}^{\infty} \|x_i\|^p < \infty$ implies (is implied by) the convergence in probability (or a.e., or in L_p: all are the same in this case) of $\sum_{i=1}^{\infty} \varepsilon_i x_i$. If the sequence $\{\varepsilon_i\}$ is replaced by a sequence of symmetric, nondegenerate stable of order p independent, equidistributed random variables in the definition of type and cotype p-Rademacher, one obtains the type and cotype p-stable spaces, $p \in (0, 2]$. For $p = 2$ both notions of type coincide and the same is true for cotype. More is true: let us say that B is of cotype $2 - \xi$ if whenever ξ_i are independent copies of ξ and $x_i \in B$ then $E\|\sum_{i=1}^{n} \xi_i x_i\|^2 \geqq K \sum_{i=1}^{n} \|x_i\|^2$; then it can be easily deduced from Maurey and Pisier (1976), Corollary 1.3, that if $E|\xi|^p < \infty$ for all $p > 0$, ξ symmetric and B is of cotype $2 - \xi$, then B is of cotype 2-Rademacher. We will use this remark in case ξ has the symmetrized Poisson distribution with parameter 1. The spaces $L_p(\Omega, \rho)$, (Ω, ρ) any measure space, are cotype 2 spaces if $p \in [1, 2)$ and type 2 if $p \in [2, \infty)$. A Banach space is type 2 and cotype 2 if and only if it is isomorphic to a Hilbert space (Kwapień (1972)).

We end up this condition with a necessary and sufficient condition for μ to be a Lévy measure which is seemingly weaker than the definition.

1.2. PROPOSITION. *In order that a σ-finite measure μ on a separable Banach space B be a Lévy measure it is sufficient (and necessary) that, for every continuous linear map π on B with finite dimensional range, $\mu \circ \pi^{-1}$ be a Lévy measure on Euclidean space and that there exist a probability measure ν such that $\nu \circ \pi^{-1}$ is a shift of $c\,\mathrm{Pois}\,(\mu \circ \pi^{-1})$. Then $\nu = s\mathrm{Pois}\,\mu$.*

PROOF. Let μ satisfy the hypothesis and let $\mu_n \uparrow \mu$, μ_n finite. Define $\bar{\mu}$, $\bar{\mu}_n$ by $\bar{\mu}(A) = \mu(-A)$, $\bar{\mu}_n(A) = \mu_n(-A)$ for every Borel set A. Then $\mu_n + \bar{\mu}_n \uparrow \mu + \bar{\mu}$, these are symmetric measures, and by the hypothesis, if $\nu = s\mathrm{Pois}\,\mu$ then $w\text{-}\lim_{n\to\infty} \mathrm{Pois}\,(\mu_n + \bar{\mu}_n) \circ f^{-1} = (\nu * \bar{\nu}) \circ f^{-1}$ for every $f \in B'$. If $\{X_i, Y_i\}_{i=1}^{\infty}$ are independent B-valued random variables with distributions $\mathscr{L}(X_1) = \mathscr{L}(Y_1) = \mathrm{Pois}\,\mu_1$ and $\mathscr{L}(X_i) = \mathscr{L}(Y_i) = \mathrm{Pois}\,(\mu_i - \mu_{i-1})$, $i = 2, \cdots,$ then $\mathscr{L}(\sum_{i=1}^{n} (X_i - Y_i)) = \mathrm{Pois}\,(\mu_n + \bar{\mu}_n)$. So, Theorem 4.1 of Itô–Nisio (1968) implies that the series $\sum_{i=1}^{\infty} (X_i - Y_i)$ converges a.s. and thus, by Fubini's theorem, that there exists a sequence $\{x_i\} \in B$ such that the sequence of random variables $\{x_n + \sum_{i=1}^{n} X_i\}$ converges a.s. Therefore, the sequence $\{(\mathrm{Pois}\,\mu_n) * \delta_{x_n}\}_{n=1}^{\infty}$ is weakly convergent. If λ is its limit, an easy argument using the definition of Lévy measure proves the existence of $u : B' \to \mathbb{R}$ linear such that

$$\lambda \circ f^{-1} = (\nu \circ f^{-1}) * \delta_{u(f)}$$

for every $f \in B'$. Moreover, u is weak-star sequentially continuous: if $f_n(x) \to f(x)$ for all $x \in B$ then $\int e^{itf_n(x)}\, d\lambda(x) \to \int e^{itf(x)}\, d\lambda(x)$ and likewise for ν by bounded convergence, and therefore $e^{itu(f_n)} \to e^{itu(f)}$ for all t in a neighborhood of zero, i.e., $u(f_n) \to u(f)$. Hence u defines a vector in B and $w\text{-}\lim_{n\to\infty} (\mathrm{Pois}\,\mu_n) * \delta_{x_n} = \lambda = \nu * \delta_u$; i.e., μ is a Lévy measure. □

In connection with integrability questions it is convenient to remark that the arguments in the proof of the first part of Parthasarathy (1967) IV.4.4 prove also that every Lévy measure μ on B gives finite mass to the complement of every neighborhood of zero. Therefore only integrability of Lévy measures near the origin will be of interest here. This fact will be used without further mention.

