

**TABLES FOR TESTING RANDOMNESS OF GROUPING
IN A SEQUENCE OF ALTERNATIVES**

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When two different kinds of objects are arranged along a line they will form two or more distinct groups of like objects. Thus, in the arrangement: *aabbbab*, there are 3 *a*'s and 4 *b*'s forming 4 groups. In general, if there are *m* objects of one kind and *n* objects of another kind, there are in all

$$C_m^{m+n} = C_n^{m+n}$$

different arrangements possible. There will be no loss of generality if we assume that $m \leq n$.

If *u* is defined to be the number of distinct groups of like objects in any one arrangement, then the proportion of arrangements yielding *u'* or less groups is¹

$$(1) \quad P\{u \leq u'\} = \frac{1}{C_m^{m+n}} \sum_{u=2}^{u'} f_u,$$

where

$$f_u = 2C_{k-1}^{m-1} \cdot C_{k-1}^{n-1}, \quad \text{when } u = 2k, \quad \text{i.e. } u \text{ is even,}$$

and

$$f_u = C_{k-1}^{m-1} \cdot C_{k-2}^{n-1} + C_{k-2}^{m-1} \cdot C_{k-1}^{n-1}, \quad \text{when } u = 2k - 1, \quad \text{i.e. } u \text{ is odd,}$$

for $k = 1, 2, \dots, m + 1$. For example, if $m = n = 5$, then

$$P\{u = 2\} = \frac{f_2}{C_5^{10}} = \frac{2\{C_0^4 \cdot C_0^4\}}{C_5^{10}} = \frac{1}{126},$$

$$P\{u = 3\} = \frac{f_3}{C_5^{10}} = \frac{C_1^4 \cdot C_0^4 + C_0^4 \cdot C_1^4}{C_5^{10}} = \frac{8}{252}.$$

In a random arrangement (1) is the probability of $u \leq u'$.

The following tables have been prepared for use in testing data for randomness and for testing whether two samples are from the same population. Table I gives $P\{u \leq u'\}$ to 7 decimal places for $m \leq n \leq 20$ with a range of *m* from 2 to 20 inclusive whereas Table II gives correct values for u_ϵ for $\epsilon = .005, .01, .025, .05, .95, .975, .99$ and $.995$, where u_ϵ is the *largest* integer, u' , for which $P\{u \leq u'\} \leq \epsilon$ when $\epsilon < .50$, and is the *smallest* integer, u' , for which $P\{u \leq u'\} \geq \epsilon$ when $\epsilon > .50$. This table was obtained from Table I and covers the same

¹ W. L. Stevens, "Distribution of Groups in a Sequence of Alternatives" (*Annals of Eugenics*, Vol. IX, Part I (1939) pp. 10-17).

A. Wald and J. Wolfowitz, "On a Test Whether Two Samples are from the Same Population" (*Annals of Math. Stat.*, Vol. XI, No. 2, June (1940) pp. 147-162).

range of values of m and n . Table III gives values of u_ϵ for $m = n$ from 10 to 100. These values of u_ϵ were obtained by using the normal approximation given on page 151 of the Wald-Wolfowitz paper together with a correction for continuity not given in their article—this correction improved the approximation for small values of m and n . The values of u_ϵ for $m = n = 10$ through 20 are included in Table III although they can be obtained from Table II in order to check on the adequacy of the approximation. These values obtained with the approximation check with those of Table II except for the five underscored values. It appears that the approximation will be adequate in general for $m = n \geq 20$.

To illustrate the use of these tables to test randomness of an arrangement,² consider a case where one might suspect nonrandomness and, more specifically, expect too few groups. The arrangement of diseased and healthy plants in a row of a field might be such a case. For example, we might have the following plant arrangement:

H H H H H H H H H D H D D D D H H H H H H H H,

where

$m = 5$, the number of diseased plants present,
 $n = 20$, the number of healthy plants present,
 $u' = 5$, the number of groups actually formed.

From Table I the probability associated with this arrangement is found to be .018,3512, which is the probability of $u \leq u'$. Since $P < .05$, we might elect to regard this as evidence of a tendency for the disease to be nonrandomly distributed among the plants in a row, knowing that if we look for an explanation of "clustering" whenever $P\{u \leq u'\} \leq .05$ we may expect to follow a false scent not more than one time in twenty in the long run.

When a control chart³ suggests the presence of assignable causes of variation in a manufactured product flowing from a production line, an examination of various types of runs, e.g. the lengths and relative frequency of runs above and below the median of a sequence of values, may assist in diagnosing the nature of the cause. Dr. Walter A. Shewhart has given us such an instance: A sequence of observations dealing with corrosion suggested the presence of an assignable cause of variation. By the use of run charts an assignable cause of variation was tracked down in the measuring apparatus and an attempt was made to eliminate it. The original sequence examined with regard to runs above and below the median of the sequence exhibited an unexpectedly large number of runs of length 7 or more and as a result a significantly low value of

² W. L. Stevens (ibid).

³ American Defense Emergency Standards Z1.1 and Z1.2 entitled "Guide for Quality Control" and "Control Chart Method of Analyzing Data" and American War Standard Z1.3 entitled "Control Chart Method of Controlling Quality During Production" (published by the American Standards Association, New York City).

u , and, if the assignable cause were not completely eliminated in the new design, we might expect too large a proportion of long runs above and below the median, and, hence, too few total runs. A sequence of 40 observations taken with the new measuring device yielded a total of 15 runs above and below the median of the sequence which is significantly fewer than would be expected to arise under a state of statistical control, since for $m = n = 20$, $P\{u \leq 15\} = .038$. This sequence is of special interest since the occurrence of too few runs suggested the assignable cause had not been entirely eliminated although no especially long runs, say of length 7 or more, occurred in this sequence, so that from the point of view of length of runs without regard to their number the assignable cause might have been judged to have been eliminated.

As an instance where too many groups would be the probable alternative to randomness consider the arrangement of occupied and unoccupied seats at a lunch counter about half an hour before the popular lunch hour begins. In such a case the critical region would be $u \geq u'$ and the appropriate probability would be $P = 1 - P\{u \leq u' - 1\}$. Such a situation was observed and yielded the following arrangement of empty and occupied seats along the lunch counter:

E O E E O E E E O E E E O E O E,

$$m = 5,$$

$$n = 11,$$

$$u' = 11,$$

$$P = 1 - .942,3077 = .057,6923;$$

and though this probability is not quite significant, the arrangement observed has the maximum number of groups of empty and occupied seats for the m and n of the size observed since no two occupied seats are adjacent. However, if another customer had entered and sat either in the 5th empty seat from the left or in the 8th empty seat, the number of groups would have been increased by two and the situation would be:

$$m = 6,$$

$$n = 10,$$

$$u' = 13,$$

$$P = 1 - .989,5105 = .010,4895.$$

This P value is significant, and for this assumed case, as well as for the actual case observed, the arrangement of E 's and O 's has the maximum number of groups of like objects. Certainly both of these cases exhibit too many groups to be considered random arrangements.

The use of these tables to test whether two samples constitute independent random samples from the same population⁴ can be illustrated by using the data of Snedecor's Example 4.11 on page 75 of his *Statistical Methods* (3d edition)

⁴A. Wald and J. Wolfowitz (ibid) have pointed out that exceptionally small values of u' are to be regarded as evidence for rejecting this null hypothesis.

which gives daily gains in two lots of steer calves on two different rations. The daily rates of gain given for the two lots are:

- I. 1.95, 2.17, 2.06, 2.11, 2.24, 2.52, 2.04, 1.95;
 V. 1.82, 1.85, 1.87, 1.74, 2.04, 1.78, 1.76, 1.86.

Arranging these rates in order of magnitude, designating a calf on ration I by italics and one from V by (), we have

(1.74), (1.76), (1.78), (1.82), (1.85), (1.86), (1.87), *1.95*, *1.95*,
 (2.04), *2.04*, *2.06*, *2.11*, *2.17*, *2.24*, *2.52*.

Whence

$$\begin{aligned}m &= 8, \\n &= 8, \\u' &= 4, \\P &= .008,8578.\end{aligned}$$

Accordingly, at either the .05 or .01 level of significance rejection of the null hypothesis that the two samples constitute independent random samples from the same population is indicated.

For these data we note the fact that having two identical values, i.e. 2.04, in the two lots did not alter the number of groups regardless of whether they were recorded as (2.04), *2.04* or as *2.04*, (2.04). However, such duplications in general may be more bothersome, since they may yield different values of u' depending on the order in which they are considered. In such instances both possible orders should be considered.

