THE TEACHING OF STATISTICS

A report of the Institute of Mathematical Statistics Committee on the Teaching of Statistics[1]

PREFATORY NOTE

This report on the teaching of statistics contains two parts. Part I is a summary of the conclusions reached by the committee concerning the appropriate content and organization of teaching in statistics. It is oriented towards the future, and is intended as a program for action. Part II, mainly the work of the chairman of the committee, is a more intensive discussion of the general problem. It surveys the present state of the teaching of statistics, probes some of the reasons for existing weaknesses in this teaching, and states more fully the basis for the conclusions summarized in Part I.

Additional material, with special reference to applied statistics, is contained in a report of The Committee on Applied Mathematical Statistics of the National Research Council, entitled *Personnel and Training Problems Created by the Recent Growth of Applied Statistics in the United States.*[2]

PART I

SUMMARY OF CONCLUSIONS

1. Who are the prospective students of statistics? A complete teaching program in statistics must be designed to meet the needs of four principal categories of students, listed here according to the amount of training in statistics that is needed to meet their requirements.

a. *All college students.* Statistical method is a vital branch of scientific method. It is widely used in most sciences, business, government, and ordinary life. Some understanding of the nature of inductive inference from quantitative data on the basis of the theory of probability as portrayed in statistical method is an indispensable part of a liberal education.

b. *Future consumers of statistics.* Some students will specialize in administration, business, or other subject-matter that will require them to understand the results of statistical analyses of special problems, although they themselves do not make these analyses. For example, business executives and government administrators must frequently base action on statistical studies. Research workers and teachers in many fields may not themselves use statistical methods, yet in order to keep abreast of their own or cognate fields they must read and understand studies using statistical methods.

c. *Future users of statistical methods.* A still smaller group of students of

[1] The Committee consists of Harold Hotelling, Chairman; Walter Bartky, W. Edwards Deming, Milton Friedman, and Paul Hoel.

[2] Copies may be obtained from the National Research Council, 2101 Constitution Ave., Washington 25.

statistics are training themselves for careers of specialization in economics, population, sociology, housing, business, business research, industrial design, industrial production, personnel, purchasing, public opinion, biology, agricultural science, metallurgy, physics, chemistry, psychology, or some other field that makes extensive use of statistics. Research in these fields often requires the use of advanced statistical techniques, and even the development of new statistical theory. Students planning to do such research need statistical theory and methods as a tool.

d. *Future producers and teachers of statistical methods.* The smallest, but in many respects most crucial group of students of statistics, are those who intend to specialize in statistical methods for the sake of statistical methodology. Many of these will become teachers or full-time research workers, though some will find posts in government and industry in high-grade statistical work, frequently requiring the development of new statistical theory and methods. These students will become tool-makers.

2. What should they be taught?

a. *All college students.*[3] The fundamental logic and philosophy of statistics can be taught at an early stage. It is perhaps an appropriate subject to include in the kind of survey courses of physical or social sciences that have become so common in recent years. Three or four weeks of lectures and discussions should suffice to acquaint the students with the broad principles of inductive inference. No mathematics need be included, although some elementary experiments may well be performed to instil the concepts of sampling variation, randomness, and statistical predictability. The student even at this stage can be made to recognize the fundamentally statistical character of most decisions, arising from the fact that they involve an element of uncertainty and a balancing of the importance of different types of errors. The student can be made to understand the fundamental difference between inductive and deductive statements, the nature of statistical estimation, and the nature of a statistical hypothesis. These concepts can be made concrete by illustrating them in terms of problems ranging from everyday questions such as whether to cross a street in the middle of the block on up to such vital problems as the construction of an appropriate social security plan, or the design of an efficient experiment for selecting the best variety of corn, or the selection of the best method of testing for the presence of a disease.

b. *Future consumers of statistics.* Future consumers of statistics need two kinds of training in statistics. First, they need some knowledge of the kind of statistical material available in their field of specialization; of the sources of such data; and of their limitations. To meet this need they require what may be called "descriptive statistics," which places special emphasis on their own field of specialization. A one-quarter or one-semester course in some department or division (e.g., in the social sciences, or biological sciences) should meet