All the Banach spaces in this paper are assumed to be *separable*.

2. Integrability of Lévy measures. Among the main tools for the next theorem we have the Lévy's and the converse Kolmogorov's inequalities for Banach space valued random variables. We refer to Kahane (1968), page 12, for the first and to de Acosta and Samur (1978) for the second. Both inequalities together give the following.

2.1. PROPOSITION. *If $\{X_i\}_{i=1}^{n}$ are n independent Banach valued random variables with $\|X_i\| \leqq c$ a.s. and $EX_i = 0$, $i = 1, \cdots, n$, then for every $p \geqq 1$ and $a > 0$,* (2.1) $$P\{\|X_1 + \cdots + X_n\| > a\} \geqq 2^{-p}[1 - ((a + c)^p + a^p(1 - 2^{1-p}))/E\|X_1 + \cdots + X_n\|^p] .$$

2.2. THEOREM. *A Banach space B is of cotype 2 if and only if every Lévy measure μ on B satisfies*

(2.2) $$\int \min(1, \|x\|^2)\, d\mu(x) < \infty .$$

PROOF. (a) Assume B is of cotype 2-Rademacher. The arguments at the beginning of the proof of Parthasarathy (1967), Theorem VI.4.6, reduce the proof of condition (2.2) for Lévy measures to showing that

(2.3) $$\sup_n \int \|x\|^2\, d\mu_n(x) < \infty$$

under the assumptions: μ_n symmetric, finite, supported by the unit ball $\{\|x\| \leqq 1\}$, $\mu_n(B) = \mu_n\{\|x\| \leqq 1\} = k_n$ integer with $k_n \to \infty$ as $n \to \infty$, and $\{\text{Pois}\, \mu_n\}$ uniformly tight.

Define $\nu_n = \mu_n/k_n$ and $\rho_n = \nu_n^{k_n}$, $n = 1, \cdots$. Then the fact that B is of cotype 2-Rademacher gives

$$\int \|x\|^2\, d\rho_n(x) \geqq K_2 k_n \int \|x\|^2\, d\nu_n(x) = K_2 \int \|x\|^2\, d\mu_n(x) .$$

So, in order to prove (2.3) we need only show that

(2.3)′ $$\sup_n \int \|x\|^2\, d\rho_n(x) < \infty .$$

Now, inequality (2.1) yields, for $\rho_n\{\|x\| \geqq a\} < \frac{1}{4}$,

(2.4) $$\int \|x\|^2\, d\rho_n(x) \leqq [(a + 1)^2 + a^2/2]/[1 - 4\rho_n\{\|x\| \geqq a\}] .$$

So, (2.3)′ holds if $\{\rho_n\}_{n=1}^\infty$ is uniformly tight. But the tightness of $\{\rho_n\} = \{\nu_n^{k_n}\}$ follows from the tightness of $\{\text{Pois}\, \mu_n\} = \{\text{Pois}\, k_n\nu_n\}$ by a direct probabilistic argument (Le Cam (1970), Proposition 3).

(b) Assume now that every Lévy measure on B satisfies (2.2). Let $\{\xi_i, \xi_i'\}_{i=1}^\infty$ be independent Pois δ_1 real random variables and $\{x_i\}_{i=1}^\infty \subset B$ be such that $\sum_{i=1}^\infty \xi_i x_i$ converges a.s. Then $\sum_{i=1}^\infty (\xi_i - \xi_i')x_i$ converges too. In particular the measures $\mu_n = \sum_{i=1}^n (\delta_{x_i} + \delta_{-x_i})$ increase to $\mu = \sum_{i=1}^\infty (\delta_{x_i} + \delta_{-x_i})$ and the sequence of probability measures Pois $\mu_n = \mathscr{L}(\sum_{i=1}^n (\xi_i - \xi_i')x_i)$, $n = 1, \cdots$, converges weakly to a Borel probability measure on B, i.e., μ is a Lévy measure (Theorem 2.3). Since the sequence $\{x_i\}$ is bounded, (2.2) implies $2 \sum_{i=1}^\infty \|x_i\|^2 = \int \|x\|^2\, d\mu(x) < \infty$. Finally, the inequality $E\|\sum_{i=1}^n \xi_i x_i\|^2 \geqq K_2 \sum_{i=1}^n \|x_i\|^2$ follows the closed graph theorem (in complete analogy with the comment before (1.4) in Hoffmann-Jørgensen and Pisier (1976)). Hence, B is of cotype $2 - \xi$ where ξ is a symmetric random variable with $E|\xi|^p < \infty$ for every $p > 0$ and therefore it is of cotype 2-Rademacher by Maurey and Pisier (1976). □