The merit of this test is that it employs a minimum of assumptions—merely that the common population be continuous, and that the samples be drawn at random independently. Its principal defect is its lack of power. As a consequence gross disparity between the samples is generally required to render $u' \leq u_c$. Therefore, when additional assumptions are tenable, tests utilizing them should be employed.

Most of the computing and checking of these tables was done by Frieda S. Swed, Philip Ritz and Beatrice E. Kelley with some assistance from Jay Grodman, Edward Halamka and Mrs. Henry Wallman. Also, Duane Borst and Francis Cox helped with the typing and the proofing of the tables.

TABLE I
 $P\{u \leq u'\}$

When $m = n$, the largest possible value of u' is $2m$; when $m < n$, the largest possible value of u' is $2m + 1$.

n	m = 4					
	u'	2	3	4	5	6
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						
16						
17						
18						
19						
20						

n	m = 5					
	u'	2	3	4	5	6
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						
16						
17						
18						
19						
20						

TABLE II
 $P\{u \leq u'\}$

When $m = n$, the largest possible value of u' is $2m$; when $m < n$, the largest possible value of u' is $2m + 1$.

n	m = 4					
	u'	2	3	4	5	6
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						
16						
17						
18						
19						
20						

n	m = 5					
	u'	2	3	4	5	6
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						
16						
17						
18						
19						
20						

TABLE I (Continued)

n = 5		n = 6		n = 6		n = 6		n = 6		n = 6	
n	u'	n	u'	n	u'	n	u'	n	u'	n	u'
5	.007, 9365	.039, 6825	.166, 6667	.357, 11429	.642, 8571	.002, 1645	.012, 9870	.067, 0996	.175, 3247	.391, 7749	
6	.004, 3590	.023, 8095	.110, 3896	.261, 9048	.521, 6450	.001, 1655	.007, 5756	.042, 5408	.121, 2121	.296, 0373	
7	.002, 5253	.015, 1515	.075, 3716	.196, 9697	.424, 2824	.000, 6660	.002, 6680	.027, 3720	.086, 2471	.226, 1072	
8	.001, 5940	.010, 1010	.053, 6131	.151, 5152	.347, 3193	.000, 3996	.002, 9970	.016, 9810	.062, 9371	.174, 8252	
9	.000, 9990	.005, 9990	.034, 9610	.118, 8811	.286, 7133	.000, 2498	.001, 9980	.013, 2368	.046, 9530	.136, 8531	
10	.000, 6660	.004, 9990	.024, 9710	.094, 9051	.238, 7612	.000, 1616	.001, 3136	.009, 4538	.039, 7113	.108, 4357	
11	.000, 4579	.003, 6650	.021, 9780	.076, 9231	.200, 5094	.000, 1077	.000, 9696	.006, 8951	.027, 6341	.086, 8686	
12	.000, 3232	.002, 7472	.016, 9683	.065, 1868	.169, 8449	.000, 0777	.000, 7003	.005, 1231	.021, 7087	.070, 3597	
13	.000, 2534	.002, 1008	.013, 9053	.052, 5210	.144, 9980	.000, 0516	.000, 5160	.003, 8700	.017, 2899	.057, 5335	
14	.000, 1720	.001, 6340	.010, 5179	.044, 1176	.124, 6130	.000, 0916	.000, 3870	.002, 9570	.013, 9319	.047, 4716	
15	.000, 0983	.001, 0520	.006, 5339	.037, 4097	.107, 8431	.000, 0868	.000, 2949	.002, 3952	.011, 3519	.039, 4971	
16	.000, 0799	.001, 0354	.005, 6960	.031, 9917	.095, 9112	.000, 0198	.000, 2787	.001, 6128	.009, 3415	.033, 1154	
17	.000, 0799	.000, 8394	.005, 6960	.027, 5689	.082, 2511	.000, 0199	.000, 1783	.001, 4414	.007, 7595	.027, 9652	
18	.000, 0799	.000, 6875	.004, 7252	.023, 9234	.072, 4241	.000, 0113	.000, 1412	.001, 1575	.006, 4935	.023, 7719	
19	.000, 0471	.000, 4617	.003, 9526	.020, 8922	.064, 0881	.000, 0087	.000, 1129	.000, 3182	.005, 4771	.020, 3318	
20	.000, 0376	.000, 1, 705	.003, 3314	.018, 3512	.056, 9735						

n = 5		n = 6		n = 6		n = 6		n = 6		n = 6	
n	u'	n	u'	n	u'	n	u'	n	u'	n	u'
5	.833, 3333	.960, 3175	.992, 0635	1.		.608, 2251	.824, 6753	.932, 9004	.987, 0130	.997, 8395	1.
6	.738, 0992	.911, 2594	.976, 1905	.997, 8395		.500, 0000	.733, 1002	.878, 7879	.966, 2005	.994, 4242	
7	.651, 5152	.853, 3194	.954, 5095	.992, 4242		.412, 5874	.646, 8876	.820, 3188	.937, 0689	.983, 6830	
8	.575, 7576	.793, 3178	.929, 2929	.983, 6830		.342, 6733	.566, 4336	.762, 2378	.902, 0979	.972, 0280	
9	.510, 4895	.734, 2657	.902, 0979	.972, 0280		.286, 1733	.496, 9035	.706, 2937	.863, 6364	.956, 0420	
10	.454, 9494	.678, 3217	.874, 1299	.958, 0480		.241, 7582	.435, 6820	.653, 4862	.823, 2894	.942, 3077	
11	.406, 9994	.626, 3736	.846, 1536	.942, 3077		.205, 9375	.383, 1610	.609, 3652	.783, 1286	.929, 3394	
12	.365, 3846	.578, 7007	.818, 6813	.929, 3394		.175, 7703	.331, 9404	.560, 3244	.743, 3658	.907, 5630	
13	.329, 8319	.535, 2474	.792, 0168	.907, 8630		.148, 4448	.299, 0196	.520, 3818	.704, 8504	.889, 3189	
14	.299, 0196	.495, 7860	.766, 3399	.889, 3189		.121, 2209	.269, 4799	.489, 4681	.667, 9587	.870, 8720	
15	.272, 1878	.460, 0103	.741, 7441	.870, 8720		.100, 4779	.231, 4278	.419, 5777	.632, 8924	.852, 4252	
16	.248, 7100	.427, 9886	.718, 2662	.852, 4252		.088, 9210	.189, 5342	.349, 7204	.599, 7504	.834, 1308	
17	.228, 0702	.398, 1924	.695, 9064	.834, 1308		.078, 4667	.170, 6381	.326, 4596	.566, 5499	.816, 1015	
18	.209, 8428	.371, 5118	.674, 6411	.816, 1015		.069, 8475	.151, 4542	.303, 2418	.539, 2430	.798, 4130	
19	.193, 6799	.347, 2614	.654, 4323	.798, 4130							
20	.179, 2772	.325, 1835	.635, 2343	.781, 1406							

TABLE I (Continued)

n = 7		3		4		5		6	
n	n'	n	n'	n	n'	n	n'	n	n'
7									
8		.004, 0783		.025, 0583		.077, 5058		.204, 6217	
9		.000, 3108		.015, 3462		.051, 2621		.149, 3841	
10		.000, 1718		.009, 1986		.034, 5492		.106, 3316	
11		.000, 1028		.004, 4713		.024, 1759		.080, 6082	
12		.000, 0628		.000, 5856		.009, 3373		.049, 2524	
13		.000, 0268		.000, 3771		.011, 2535		.042, 6889	
14		.000, 0172		.000, 2880		.002, 1164		.025, 9177	
15		.000, 0117		.000, 1806		.007, 2279		.027, 6144	
16		.000, 0082		.000, 1280		.001, 1113		.021, 4816	
17		.000, 0058		.000, 0938		.000, 6280		.005, 3176	
18		.000, 0042		.000, 0631		.003, 3978		.011, 1993	
19		.000, 0030		.000, 0520		.000, 4764		.003, 1180	
20		.000, 0023		.000, 0388		.000, 3879		.002, 1719	
				.000, 0304		.000, 2872		.001, 1535	
								.007, 5403	

n = 7		12		13		14	
n	n'	n	n'	n	n'	n	n'
7							
8		.995, 9207		.999, 4172		1.	
9		.987, 8788		.997, 6690		.999, 6416	
10		.974, 8652		.994, 4056		.999, 3007	
11		.955, 1164		.989, 5105		.998, 1189	
12		.935, 5804		.983, 0317		.996, 2293	
13		.910, 9312		.975, 1131		.993, 4908	
14		.884, 2105		.965, 9443		.989, 7853	
15		.859, 1146		.955, 7276		.985, 2825	
16		.827, 2704		.944, 6594		.979, 6762	
17		.794, 1785		.932, 5805		.973, 7515	
18		.763, 2294		.920, 6712		.966, 9464	
19		.740, 7031		.908, 0908		.959, 5423	
20		.712, 8246		.895, 1779		.951, 6206	
				.882, 1556		.943, 2987	