[3] This recommendation is almost an exact parallel of one made by a committee on the teaching of statistics, appointed by the Royal Statistical Society and published by the Society in 1947 as a report to the Council; later published in the Journal of the Royal Statistical Society, vol. cx, Part I, 1947.

this need. In addition, they need a reasonably thorough understanding of what statistics can and cannot do, what the major statistical techniques are, and how to interpret the results obtained by the application of such techniques. This need may be met for those students who have some mathematical background by all or part of the fundamental one-year course discussed in the next section. For students lacking this background, special courses along similar lines will be required.

c. *Future users of statistical methods.* It is essential for fruitful application that users of statistical methods should not mechanically apply procedures learned by rote or taken from a manual. Since few research problems fit perfectly into clearly defined patterns, nothing is so important to the successful collection and analysis of statistical data as adaptability and flexibility in using techniques. These require a thorough comprehension of the logical foundations of statistics, especially of the assumptions underlying its various technical devices, and sufficient knowledge of the derivations of these devices to be able to adapt them to the special circumstances that inevitably develop. To provide this background, a minimum of a full year fundamental course in statistical methods is essential, followed by courses of application. It is highly desirable that this fundamental course be based on calculus as a prerequisite, because without it a proper understanding of the development of statistical techniques cannot be attained. But this is probably impossible at present, in view of the unfortunately low level of mathematical training of most college students. As an expedient, and it is hoped a temporary expedient, it is recommended that the fundamental course be given in two sections, one requiring calculus, the other only a knowledge of first-year college algebra. A single course (or pair of courses, in line with the temporary expedient just mentioned) *should suffice for all departments,* because the core of statistical methods is common to all fields of study. Given in this way, the fundamental course can have the advantage of being taught by the most competent statisticians in the institution.

In addition to a thorough training in theory and methods, users of statistical methods need training in applications. This can be provided by courses in various applied fields. It is usually advisable that these courses be given in the department of application (agriculture, population, engineering, economics, psychology, etc.), and require the fundamental one-year course as a prerequisite.

d. *Future research workers and teachers of statistical method.* The future research workers and teachers of statistical method clearly require far more intensive training in theory than has so far been suggested. A fundamental prerequisite to such training is knowledge of some advanced mathematics. It is difficult to specify exactly what or how much mathematics is necessary, but something of the algebra of matrices and of the theory of functions are minimum necessities, and a good deal of additional knowledge of algebra, geometry, and analysis add richness and power to the work of the statistical theorist.

In addition to advanced mathematics and advanced work in statistical method, the future statistical theorist needs a good deal of work on applications, in the form either of experience or courses. He will be a tool-maker, and needs to

know by personal experience something of the problems of those who use his tools. One satisfactory arrangement is an internship in statistical research, as is currently provided by some institutions. By this arrangement, interns work under competent leadership in various government or private agencies that are engaged in large-scale statistical studies. The interns do research in theory, adapt the physical circumstances to theory and vice versa, and have actual practice in the design of experiments, the construction of questionnaires, writing of instructions, planning tabulations, analyzing the results, and examining sampling variances.

It is obvious that proper advanced courses in statistics will for many years be the province of a few institutions only, as there does not exist at present an adequate professional body to man more than a few.

3. Who should teach statistics? It is clear from the preceding section that two different kinds of courses are required to meet the needs of students of statistics: first, courses in statistical method and methodology; and second, courses in applications of statistical methods to particular fields.

The most important requirement for a successful university program in statistics is that courses in statistical method and methodology should be taught by a statistical theorist, a man who has had the training outlined in Art. 2d above, is specializing in statistics, is doing research in statistical method, and who has had some first-hand acquaintance with applications of statistical techniques. This is the only way such courses can be kept abreast of developments and sufficiently broad to meet the needs of all departments. This recommendation may seem to belabor the obvious, but a glance at the qualifications of most people currently teaching statistical methods will show why it is necessary.

Most courses in applications should be taught by people thoroughly conversant with the relevant subject-matter fields as well as statistical methodology. Some courses in applications may be taught by statistical theorists, particularly new applications or applications that are common to many fields.

4. How should the teaching of statistics be organized? The teaching program in statistics should be organized around a separate administrative unit, an Institute or Department of Statistics. This department should be primarily responsible for the teaching of courses in statistical methods: the fundamental course in statistical method described above, specialized methods for particular fields of application (e.g., factor analysis, time-series analysis), and advanced courses in statistical theory.

