and the points $P(t)$ take up the whole curve (C), e.g. between $t = 0$ and ∞. Then the relations between the given S_ρ become *non-linear* inequalities, well known for the problem of moments.

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AN INEQUALITY FOR MILL'S RATIO

BY Z. W. BIRNBAUM

University of Washington

Mr. R. D. Gordon[1] recently proved the inequalities

$$\frac{x}{x^2+1}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2} \leq \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{1}{2}t^2}\,dt \leq \frac{1}{x}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}, \quad \text{for } x > 0.$$

In the present note we show that the lower inequality can be replaced by the better estimate

$$\frac{\sqrt{4+x^2}-x}{2}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2} \leq \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{1}{2}t^2}\,dt.$$

PROOF: According to a well-known theorem of Jensen[2], for $f(t)$ convex and $g(t) \geqq 0$ in the interval (a, b), the following inequality holds

$$f\left[\int_a^b tg(t)\,dt\Big/\int_a^b g(t)\,dt\right] \leq \int_a^b f(t)\,g(t)\,dt\Big/\int_a^b g(t)\,dt.$$

For $a = x, b = \infty, f(t) = 1/t, g(t) = te^{-t^2/2}$, this inequality gives

$$\int_x^\infty te^{-\frac{1}{2}t^2}\,dt\Big/\int_x^\infty t^2e^{-\frac{1}{2}t^2}\,dt \leq \int_x^\infty e^{-\frac{1}{2}t^2}\,dt\Big/\int_x^\infty te^{-\frac{1}{2}t^2}\,dt.$$

Since

$$\int_x^\infty te^{-\frac{1}{2}t^2}\,dt = e^{-\frac{1}{2}x^2} \qquad \text{and} \qquad \int_x^\infty t^2e^{-\frac{1}{2}t^2}\,dt = xe^{-\frac{1}{2}x^2} + \int_x^\infty e^{-\frac{1}{2}t^2}\,dt,$$

[1] R. D. Gordon, "Values of Mill's ratio of area to bounding ordinate of the normal probability integral for large values of the argument," *Annals of Math. Stat.*, Vol. 12 (1941), pp. 364–366.

[2] See for example: G. H. Hardy, J. E. Littlewood and G. Pólya, *Inequalities*, Cambridge, 1934, p. 150–151.

we find

$$(e^{-\frac{1}{2}x^2})^2 \leq xe^{-\frac{1}{2}x^2} \int^{\infty} e^{-\frac{1}{2}t^2}\, dt + \left(\int_x^{\infty} e^{-\frac{1}{2}t^2}\, dt\right)^2,$$

and hence

$$\frac{\sqrt{4+x^2} - x}{2} \cdot e^{-\frac{1}{2}x^2} \leq \int_x^{\infty} e^{-\frac{1}{2}t^2}\, dt.$$