n = 7		7		8		9		10		11	
n	n'	n	n'	n	n'	n	n'	n	n'	n	n'
7											
8		.383, 4499		.616, 5501		.791, 3753		.922, 4942		.974, 9417	
9		.230, 7692		.513, 5975		.703, 9627		.867, 1329		.948, 7179	
10		.181, 8182		.426, 5774		.622, 3776		.805, 9441		.916, 6839	
11		.144, 7964		.354, 5856		.518, 9510		.713, 3155		.879, 3706	
12		.116, 5158		.295, 6899		.468, 1629		.682, 1267		.840, 4977	
13		.094, 5852		.247, 4994		.427, 6018		.624, 0772		.800, 9050	
14		.077, 6574		.208, 2043		.378, 4850		.570, 0464		.761, 6099	
15		.063, 5698		.176, 0406		.335, 9133		.520, 3818		.723, 2972	
16		.051, 6038		.149, 6153		.299, 0196		.475, 1042		.686, 4035	
17		.044, 0698		.127, 8079		.257, 0044		.444, 0402		.651, 1796	
18		.034, 0695		.109, 7242		.219, 1653		.356, 9212		.563, 2284	
19		.032, 4111		.082, 0310		.191, 6759		.313, 2320		.556, 5217	
20		.027, 7592		.071, 4064		.175, 0684		.306, 0099		.528, 6105	

n = 8		2		3		4		5		6	
n	n'	n	n'	n	n'	n	n'	n	n'	n	n'
8											
9		.000, 1954		.001, 2452		.008, 8978		.031, 7016		.100, 2331	
10		.000, 0823		.000, 6953		.005, 3065		.020, 2797		.068, 6549	
11		.000, 0497		.000, 4114		.003, 2908		.013, 3690		.047, 9227	
12		.000, 0265		.000, 2914		.002, 1057		.009, 0498		.034, 0957	
13		.000, 0159		.000, 1548		.001, 3813		.006, 2713		.024, 6090	
14		.000, 0098		.000, 1032		.000, 9288		.004, 4376		.018, 0599	
15		.000, 0063		.000, 0686		.000, 6360		.003, 1952		.013, 4440	
16		.000, 0041		.000, 0467		.000, 4467		.002, 3454		.010, 1404	
17		.000, 0027		.000, 0282		.000, 3182		.001, 7458		.007, 7420	
18		.000, 0018		.000, 0231		.000, 2902		.001, 5175		.005, 9774	
19		.000, 0013		.000, 0166		.000, 1690		.001, 0069		.004, 6651	
20		.000, 0009		.000, 0122		.000, 1257		.000, 7784		.003, 6728	
		.000, 0006		.000, 0090		.000, 0946		.000, 6081		.002, 9188	

TABLE I (Continued)

n = 10		3		4		5		6	
μ^*									
10	.000,0108	.000,1053	.000,9851	.000,8924	.018,5217				
11	.000,0057	.000,0395	.000,5695	.002,7387	.011,3846				
12	.000,0031	.000,0190	.000,3026	.001,4820	.007,8640				
13	.000,0017	.000,0093	.000,2086	.000,9571	.005,2933				
14	.000,0010	.000,0052	.000,1487	.000,7282	.003,7316				
15	.000,0006	.000,0034	.000,1047	.000,5695	.002,5319				
16	.000,0004	.000,0024	.000,0857	.000,4385	.001,9586				
17	.000,0002	.000,0016	.000,0671	.000,3375	.001,4716				
18	.000,0001	.000,0010	.000,0502	.000,2645	.001,0857				
19	.000,0001	.000,0006	.000,0371	.000,2187	.000,8487				
20	.000,0001	.000,0004	.000,0274	.000,1864	.000,6982				

n = 9		13		14		15		16	
μ^*									
9	.891,0125	.995,5327	.987,7628	.996,9971	.999,6298				
10	.834,1705	.923,2826	.974,2038	.992,3800	.998,6262				
11	.773,0650	.885,0917	.955,1024	.985,1155	.996,5468				
12	.711,0502	.843,0817	.931,1026	.975,1131	.993,0766				
13	.650,4634	.799,0712	.903,0960	.962,1137	.988,0141				
14	.592,7985	.754,4892	.872,0824	.947,6780	.981,2761				
15	.538,9016	.710,3917	.839,0093	.930,8790	.972,8766				
16	.489,1695	.667,5192	.804,7113	.912,5050	.962,9021				
17	.443,6966	.626,3616	.769,6856	.892,9062	.951,4874				
18	.402,3771	.587,2186	.735,0919	.872,4027	.938,7944				
19	.365,0025	.550,2503	.700,7642	.851,2780	.924,9591				
20	.331,2952	.515,5164	.667,2279	.829,7760	.910,2760				

n = 10		8		9		10		11	
μ^*									
10	.051,2565	.127,6186	.282,2113	.414,0704	.585,9256				
11	.034,6893	.092,0457	.184,9250	.324,9607	.500,0000				
12	.024,1724	.067,0398	.142,0576	.270,6597	.424,9821				
13	.017,0279	.049,3337	.109,9071	.218,9393	.350,6811				
14	.012,1820	.036,6806	.085,6777	.177,5474	.306,1650				
15	.008,8413	.027,5493	.067,3038	.144,4744	.260,2702				
16	.006,5028	.020,6915	.053,2728	.118,0313	.221,6449				
17	.004,8423	.015,9941	.042,4797	.096,6449	.189,2857				
18	.003,6475	.012,5877	.034,1158	.079,8184	.162,0831				
19	.002,7769	.009,6211	.027,5869	.066,0851	.139,2318				
20	.002,1353	.007,5536	.022,4538	.054,9634	.119,9827				

n = 9		17		18	
μ^*					
9	.999,9589				
10	.999,7943	1.			
11	.999,4046	.999,9892			
12	.998,6902	.999,9405			
13	.997,5674	.999,8129			
14	.995,9752	.999,1251			
15	.993,8754	.998,4688			
16	.991,2505	.997,5501			
17	.988,1007	.996,3387			
18	.984,4394	.994,6131			
19	.980,2699	.992,9607			
20	.975,6823	.990,7760			

n = 10		13		14		15		16	
μ^*									
10	.757,1887	.872,3614	.946,4532	.981,4783	.995,5076				
11	.680,0429	.815,0750	.915,0988	.965,1107	.989,6064				
12	.605,0250	.755,0607	.875,0893	.943,6771	.980,4205				
13	.525,1326	.695,0464	.830,7309	.917,9567	.967,7999				
14	.471,5305	.636,8959	.785,8875	.888,8814	.951,8778				
15	.444,5713	.581,7741	.736,1152	.857,8832	.932,9789				
16	.424,1136	.530,3271	.688,6257	.824,3101	.911,5359				
17	.419,7420	.482,8375	.642,3087	.790,3890	.888,0244				
18	.280,9098	.439,3464	.597,7810	.756,2166	.862,9182				
19	.247,0268	.399,7545	.555,4405	.722,2661	.836,6609				
20	.217,1115	.363,2048	.515,5164	.688,2010	.809,5510				

TABLE I (Continued)

n = 11		12	13	14	15	16
n	n'					
11		.590,0214	.710,0572	.865,0869	.936,5325	.977,3586
12		.507,1756	.665,0793	.806,5517	.900,8097	.959,3462
13		.433,4692	.593,2631	.748,7549	.859,8062	.935,9756
14		.369,0039	.526,6923	.685,3430	.815,3856	.907,7803
15		.313,6559	.466,0183	.625,2637	.769,1883	.875,7976
16		.266,4962	.411,6032	.572,6332	.722,5468	.841,0015
17		.226,5446	.363,2342	.519,6576	.676,4611	.804,3369
18		.192,6144	.320,2187	.470,7594	.631,7315	.766,6415
19		.164,3400	.282,9585	.445,6872	.588,8056	.728,6214
20		.140,4323	.250,0211	.384,6040	.548,0260	.690,8486

n = 10		17	18	19	20
n	n'				
10		.999,0149	.999,8917	.999,9892	1.
11		.997,2613	.999,5977	.999,9105	.999,9972
12		.994,3393	.998,7922	.999,5129	.999,9830
13		.989,5664	.997,3944	.999,5977	.999,9423
14		.983,3760	.995,1878	.999,1251	.999,8942
15		.975,5014	.992,0380	.998,4688	.999,6938
16		.966,0920	.987,8984	.997,5901	.999,4346
17		.955,1487	.982,6087	.996,3387	.999,0908
18		.942,9443	.976,2895	.994,8131	.998,5180
19		.929,6066	.968,9298	.992,9607	.997,8154
20		.915,3073	.960,5885	.990,7760	.996,9253