In addition, the department of statistics should offer its services as a consulting centre on problems in statistics arising in research in other departments of the institution, both as a service to these other departments and because research in statistical methods peculiarly requires stimulation from close communication with applications. The department of statistics might also provide laboratory facilities for itself and other departments,[4] and might undertake directly, or

[4] See the interesting suggestions on this point on p. 14 in *Personnel and Training Problems*, loc. cit.

through an associated research staff, special assignments involving the application of statistical methods to concrete problems.

Intermediate courses dealing primarily with applications ordinarily belong in other departments (agriculture, economics, demography, engineering, biology, etc.), although some may be given in the department of statistics. The exact location of courses in application will depend on the accident of the departmental affiliation of the persons competent to teach them. Coordination of the teaching program in statistics can be achieved by an interdepartmental committee. The department of statistics should not, however, consist of such a committee under a different name. It should be a thoroughly independent department, with all or most of its members entirely in the department.

The recommendation that the responsibility for teaching statistical methods be centered in a separate department is based on the belief that the teaching of statistical methods without theory can only be uninspiring and harmful; that a separate department of statistics offers the only arrangement that can assure statistical theory being taught by competent theorists, and the only satisfactory arrangement for ensuring the strong incentive for statistical research, with appropriate recognition and advancement, which is as necessary for the teaching of statistics as for the teaching of any other subject.

5. What should be done about adult education? The preceding recommendations are all directed toward the teaching of statistics to undergraduate and graduate students. There is an additional need that these do not meet, namely, the provision of training to mature research workers in various fields already established in their professions. This need arises in part from the inadequate teaching of statistics in the past, but even more from the extremely rapid advance in the theory and practice of statistics which have made it difficult for any but the specialist to keep abreast of developments. Some institutions are making efforts to meet this need by providing evening and late-afternoon classes for employed research workers. Such classes are feasible only in the larger centres of statistical activity. There is also the need of providing advanced research workers in particular fields with highly specialized guidance in selected topics. A department of statistics organized along the lines suggested above can contribute toward meeting this need by effective counseling of colleagues in other departments, and by organizing special seminars and lectures for them. The professional statistical associations are also contributing by arranging special expository programs.

PART II

THE PLACE OF STATISTICS IN THE UNIVERSITY[5]

Contents

A. MINOR NUISANCES AND INEFFICIENCIES IN STATISTICAL TEACHING

6. Lack of coordination among departments. Lack of advanced courses and laboratory facilities. The teaching of statistics in American colleges and universities, which has for the most part been a development since the first world war and has now reached large proportions, presents a number of unsatisfactory features. Courses in statistical methods are taught in various departments without coordination or inter-communication. These courses cover what is to a large extent the same material, but with many variations in the selection of subjects according to the ideas and abilities of individual instructors, and with

[5] An earlier version of this part, prepared entirely by the chairman, is being published by the University of California Press in a report of a symposium on probability and statistics. The Committee as a whole made and adopted the present condensation, with W. Edwards Deming and Milton Friedman contributing most of it. Publication of the Berkeley symposium, including the more detailed original, has been delayed, but it is expected to appear soon.

illustrative examples drawn in each case from material pertaining to the department in which the course is taught. Thus a student desiring to learn more about statistics than he can obtain in one department must, in taking courses in other departments, repeat a great deal of what he has previously covered.

There is a plethora of elementary courses and a dearth of advanced ones. Some departments have excellent statistical laboratories which they reserve for the use of their own students, each with an attendant to keep others away, while other departments have none. Some classes in elementary statistics are too large and some too small, with no one in a position to equalize the sections between different departments.

7. Inefficient decentralization of libraries. The library situation is confused. Books on statistical methods are catalogued and shelved under Sociology, Economics, Business, Psychology, Zoology, Botany, Engineering, and Medicine. Books on probability are divided between Philosophy, Mathematics, Physics, and Chemistry. Books on the method of least squares are for the most part divided between Mathematics, Astronomy, and Civil Engineering, though some get into the Economics, Geology, and Physics reading-rooms. Works on the analysis of variance and design of experiments are likely to be concentrated under Agriculture, while methods of approximate evaluation of multiple integrals and similar purely mathematical subjects of use in statistics are, at least in one of our largest universities, to be found only in the library of Biology.