REMARK. The last proof can be carried out for some $p \neq 2$. Formally the conclusions are: (a) if B is of cotype p-Rademacher then every Lévy measure on B integrates the function $\min(1, \|x\|^p)$, $p \geqq 1$, and (b) if every Lévy measure on B integrates $\min(1, \|x\|^p)$, then B is of cotype p-stable, $p \leqq 2$. But the first assertion has a meaning only for $p \geqq 2$ and the second for $p = 2$ (every Banach space is of cotype p-stable for $p < 2$ and the only cotype p-Rademacher space for $p < 2$ is $\{0\}$). These results for l_p spaces, $p \geqq 2$ (which are of cotype p) are contained in Yurinskii (1974).

For type p spaces the situation is as follows:

2.3. THEOREM. *A Banach space B is of type p-Rademacher, $p \in [1, 2]$, if and only if every measure on B which integrates the function* $\min(1, \|x\|^p)$ *and gives zero mass to* $\{0\}$ *is a Lévy measure.*

PROOF. Assume B is of the type p-Rademacher. Let μ be a measure on B satisfying (2.2), $\lambda = \mu + \bar{\mu}$, $\{X_i\}_{i=1}^\infty$ independent B-valued random variables with distributions $\mathscr{L}(X_1) = \text{Pois}\,(\lambda|_{\{\|x\|>1\}})$ and $\mathscr{L}(X_i) = \text{Pois}\,(\lambda|_{\{1/i<\|x\|\leqq 1/(i-1)\}})$, $i = 2, \cdots$. Then, B being of type p, we have

$$\begin{aligned} \int \|x\|^p \, d\text{Pois}\,\rho(x) &= e^{-|\rho|} \textstyle\sum_{n=0}^\infty |\rho|^n \int \|x\|^p \, d(\rho/|\rho|)^n(x)/n! \\ &\leqq K_p e^{-|\rho|} \textstyle\sum_{n=0}^\infty |\rho|^n n \int \|x\|^p \, d(\rho/|\rho|)(x)/n! \\ &= K_p \int \|x\|^p \, d\rho(x) \end{aligned}$$

for every finite measure ρ. So,

$$E\|\textstyle\sum_{j=r}^s X_j\|^p \leqq K_p \int_{1/s<\|x\|\leqq 1/r} \|x\|^p \, d\lambda(x) \to 0 \qquad \text{as} \quad r, s \to \infty$$

and therefore, $\sum_{j=1}^\infty X_j$ converges in probability. In particular,

$$w\text{-}\lim_{n\to\infty} \text{Pois}\,(\lambda|_{\{\|x\|\geqq 1/n\}}) = \mathscr{L}(\textstyle\sum_{j=1}^\infty X_j)\,.$$

Hence, $\{(\text{Pois}\,(\mu|_{\{\|x\|\geqq 1/n\}})) * \delta_{x_n}\}$ is weakly convergent for some choice of $\{x_n\}$ by Fubini's theorem (note that $\sum_{j=1}^\infty X_j$ converges a.s. by the previously mentioned Itô and Nisio's result). So, μ is a Lévy measure.

Assume now that every measure satisfying (2.2) is Lévy. Then, if $\sum_{i=1}^\infty \|x_i\|^p < \infty$ the measure $\sum_{i=1}^\infty (\delta_{x_i} + \delta_{-x_i})$ is a Lévy measure. Hence $\sum_{i=1}^\infty (\xi_i - \xi_i')x_i$, where the ξ_i, ξ_i' are independent Pois δ_1 random variables, converges a.s. (Itô and Nisio (1968)). Then, the already mentioned result of Jain and Marcus (1975) yields the a.s. convergence of $\sum_{i=1}^\infty \varepsilon_i x_i$. By Kahane (1968), page 17, this last series converges in L_p and therefore B is of type p-Rademacher. □

Since every Banach space is of type 1-Rademacher, this theorem proves that a sufficient condition for μ on a general Banach space B to be a Lévy measure is that $\int \min(1, \|x\|) \, d\mu(x) < \infty$. This result is proved in Araujo (1975a) and in Yurinskii (1974).

In view of Kwapień (1972), Proposition 3.1, the last two theorems yield

2.4. COROLLARY. *A Banach space B is isomorphic to Hilbert space if and only if the set of Lévy measures on B coincides with the set of measures which integrate the function* $\min(1, \|x\|^2)$.