n = 11		17	18	19	20	21
n	n'					
11		.992,6683	.998,4095	.999,6853	.999,9688	.999,9972
12		.985,0134	.995,9985	.999,0474	.999,8610	.999,9830
13		.974,0303	.991,8779	.997,8270	.999,5898	.999,9423
14		.959,7523	.985,7383	.995,8400	.999,0521	.999,8542
15		.942,4283	.971,4095	.992,9587	.998,1390	.999,6938
16		.922,4391	.946,8596	.989,0699	.996,7475	.999,4346
17		.900,2288	.924,8596	.984,1342	.994,7889	.999,0908
18		.876,2529	.903,1949	.978,1410	.992,1941	.998,5180
19		.850,9402	.882,0528	.971,1184	.988,9151	.997,8154
20		.824,7448	.860,0625	.963,1042	.984,9243	.996,9253

n = 11		2	3	4	5	6
n	n'					
11		.000,0028	.000,0312	.000,3147	.001,5905	.007,3317
12		.000,0015	.000,0170	.000,1797	.000,9528	.004,6136
13		.000,0008	.000,0096	.000,1098	.000,5865	.002,9632
14		.000,0004	.000,0056	.000,0639	.000,3702	.001,9451
15		.000,0003	.000,0034	.000,0396	.000,2389	.001,2590
16		.000,0002	.000,0021	.000,0291	.000,1574	.000,8822
17		.000,0001	.000,0013	.000,0162	.000,1096	.000,6085
18		.000,0001	.000,0008	.000,0107	.000,0721	.000,4259
19		.000,0000	.000,0005	.000,0071	.000,0500	.000,3020
20		.000,0000	.000,0004	.000,0049	.000,0351	.000,2165

n = 11		22
n	n'	
11		.999,9993
12		.999,9993
13		.999,9995
14		.999,9995
15		.999,9999
16		.999,9999
17		.999,9999
18		.999,9999
19		.999,9999
20		.999,9999

n = 11		7	8	9	10	11
n	n'					
11		.022,6414	.063,4675	.134,9131	.259,9428	.409,9786
12		.014,9866	.044,2748	.099,1903	.201,6892	.334,9607
13		.010,1052	.031,2578	.073,5631	.156,8515	.273,4654
14		.008,9322	.022,3314	.055,0946	.122,4256	.221,8423
15		.004,8324	.016,1395	.041,5803	.095,9954	.183,0986
16		.003,4190	.011,7946	.031,6866	.075,6585	.150,4106
17		.002,4626	.008,7113	.024,3580	.059,9942	.124,0275
18		.001,7820	.006,4992	.018,8817	.047,7741	.102,6697
19		.001,3103	.004,8954	.014,7542	.038,2809	.085,3343
20		.000,9742	.003,7208	.011,6173	.030,8434	.071,2182

TABLE I (Continued)

n = 12												
n _i		17	18	19	20	21						21
12		.970,3693	.990,5050	.997,2169	.999,4642	.999,9016						.999,9016
13		.980,2936	.981,6053	.993,8609	.998,6314	.999,6779						.999,6779
14		.985,1043	.984,0866	.988,6857	.996,7694	.999,2125						.999,2125
15		.989,7868	.992,7931	.981,9643	.993,9643	.998,3982						.998,3982
16		.985,2117	.993,0154	.971,7952	.989,8925	.997,1314						.997,1314
17		.988,5059	.910,1487	.980,1613	.984,4098	.995,3216						.995,3216
18		.791,9371	.884,6877	.946,5215	.977,4363	.989,8968						.989,8968
19		.794,8369	.897,1617	.931,0690	.968,9611	.989,6051						.989,6051
20		.717,6275	.828,0928	.914,0089	.959,0129	.986,0153						.986,0153

n = 12												
n _i		2	3	4	5	6						6
12		.000,0007	.000,0089	.000,0961	.000,5456	.002,7631						.002,7631
13		.000,0004	.000,0018	.000,0556	.000,3221	.001,7182						.001,7182
14		.000,0002	.000,0027	.000,0752	.000,4936	.001,8970						.001,8970
15		.000,0001	.000,0031	.000,0753	.000,1211	.000,4595						.000,4595
16		.000,0000	.000,0009	.000,0769	.000,0769	.000,3041						.000,3041
17		.000,0000	.000,0003	.000,0417	.000,0437	.000,2057						.000,2057
18		.000,0000	.000,0000	.000,0047	.000,0328	.000,1412						.000,1412
19		.000,0000	.000,0002	.000,0030	.000,0220	.000,1142						.000,1142
20		.000,0000	.000,0001	.000,0020	.000,0150	.000,0985						.000,0985

n = 12												
n _i		22	23	24							24	
12		.999,9911	.999,9999	1.	.999,9996							.999,9996
13		.999,9971	.999,9992	.999,9985	.999,9987							.999,9987
14		.999,9940	.999,9925	.999,9946	.999,9946							.999,9946
15		.999,9950	.999,9925	.999,9950	.999,9950							.999,9950
16		.999,9970	.999,9973	.999,9973	.999,9973							.999,9973
17		.999,9920	.999,9956	.999,9949	.999,9949							.999,9949
18		.997,4925	.999,9925	.999,9925	.999,9925							.999,9925
19		.994,6286	.999,9914	.999,9914	.999,9914							.999,9914
20		.999,0161	.999,1074	.999,1768	.999,1768							.999,1768

n = 12												
n _i		7	8	9	10	11						11
12		.009,4090	.029,6307	.069,9020	.150,4447	.261,2045						.261,2045
13		.006,1391	.020,0998	.049,7634	.112,5897	.204,6946						.204,6946
14		.004,0450	.013,6175	.035,8056	.084,6582	.152,6402						.152,6402
15		.002,7123	.009,6222	.026,0375	.064,0375	.122,6402						.122,6402
16		.001,8486	.006,7842	.019,1233	.048,7346	.102,0312						.102,0312
17		.001,2791	.004,8401	.014,1876	.037,1346	.081,3130						.081,3130
18		.000,8976	.003,4020	.010,6267	.028,4770	.065,1080						.065,1080
19		.000,6383	.002,5463	.008,0322	.022,3434	.052,3069						.052,3069
20		.000,4693	.001,8755	.006,1241	.017,4538	.042,1791						.042,1791

n = 13												
n _i		2	3	4	5	6						6
13		.000,0002	.000,0005	.000,0302	.000,1895	.001,0201						.001,0201
14		.000,0001	.000,0013	.000,0169	.000,1083	.000,6196						.000,6196
15		.000,0001	.000,0007	.000,0097	.000,0636	.000,3644						.000,3644
16		.000,0000	.000,0004	.000,0057	.000,0389	.000,2431						.000,2431
17		.000,0000	.000,0003	.000,0035	.000,0243	.000,1586						.000,1586
18		.000,0000	.000,0002	.000,0021	.000,0155	.000,1025						.000,1025
19		.000,0000	.000,0001	.000,0013	.000,0100	.000,0682						.000,0682
20		.000,0000	.000,0001	.000,0009	.000,0066	.000,0460						.000,0460

n = 12												
n _i		12	13	14	15	16						16
12		.421,0682	.576,9318	.736,7955	.849,5553	.930,0980						.930,0980
13		.347,5488	.500,0000	.664,1782	.793,1754	.893,6927						.893,6927
14		.285,9820	.429,5379	.593,6161	.734,6403	.851,8105						.851,8105
15		.235,0563	.368,0711	.527,6888	.675,9062	.806,2042						.806,2042
16		.193,2517	.314,4652	.466,8820	.618,8989	.758,8068						.758,8068
17		.159,0836	.269,2601	.411,8414	.564,4072	.710,0983						.710,0983
18		.131,2154	.230,3743	.362,5861	.513,6853	.662,0863						.662,0863
19		.108,4967	.197,3213	.318,8709	.466,4668	.615,3029						.615,3029
20		.089,9637	.169,2714	.280,3023	.423,0562	.570,3420						.570,3420

TABLE I (Continued)

n = 14		12	13	14	15	16
n	n'					
14		.169,6893	.279,7927	.426,5976	.573,4024	.720,2073
15		.130,6149	.224,7627	.351,6246	.500,0000	.651,2671
16		.100,7163	.180,4465	.272,6252	.374,3591	.485,4252
17		.077,8803	.145,0107	.226,6924	.314,4397	.422,2555
18		.060,4353	.116,7457	.186,6312	.252,5788	.344,2682
19		.047,0863	.094,2243	.142,0214	.211,5305	.311,0197
20		.036,6049	.071,2651	.113,1167	.176,7258	.265,1965

n = 14		17	18	19	20	21	22
n	n'						
14		.430,3109	.912,8687	.948,7652	.968,2522	.994,4469	.998,5248
15		.775,2591	.874,9219	.915,8269	.972,7391	.989,3495	.996,7230
16		.715,3049	.852,2093	.906,1428	.957,3590	.981,9671	.993,7790
17		.661,3588	.786,2817	.876,5039	.918,1942	.972,1259	.989,3972
18		.605,3779	.736,5681	.841,8030	.915,5423	.959,7458	.983,3824
19		.552,7452	.690,3024	.804,5934	.889,8053	.945,0190	.975,6062
20		.502,7284	.642,4694	.766,7231	.861,6246	.928,0556	.966,0162

n = 14		23	24	25	26	27	28
n	n'						
14		.999,6170	.999,9403	.999,9909	.999,9993	.999,9999	1.
15		.999,0809	.999,6121	.999,8662	.999,9861	.999,9996	.999,9999
16		.999,0712	.999,0319	.999,2006	.999,8679	.999,9985	.999,9999
17		.999,1645	.999,0397	.999,1826	.999,8640	.999,9953	.999,9999
18		.999,1037	.999,0211	.999,1648	.999,8697	.999,9885	.999,9986
19		.999,4904	.999,0513	.999,2272	.999,8616	.999,9753	.999,9963
20		.999,7220	.999,1365	.999,7213	.999,6631	.999,9527	.999,9916

n = 15		2	3	4	5	6
n	n'					
15		.000,0000	.000,0002	.000,0027	.000,0131	.000,1289
16		.000,0000	.000,0001	.000,0015	.000,0109	.000,0745
17		.000,0000	.000,0001	.000,0008	.000,0064	.000,0450
18		.000,0000	.000,0000	.000,0005	.000,0038	.000,0277
19		.000,0000	.000,0000	.000,0003	.000,0023	.000,0173
20		.000,0000	.000,0000	.000,0002	.000,0014	.000,0110

n = 15		7	8	9	10	11
n	n'					
15		.000,5530	.002,2613	.006,9592	.019,8785	.046,7170
16		.000,3395	.001,4416	.004,6103	.013,7030	.032,7177
17		.000,2123	.000,9329	.003,6948	.009,5355	.023,7050
18		.000,1351	.000,6124	.002,1040	.006,6960	.017,2653
19		.000,0873	.000,4074	.001,4477	.004,7464	.012,6703
20		.000,0573	.000,2745	.001,0076	.003,3967	.009,3695

n = 15		12	13	14	15	16
n	n'					
15		.097,1910	.174,9095	.291,1827	.424,0664	.575,9316
16		.072,6057	.136,1517	.236,1717	.357,6246	.504,5928
17		.054,6203	.106,1477	.191,1627	.300,4703	.439,2736
18		.041,1534	.082,9953	.151,6265	.251,8852	.380,5976
19		.031,1546	.065,0425	.125,1165	.210,9365	.328,6325
20		.023,7043	.051,1792	.101,3509	.176,6085	.283,0953

TABLE I (Continued)

n = 15		17		18		19		20		21	
n	u										
15		.704, 8173	.825, 0905	.902, 0660	.954, 2930	.980, 1215					
16		.642, 3754	.770, 9725	.863, 8483	.930, 5283	.967, 2023					
17		.574, 0768	.714, 7113	.828, 9225	.901, 5911	.950, 5323					
18		.517, 9504	.658, 1253	.775, 0377	.869, 2877	.930, 5901					
19		.451, 0404	.602, 6434	.728, 5128	.833, 4039	.906, 8276					
20		.405, 5445	.549, 3124	.681, 3118	.795, 1935	.880, 6049					

n = 15		22		23		24		25		26	
n	u										
15		.993, 0408	.997, 7387	.999, 4470	.999, 8741	.999, 9809					
16		.997, 2963	.999, 3897	.999, 6962	.999, 6623	.999, 9863					
17		.997, 8713	.999, 1288	.999, 3736	.999, 2412	.999, 5227					
18		.997, 8201	.998, 6941	.999, 2870	.999, 6412	.999, 6375					
19		.997, 9265	.999, 1746	.999, 2915	.999, 4687	.999, 2792					
20		.997, 5457	.999, 1388	.998, 1337	.999, 5984	.998, 7233					

n = 15		27		28		29		30	
n	u								
15		.999, 9973	.999, 9798	.999, 9999	.999, 9999	1.			
16		.999, 9991	.999, 9982	.999, 9993	1.				
17		.999, 9979	.999, 9785	.999, 9725	1.				
18		.999, 9228	.999, 9271	.999, 9982	.999, 9999				
19		.999, 8193	.999, 9686	.999, 9963	.999, 9996				
20		.999, 6223	.999, 9132	.999, 9916	.999, 9988				

n = 16		7		8		9		10		11	
n	u										
16		.000, 2017	.000, 5905	.002, 9571	.009, 1586	.022, 7957					
17		.000, 1222	.000, 5590	.001, 5236	.006, 1821	.015, 9762					
18		.000, 0734	.000, 3362	.001, 6387	.004, 2167	.011, 6281					
19		.000, 0473	.000, 2302	.000, 8473	.002, 9051	.006, 0482					
20		.000, 0502	.000, 1599	.000, 5732	.002, 0211	.005, 7856					

n = 16		12		13		14		15		16	
n	u										
16		.052, 8017	.102, 8117	.186, 1617	.293, 1260	.431, 1087					
17		.018, 4600	.077, 8067	.146, 4071	.239, 7439	.365, 9287					
18		.028, 1849	.059, 0702	.115, 2782	.192, 5782	.309, 3465					
19		.020, 7442	.045, 0178	.090, 7888	.153, 4445	.260, 1267					
20		.015, 3423	.034, 4574	.071, 6196	.110, 0203	.218, 7595					

TABLE I (Continued)

m = 16		17	18	19	20	21
u	v					
15		.568, 8313	.706, 6740	.813, 8383	.897, 1883	.947, 1983
17		.500, 0000	.641, 9579	.750, 2561	.858, 3998	.922, 1933
18		.436, 9076	.578, 8555	.705, 0503	.815, 4520	.892, 7501
19		.380, 1245	.518, 8376	.649, 8644	.769, 7200	.859, 5265
20		.329, 6833	.462, 7919	.595, 9004	.722, 4357	.823, 6640

m = 16		22	23	24	25	26
u	v					
16		.977, 2043	.930, 8433	.937, 0429	.999, 1095	.999, 7983
17		.963, 4136	.984, 0238	.994, 2437	.998, 0762	.999, 4956
18		.945, 7478	.974, 6555	.989, 9854	.996, 3129	.998, 9279
19		.924, 3594	.962, 6106	.984, 0099	.993, 8179	.997, 9789
20		.899, 5852	.947, 8986	.976, 1338	.990, 2814	.996, 5258

m = 16		27	28	29	30	31	32
u	v						
16		.999, 9573	.999, 9340	.999, 9992	.999, 9999	1.	1.
17		.999, 8778	.999, 9786	.999, 9956	.999, 9997	1.	1.
18		.999, 7140	.999, 9408	.999, 9894	.999, 9986	.999, 9999	1.
19		.999, 4192	.999, 8624	.999, 9732	.999, 9958	.999, 9996	1.
20		.998, 9391	.999, 7188	.999, 9415	.999, 9893	.999, 9988	.999, 9999

m = 17		2	3	4	5	6
u	v					
17		.000, 0000	.000, 0000	.000, 0002	.000, 0019	.000, 0142
18		.000, 0000	.000, 0000	.000, 0001	.000, 0011	.000, 0083
19		.000, 0000	.000, 0000	.000, 0001	.000, 0005	.000, 0049
20		.000, 0000	.000, 0000	.000, 0000	.000, 0004	.000, 0023

m = 17		7	8	9	10	11
u	v					
17		.000, 0718	.000, 3406	.001, 2141	.004, 0520	.010, 8652
18		.000, 0470	.000, 2109	.000, 7773	.002, 6855	.007, 4596
19		.000, 0262	.000, 1325	.000, 5046	.001, 8001	.005, 1585
20		.000, 0163	.000, 0845	.000, 3318	.001, 2189	.003, 6139

m = 17		12	13	14	15	16
u	v					
17		.027, 2181	.057, 1966	.112, 1570	.190, 6720	.302, 8353
18		.019, 3731	.042, 2073	.085, 8902	.151, 4145	.249, 4781
19		.013, 8746	.031, 8866	.065, 8689	.120, 2125	.201, 8018
20		.010, 0005	.023, 3061	.050, 6268	.095, 6108	.167, 5444

m = 17		17	18	19	20	21
u	v					
17		.429, 0211	.570, 9789	.697, 1637	.809, 3280	.887, 8130
18		.365, 9287	.503, 8306	.634, 0713	.756, 6808	.848, 6851
19		.310, 7679	.441, 7747	.572, 7816	.702, 1711	.816, 6822
20		.263, 1290	.385, 4450	.514, 5563	.647, 4428	.760, 3563