3. The Lévy-Khintchine representation. The following theorem about the structure of infinitely divisible distributions on Banach spaces is an easy consequence of Tortrat (1967) Corollary I.4.4 and (1969) Theorem II and Corollary II.3, and the previous Proposition 1.2.

3.1. THEOREM. *Let ρ be an infinitely divisible probability measure on a (separable) Banach space B. Then, there exists a unique decomposition of ρ into*

$$\rho = \eta * s\text{Pois}\,\mu * \delta_x \tag{3.1}$$

where η is centered Gaussian, μ is a Lévy measure with $\mu\{0\} = 0$ and $x \in B$.

If a Lévy measure μ satisfies the integrability condition (2.2) with $p = 2$ then the characteristic function of sPois μ can be written explicitly:

3.2. PROPOSITION. *If μ is a Lévy measure on B such that $\int \min(1, \|x\|^2)\, d\mu(x) < \infty$, then the* ch.f. *$\phi$ of sPois μ is*

$$\phi(y) = \exp\{iy(x_0) + \int (e^{iy(x)} - 1 - iy(x)/(1 + \|x\|^2))\, d\mu(x)\}\,, \quad y \in B'\,. \tag{3.2}$$

PROOF. If $\int \min(1, \|x\|^2)\, d\mu(x) < \infty$, the proof of the second part of Theorem VI.4.7 in Parthasarathy (1967) implies that if $\mu_n \uparrow \mu$ and $c_n = -\int x/(1 + \|x\|^2)\, d\mu_n(x)$, then $\{(\text{Pois}\,\mu_n) * \delta_{c_n}\}$ converges weakly. Since the ch.f. of Pois ρ is $\exp(\hat\rho - \hat\rho(0))$, the proposition follows by the Lebesgue dominated convergence theorem applied to $\int_{\|x\| \geq 1/n} (e^{iy(x)} - 1 - iy(x)/(1 + \|x\|^2))\, d\mu(x)$. □

3.3. THEOREM. *On a cotype 2 Banach space B a distribution ρ with* ch.f. *ϕ is infinitely divisible if and only if there exist a covariance Φ of a Gaussian Borel probability measure on B, a Lévy measure μ and a vector $x_0 \in B$ such that for every $f \in B'$,*

$$\phi(y) = \exp\{iy(x_0) - \tfrac{1}{2}\Phi(y, y) + \int (e^{iy(x)} - 1 - iy(x)/(1 + \|x\|^2))\, d\mu(x)\}\,. \tag{3.3}$$

PROOF. If ρ is infinitely divisible then (3.3) is a direct consequence of the Theorems 3.1, 3.2 and 2.2. If (3.3) holds then $\rho = (\rho_n)^n$, the ch.f. of ρ_n being

$$\phi_n(y) = \exp\{iy(x_0)/n - \tfrac{1}{2}\Phi(y, y)/n + \int (e^{iy(x)} - 1 - iy(x)/(1 + \|x\|^2)\, d\mu(x)/n\}:$$

the covariance Φ/n is obviously that of a Gaussian Borel measure on B; so, in order to prove that ϕ_n is the ch.f. of a Borel measure on B we need only see that μ/n is a Lévy measure; if $\mu_k \uparrow \mu$, μ_k finite measures, then $\mu_k/n \uparrow \mu/n$ and $\{\text{Pois}\,(\mu_k/n)\}$ is relatively shift compact as it is a subset of factors of $\{\text{Pois}\,\mu_k\}$ (Parthasarathy (1967) Theorem III.5.1), and therefore, μ/n is a Lévy measure. □

It is possible to prove a Lévy–Khintchine representation for general Banach spaces which has a different kernel (see, e.g., Araujo (1975b) or Dettweiler (1976)). The function $y(x)/(1 + \|x\|^2)$ appearing in (3.3) is replaced in their representation by a function exactly of the form $y(x)$ in a neighborhood of zero or in an appropriate compact convex symmetric set; then (3.3) follows directly from their representations and Theorem 2.1. One might conjecture that the representation (3.3) is valid in general Banach spaces, but Araujo (1975b) has a counterexample.

The next result is just a reformulation of Theorems 3.1 and 2.3.

3.4. THEOREM. *If B is a type 2 space, every function of the form* (3.3), *where now μ is just a measure which integrates the function* $\min(1, \|x\|^2)$, *is the* ch.f. *of an* *infinitely divisible probability measure; and conversely, if this is true then B is a type 2 space.*

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Note added in proof. E. Dettweiler, V. I. Mandrekar and V. Paulauskas have obtained independently some of the results contained in this note. Theorem 3.3 has also been obtained by de Acosta and Samur (1978).

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