TABLE I (Continued)

n = 17		22		23		24		25		26	
n	u	u	u	u	u	u	u	u	u	u	u
17	.942, 8034	.972, 7619	.969, 1336	.995, 9471	.998, 7659						
18	.917, 2300	.957, 7927	.981, 6197	.992, 9424	.997, 4044						
19	.887, 1980	.939, 0714	.971, 4081	.987, 5764	.995, 1761						
20	.853, 4169	.916, 8400	.958, 3533	.980, 6596	.992, 3711						

n = 17		27		28		29		30		31	
n	u	u	u	u	u	u	u	u	u	u	u
17	.993, 6594	.999, 9282	.999, 9858	.999, 9981	.999, 9998						
18	.999, 2227	.999, 6101	.999, 9570	.999, 9929	.999, 9989						
19	.998, 4590	.999, 5751	.999, 6540	.999, 9765	.999, 9969						
20	.997, 2498	.999, 1604	.999, 7745	.999, 9499	.999, 9989						

n = 17		32		33		34	
n	u	u	u	u	u	u	u
17	1.	1.	1.	1.	1.	1.	1.
18	.999, 9999	1.	1.	1.	1.	1.	1.
19	.999, 9996	1.	1.	1.	1.	1.	1.
20	.999, 9986	1.	.999, 9999	1.	1.	1.	1.

n = 18		2		3		4		5		6	
n	u	u	u	u	u	u	u	u	u	u	u
18	.000, 0000	.000, 0000	.000, 0000	.000, 0001	.000, 0006	.000, 0047					
19	.000, 0000	.000, 0000	.000, 0000	.000, 0000	.000, 0003	.000, 0027					
20	.000, 0000	.000, 0000	.000, 0000	.000, 0000	.000, 0002	.000, 0016					

n = 18		7		8		9		10		11	
n	u	u	u	u	u	u	u	u	u	u	u
18	.000, 0250	.000, 1269	.000, 4836	.001, 7319	.004, 9776						
19	.000, 0148	.000, 0776	.000, 3053	.001, 1295	.003, 3548						
20	.000, 0090	.000, 0462	.000, 1954	.000, 7448	.002, 2833						

n = 18		12		13		14		15		16	
n	u	u	u	u	u	u	u	u	u	u	u
18	.013, 4163	.030, 2938	.064, 0488	.117, 0923	.200, 4463						
19	.009, 3540	.021, 8563	.047, 8555	.090, 5705	.150, 6125						
20	.006, 5591	.015, 8550	.035, 8533	.070, 1416	.128, 5999						

n = 18		17		18		19		20		21	
n	u	u	u	u	u	u	u	u	u	u	u
18	.304, 6189	.434, 8796	.565, 1204	.695, 3611	.799, 5537						
19	.252, 5426	.372, 9273	.500, 0000	.633, 7607	.747, 4674						
20	.208, 7664	.318, 2070	.439, 8077	.573, 5684	.693, 9531						

n = 18		22		23		24		25		26	
n	u	u	u	u	u	u	u	u	u	u	u
18	.882, 9077	.935, 9512	.969, 7062	.986, 5837	.995, 0224						
19	.813, 7651	.909, 4295	.954, 0017	.978, 1449	.991, 1451						
20	.800, 9617	.878, 7861	.934, 5013	.967, 0019	.985, 5736						

n = 18		27		28		29		30		31	
n	u	u	u	u	u	u	u	u	u	u	u
18	.998, 2681	.999, 5164	.999, 8731	.999, 9790	.999, 9954						
19	.996, 6452	.998, 9302	.999, 6947	.999, 9852	.999, 9852						
20	.994, 1452	.997, 9914	.999, 3651	.999, 8360	.999, 9616						

n = 18		32		33		34		35		36	
n	u	u	u	u	u	u	u	u	u	u	u
18	.999, 9994	.999, 9999	1.	1.	1.	1.	1.	1.	1.	1.	1.
19	.999, 9977	.999, 9997	1.	1.	1.	1.	1.	1.	1.	1.	1.
20	.999, 9930	.999, 9989	.999, 9999	.999, 9999	.999, 9999	1.	1.	1.	1.	1.	1.

TABLE I (Concluded)

$n = 19$

u'	2	3	4	5	6
19	.000,0000	.000,0000	.000,0000	.000,0002	.000,0015
20	.000,0000	.000,0000	.000,0000	.000,0001	.000,0009

$n = 19$

u'	7	8	9	10	11
19	.000,0086	.000,0462	.000,1875	.000,7174	.002,2009
20	.000,0050	.000,0280	.000,1169	.000,4611	.001,4591

$n = 19$

u'	12	13	14	15	16
19	.006,3548	.015,3550	.034,8553	.068,2844	.125,5915
20	.004,3501	.010,8549	.025,4705	.051,5699	.098,1013

$n = 19$

u'	17	18	19	20	21
19	.204,3888	.312,7350	.433,1196	.566,8804	.687,2650
20	.164,9901	.260,9611	.372,9273	.503,2583	.627,0727

$n = 19$

u'	22	23	24	25	26
19	.795,5112	.874,4085	.931,7156	.965,1447	.984,6450
20	.744,3706	.835,0099	.904,8070	.948,4301	.975,5734

$n = 19$

u'	27	28	29	30	31
19	.993,6452	.997,7991	.999,2826	.999,8125	.999,9538
20	.989,1451	.995,8308	.998,5409	.999,5734	.999,8831

$n = 19$

u'	32	33	34	35	36
19	.999,9914	.999,9985	.999,9998	.999,9999	1.
20	.999,9749	.999,9950	.999,9993	.999,9999	1.

$n = 20$

u'	2	3	4	5	6
20	.000,0000	.000,0000	.000,0000	.000,0001	.000,0005

$n = 20$

u'	7	8	9	10	11
20	.000,0029	.000,0165	.000,0710	.000,2890	.000,9429

$n = 20$

u'	12	13	14	15	16
20	.002,9046	.007,4821	.018,1627	.037,9982	.074,8556

$n = 20$

u'	17	18	19	20	21
20	.130,0916	.212,9756	.314,2784	.438,0928	.561,9072

$n = 20$

u'	22	23	24	25	26
20	.685,7216	.787,0244	.869,9084	.925,1644	.962,0018

$n = 20$

u'	27	28	29	30	31
20	.981,8373	.992,5179	.997,0954	.999,0571	.999,7110

$n = 20$

u'	32	33	34	35	36
20	.999,9290	.999,9835	.999,9971	.999,9995	.999,9999

$n = 20$

u'	37	38	39	40
20	1.	1.	1.	1.

TABLE II

Significance Levels of u

When $\Sigma < .50$, $u \in$ is the largest integer, u' , for which
 $P\{u \leq u'\} \leq \epsilon$; when $\Sigma > .50$, u is the smallest integer, u' , for which
 $P\{u \leq u'\} \geq \epsilon$.

$n = 2$

n	$u_{.005}$	$u_{.01}$	$u_{.025}$	$u_{.05}$	$u_{.95}$	$u_{.975}$	$u_{.99}$	$u_{.995}$
2				4	4	4	4	4
3				5	5	5	5	5
4				5	5	5	5	5
5				5	5	5	5	5
6				5	5	5	5	5
7				5	5	5	5	5
8				5	5	5	5	5
9			2	5	5	5	5	5
10			2	5	5	5	5	5
11			2	5	5	5	5	5
12			2	5	5	5	5	5
13			2	5	5	5	5	5
14			2	5	5	5	5	5
15			2	5	5	5	5	5
16			2	5	5	5	5	5
17			2	5	5	5	5	5
18			2	5	5	5	5	5
19			2	5	5	5	5	5
20	2	2	2	5	5	5	5	5

$n = 4$

n	$u_{.005}$	$u_{.01}$	$u_{.025}$	$u_{.05}$	$u_{.95}$	$u_{.975}$	$u_{.99}$	$u_{.995}$
4				2	7	8	8	8
5				2	8	8	8	8
6				2	8	8	8	8
7		2	2	2	9	9	9	9
8		2	2	2	9	9	9	9
9	2	2	2	2	9	9	9	9
10	2	2	2	2	9	9	9	9
11	2	2	2	2	9	9	9	9
12	2	2	2	2	9	9	9	9
13	2	2	2	2	9	9	9	9
14	2	2	2	2	9	9	9	9
15	2	2	2	2	9	9	9	9
16	2	2	2	2	9	9	9	9
17	2	2	2	2	9	9	9	9
18	2	2	2	2	9	9	9	9
19	2	2	2	2	9	9	9	9
20	2	2	2	2	9	9	9	9

$n = 3$

n	$u_{.005}$	$u_{.01}$	$u_{.025}$	$u_{.05}$	$u_{.95}$	$u_{.975}$	$u_{.99}$	$u_{.995}$
3				6	6	7	7	7
4				7	7	7	7	7
5				7	7	7	7	7
6				7	7	7	7	7
7				7	7	7	7	7
8				7	7	7	7	7
9				7	7	7	7	7
10			2	7	7	7	7	7
11			2	7	7	7	7	7
12			2	7	7	7	7	7
13			2	7	7	7	7	7
14			2	7	7	7	7	7
15			2	7	7	7	7	7
16			2	7	7	7	7	7
17			2	7	7	7	7	7
18			2	7	7	7	7	7
19			2	7	7	7	7	7
20	2	2	2	7	7	7	7	7

$n = 5$

n	$u_{.005}$	$u_{.01}$	$u_{.025}$	$u_{.05}$	$u_{.95}$	$u_{.975}$	$u_{.99}$	$u_{.995}$
5				3	8	9	9	10
6				3	9	10	10	10
7				3	10	10	10	11
8				3	10	11	11	11
9				3	10	11	11	11
10				3	10	11	11	11
11				3	11	11	11	11
12				3	11	11	11	11
13				3	11	11	11	11
14				3	11	11	11	11
15				3	11	11	11	11
16				3	11	11	11	11
17				3	11	11	11	11
18				3	11	11	11	11
19				3	11	11	11	11
20				3	11	11	11	11

TABLE II (Continued)

n = 8

n	$\alpha_{.005}$	$\alpha_{.01}$	$\alpha_{.025}$	$\alpha_{.05}$	$\alpha_{.95}$	$\alpha_{.975}$	$\alpha_{.99}$	$\alpha_{.995}$
8	3	4	4	5	12	13	13	14
9	3	4	5	5	13	14	14	14
10	4	4	5	6	13	14	14	15
11	4	5	5	6	14	14	15	15
12	4	5	6	6	14	15	15	16
13	5	5	6	6	14	15	16	16
14	5	5	6	7	15	15	16	16
15	5	6	6	7	15	16	16	17
16	5	6	6	7	15	16	17	17
17	6	6	7	7	15	16	17	17
18	6	6	7	8	15	16	17	17
19	6	6	7	8	15	16	17	17
20	6	6	7	8	16	16	17	17

n = 6

n	$\alpha_{.005}$	$\alpha_{.01}$	$\alpha_{.025}$	$\alpha_{.05}$	$\alpha_{.95}$	$\alpha_{.975}$	$\alpha_{.99}$	$\alpha_{.995}$
6	2	2	3	3	10	10	11	11
7	2	3	3	4	10	11	11	12
8	3	3	3	4	11	11	12	12
9	3	3	4	4	11	12	12	13
10	3	4	4	5	11	12	13	13
11	3	4	4	5	12	12	13	13
12	3	4	4	5	12	12	13	13
13	3	4	5	5	12	13	13	13
14	4	4	5	5	12	13	13	13
15	4	4	5	6	13	13	13	13
16	4	5	5	6	13	13	13	13
17	4	5	5	6	13	13	13	13
18	4	5	6	6	13	13	13	13
19	4	5	6	6	13	13	13	13
20	4	5	6	6	13	13	13	13

n = 9

n	$\alpha_{.005}$	$\alpha_{.01}$	$\alpha_{.025}$	$\alpha_{.05}$	$\alpha_{.95}$	$\alpha_{.975}$	$\alpha_{.99}$	$\alpha_{.995}$
9	4	4	5	6	13	14	15	15
10	4	5	5	6	14	15	15	16
11	5	5	6	6	14	15	16	16
12	5	5	6	7	15	15	16	17
13	5	6	6	7	15	16	17	17
14	5	6	7	7	16	16	17	17
15	6	6	7	8	16	17	17	18
16	6	6	7	8	16	17	17	18
17	6	7	7	8	16	17	18	18
18	6	7	8	8	17	17	18	19
19	6	7	8	8	17	17	18	19
20	7	7	8	9	17	17	18	19

n = 7

n	$\alpha_{.005}$	$\alpha_{.01}$	$\alpha_{.025}$	$\alpha_{.05}$	$\alpha_{.95}$	$\alpha_{.975}$	$\alpha_{.99}$	$\alpha_{.995}$
7	3	3	3	4	11	12	12	12
8	3	3	4	4	12	12	13	13
9	3	4	4	5	12	13	13	14
10	3	4	4	5	12	13	14	14
11	4	4	5	5	13	13	14	14
12	4	4	5	6	13	13	14	15
13	4	5	5	6	13	14	15	15
14	4	5	6	6	13	14	15	15
15	4	5	6	6	14	14	15	15
16	5	5	6	6	14	15	15	15
17	5	5	6	7	14	15	15	15
18	5	6	6	7	14	15	15	15
19	5	6	6	7	14	15	15	15
20	5	6	6	7	14	15	15	15

TABLE II (Continued)

■ = 12

n	u _{.005}	u _{.01}	u _{.025}	u _{.05}	u _{.95}	u _{.975}	u _{.99}	u _{.995}
12	6	7	7	8	17	18	18	19
13	6	7	8	9	17	18	19	20
14	7	7	8	9	18	19	20	20
15	7	8	8	9	18	19	20	21
16	7	8	9	10	19	20	21	21
17	8	8	9	10	20	20	21	21
18	8	8	9	10	20	20	21	22
19	8	9	10	10	20	21	22	22
20	8	9	10	11	20	21	22	22

■ = 10

n	u _{.005}	u _{.01}	u _{.025}	u _{.05}	u _{.95}	u _{.975}	u _{.99}	u _{.995}
10	5	5	6	6	15	15	16	16
11	5	5	6	7	15	16	17	17
12	5	6	7	7	16	17	18	18
13	5	6	7	8	16	17	18	19
14	6	6	7	8	16	17	18	19
15	6	7	7	8	17	18	19	19
16	6	7	8	8	17	18	19	20
17	7	7	8	9	17	18	19	20
18	7	7	8	9	18	19	20	20
19	7	8	8	9	18	19	20	20
20	7	8	9	9	18	19	20	20

■ = 13

n	u _{.005}	u _{.01}	u _{.025}	u _{.05}	u _{.95}	u _{.975}	u _{.99}	u _{.995}
13	7	7	8	9	18	19	20	20
14	7	8	9	9	19	19	20	21
15	7	8	9	10	19	20	21	21
16	8	8	9	10	20	20	21	22
17	8	9	10	10	20	21	22	22
18	8	9	10	11	20	21	22	23
19	9	9	10	11	21	22	23	23
20	9	10	10	11	21	22	23	23

■ = 11

n	u _{.005}	u _{.01}	u _{.025}	u _{.05}	u _{.95}	u _{.975}	u _{.99}	u _{.995}
11	5	6	7	7	16	16	17	18
12	6	6	7	8	16	17	18	18
13	6	6	7	8	17	18	18	19
14	6	7	8	8	17	18	19	19
15	7	7	8	9	18	18	19	20
16	7	7	8	9	18	19	20	20
17	7	8	9	9	18	19	20	21
18	7	8	9	10	19	19	20	21
19	8	8	9	10	19	20	21	21
20	8	8	9	10	19	20	21	21

TABLE II (Continued)

$n = 14$										
n	$u_{.005}$	$u_{.01}$	$u_{.025}$	$u_{.05}$	$u_{.95}$	$u_{.975}$	$u_{.99}$	$u_{.995}$		
14	7	8	9	10	19	20	21	22		
15	8	8	9	10	20	21	22	22		
16	8	9	10	11	20	21	22	23		
17	8	9	10	11	21	22	23	23		
18	9	9	10	11	21	22	23	24		
19	9	10	11	12	22	22	23	24		
20	9	10	11	12	22	23	24	24		

$n = 15$										
n	$u_{.005}$	$u_{.01}$	$u_{.025}$	$u_{.05}$	$u_{.95}$	$u_{.975}$	$u_{.99}$	$u_{.995}$		
15	8	9	10	11	20	21	22	23		
16	9	10	11	11	21	22	23	23		
17	9	10	11	12	21	22	23	24		
18	9	10	11	12	22	23	24	24		
19	10	10	11	12	22	23	24	25		
20	10	11	12	12	23	24	25	25		

$n = 16$										
n	$u_{.005}$	$u_{.01}$	$u_{.025}$	$u_{.05}$	$u_{.95}$	$u_{.975}$	$u_{.99}$	$u_{.995}$		
16	9	10	11	11	22	22	23	24		
17	9	10	11	12	22	23	24	25		
18	10	10	11	12	23	24	25	25		
19	10	11	12	13	23	24	25	26		
20	10	11	12	13	24	24	25	26		

$n = 17$										
n	$u_{.005}$	$u_{.01}$	$u_{.025}$	$u_{.05}$	$u_{.95}$	$u_{.975}$	$u_{.99}$	$u_{.995}$		
17	10	10	11	12	23	24	25	25		
18	10	11	12	13	24	25	26	26		
19	10	11	12	13	24	25	26	26		
20	11	11	13	13	24	25	26	27		

$n = 18$										
n	$u_{.005}$	$u_{.01}$	$u_{.025}$	$u_{.05}$	$u_{.95}$	$u_{.975}$	$u_{.99}$	$u_{.995}$		
18	11	11	12	13	24	25	26	26		
19	11	12	13	14	24	25	26	27		
20	11	12	13	14	25	26	27	28		

$n = 19$										
n	$u_{.005}$	$u_{.01}$	$u_{.025}$	$u_{.05}$	$u_{.95}$	$u_{.975}$	$u_{.99}$	$u_{.995}$		
19	11	12	13	14	25	26	27	28		
20	12	12	13	14	26	26	28	28		

$n = 20$										
n	$u_{.005}$	$u_{.01}$	$u_{.025}$	$u_{.05}$	$u_{.95}$	$u_{.975}$	$u_{.99}$	$u_{.995}$		
19										
20	12	13	14	15	26	27	28	29		

TABLE III

Significance Levels of u

$$P \{ u \leq u_g \} = \epsilon$$

For definition of u_g , see note to Table II.

$m=n$	$u_{.005}$	$u_{.01}$	$u_{.025}$	$u_{.05}$	$u_{.075}$	$u_{.09}$	$u_{.905}$	$u_{.91}$	$u_{.925}$	$u_{.95}$	$u_{.975}$	$u_{.99}$	$u_{.995}$
10	4	5	6	7	8	9	10	11	12	13	14	15	16
11	5	6	7	8	9	10	11	12	13	14	15	16	17
12	6	7	8	9	10	11	12	13	14	15	16	17	18
13	7	8	9	10	11	12	13	14	15	16	17	18	19
14	8	9	10	11	12	13	14	15	16	17	18	19	20
15	9	10	11	12	13	14	15	16	17	18	19	20	21
16	10	11	12	13	14	15	16	17	18	19	20	21	22
17	11	12	13	14	15	16	17	18	19	20	21	22	23
18	12	13	14	15	16	17	18	19	20	21	22	23	24
19	13	14	15	16	17	18	19	20	21	22	23	24	25
20	14	15	16	17	18	19	20	21	22	23	24	25	26
21	15	16	17	18	19	20	21	22	23	24	25	26	27
22	16	17	18	19	20	21	22	23	24	25	26	27	28
23	17	18	19	20	21	22	23	24	25	26	27	28	29
24	18	19	20	21	22	23	24	25	26	27	28	29	30
25	19	20	21	22	23	24	25	26	27	28	29	30	31
26	20	21	22	23	24	25	26	27	28	29	30	31	32
27	21	22	23	24	25	26	27	28	29	30	31	32	33
28	22	23	24	25	26	27	28	29	30	31	32	33	34
29	23	24	25	26	27	28	29	30	31	32	33	34	35
30	24	25	26	27	28	29	30	31	32	33	34	35	36
31	25	26	27	28	29	30	31	32	33	34	35	36	37
32	26	27	28	29	30	31	32	33	34	35	36	37	38
33	27	28	29	30	31	32	33	34	35	36	37	38	39
34	28	29	30	31	32	33	34	35	36	37	38	39	40
35	29	30	31	32	33	34	35	36	37	38	39	40	41
36	30	31	32	33	34	35	36	37	38	39	40	41	42
37	31	32	33	34	35	36	37	38	39	40	41	42	43
38	32	33	34	35	36	37	38	39	40	41	42	43	44
39	33	34	35	36	37	38	39	40	41	42	43	44	45
40	34	35	36	37	38	39	40	41	42	43	44	45	46
41	35	36	37	38	39	40	41	42	43	44	45	46	47
42	36	37	38	39	40	41	42	43	44	45	46	47	48
43	37	38	39	40	41	42	43	44	45	46	47	48	49
44	38	39	40	41	42	43	44	45	46	47	48	49	50
45	39	40	41	42	43	44	45	46	47	48	49	50	51
46	40	41	42	43	44	45	46	47	48	49	50	51	52
47	41	42	43	44	45	46	47	48	49	50	51	52	53
48	42	43	44	45	46	47	48	49	50	51	52	53	54
49	43	44	45	46	47	48	49	50	51	52	53	54	55
50	44	45	46	47	48	49	50	51	52	53	54	55	56
51	45	46	47	48	49	50	51	52	53	54	55	56	57
52	46	47	48	49	50	51	52	53	54	55	56	57	58
53	47	48	49	50	51	52	53	54	55	56	57	58	59
54	48	49	50	51	52	53	54	55	56	57	58	59	60
55	49	50	51	52	53	54	55	56	57	58	59	60	61
56	50	51	52	53	54	55	56	57	58	59	60	61	62
57	51	52	53	54	55	56	57	58	59	60	61	62	63
58	52	53	54	55	56	57	58	59	60	61	62	63	64
59	53	54	55	56	57	58	59	60	61	62	63	64	65
60	54	55	56	57	58	59	60	61	62	63	64	65	66
61	55	56	57	58	59	60	61	62	63	64	65	66	67
62	56	57	58	59	60	61	62	63	64	65	66	67	68
63	57	58	59	60	61	62	63	64	65	66	67	68	69
64	58	59	60	61	62	63	64	65	66	67	68	69	70
65	59	60	61	62	63	64	65	66	67	68	69	70	71
66	60	61	62	63	64	65	66	67	68	69	70	71	72
67	61	62	63	64	65	66	67	68	69	70	71	72	73
68	62	63	64	65	66	67	68	69	70	71	72	73	74
69	63	64	65	66	67	68	69	70	71	72	73	74	75
70	64	65	66	67	68	69	70	71	72	73	74	75	76
71	65	66	67	68	69	70	71	72	73	74	75	76	77
72	66	67	68	69	70	71	72	73	74	75	76	77	78
73	67	68	69	70	71	72	73	74	75	76	77	78	79
74	68	69	70	71	72	73	74	75	76	77	78	79	80
75	69	70	71	72	73	74	75	76	77	78	79	80	81
76	70	71	72	73	74	75	76	77	78	79	80	81	82
77	71	72	73	74	75	76	77	78	79	80	81	82	83
78	72	73	74	75	76	77	78	79	80	81	82	83	84
79	73	74	75	76	77	78	79	80	81	82	83	84	85
80	74	75	76	77	78	79	80	81	82	83	84	85	86
81	75	76	77	78	79	80	81	82	83	84	85	86	87
82	76	77	78	79	80	81	82	83	84	85	86	87	88
83	77	78	79	80	81	82	83	84	85	86	87	88	89
84	78	79	80	81	82	83	84	85	86	87	88	89	90
85	79	80	81	82	83	84	85	86	87	88	89	90	91
86	80	81	82	83	84	85	86	87	88	89	90	91	92
87	81	82	83	84	85	86	87	88	89	90	91	92	93
88	82	83	84	85	86	87	88	89	90	91	92	93	94
89	83	84	85	86	87	88	89	90	91	92	93	94	95
90	84	85	86	87	88	89	90	91	92	93	94	95	96
91	85	86	87	88	89	90	91	92	93	94	95	96	97
92	86	87	88	89	90	91	92	93	94	95	96	97	98
93	87	88	89	90	91	92	93	94	95	96	97	98	99
94	88	89	90	91	92	93	94	95	96	97	98	99	100
95	89	90	91	92	93	94	95	96	97	98	99	100	
96	90	91	92	93	94	95	96	97	98	99	100		
97	91	92	93	94	95	96	97	98	99	100			
98	92	93	94	95	96	97	98	99	100				
99	93	94	95	96	97	98	99	100					
100	94	95	96	97	98	99	100						