B. THE MAJOR EVIL: FAILURE TO RECOGNIZE STATISTICAL METHOD AS A SCIENCE, REQUIRING SPECIALISTS TO TEACH IT

8. Too many teachers not specialists. The above nuisances are but minor. The major evil is that those attempting to teach statistical method are all too often not specialists in the subject. Their original selection was seldom on the basis of scholarship in this field; they are not encouraged to make advanced studies in it; and their environment is such as to draw their attention in every direction except to the central truths and problems of their science. Frequently they lack the knowledge of mathematics necessary to begin to read the more serious literature of the subject that they are teaching. Many have been utterly unable to keep up with the rapid progress which has been taking place in statistical methods and theory, *progress which affects even the most elementary things to be taught.*

9. Results: students ill equipped. There results a widespread teaching of wrong theories and inefficient methods. Students are sent to the government service and to industrial and commercial statistical positions equipped with the skill that results from careful drilling in methods that ought never to be used. Some of these same students are encouraged and assisted to become college and university teachers of statistics without ever making thorough-going studies of the fundamentals of the subject, or exhibiting any power of making original contributions to it, or studying any graduate mathematics. Through the method of selection of teachers in general use, and through textbooks written by individuals of this type, there is a perpetuation of obsolete ideas and unsound methods.

All this does not mean that any considerable number of people teaching statis-

tics are unworthy or objectionable members of the academic community. Many, indeed, are of superior intellect, upright character, personal charm, and undoubted teaching ability. Some are making creative contributions to other subjects. The only trouble is that they are teaching a subject in which they are not specialists, and which progresses so fast that only specialists can keep up with it.

10. Reasons why teachers of statistics are often not specialists. The chief reasons for the extensive teaching of statistical method by people who are not specialists in it appear to be the following:

a. *The rapid growth of the subject* and multiplication of its applications, creating a very large and very urgent demand for teaching it that could not be met immediately by the small existing number of scholars specializing in statistical method. This difficulty is aggravated by the paucity of university facilities for training advanced scholars in the field, so that even now the available number of such scholars cannot be expanded with sufficient rapidity to meet the current need. As specialists have not been available in anything like sufficient numbers, statistical method has inevitably been taught largely by non-specialists.

b. *Confusion between statistical method and applied statistics.* Statistical method is a coherent, unified science. "Applied statistics" may mean any of thousands of diverse things. Any particular study in applied statistics will ordinarily utilize some few of the results obtained by the science of statistical method, but will be largely concerned with matters peculiar to the particular application in view and others closely related to it. For example, studies of business cycles utilize statistical methods, good or bad, with a view to drawing inferences from existing data on prices, production, incomes, interest rates, bank reserves and the like. The main job of the applied statistician in this field is to study the sources and nature of the various series of observations, keeping in mind incidental events which may break the continuity of a series, and watching, with a background of economic theory and knowledge of the facts, for explanations. He should also be well acquainted with statistical theory, since otherwise there is grave danger of wasting or misinterpreting the laboriously accumulated observations. Indeed, an organization studying business cycles, or solar cycles, or rat psychology or cancer or practically anything else, would almost certainly benefit from participation by a specialist in statistical method.

However, the chief attention in any such study will not be on statistical method but on features peculiar to its own scope. The specialist in statistical method will do well to participate occasionally in such a study, but if he does so too extensively the needs of the application will so engross his attention that he cannot keep up with the progress of statistical method itself.

The call of applications is enticing, and has led many young scholars to forsake the cultivation of statistical theory. The applications have benefited greatly by the process. Moreover, problems brought back in this way from applications have provided valuable inspiration in developing theory. The mistake lies in supposing that participation in applied statistics is equivalent to specialization in statistical method and theory, and the consequent appointment to teach the latter of persons whose sole concern is with the former.

c. *Failure to recognize the need for continuing research* in the theory of statistics by those who teach it. There is an easy tendency to assume that all the requisite ideas and formulae can be found in some book, and that the duty of the teacher of statistics is simply to transfer this established book-knowledge to the minds of the students and impart to them skill in applying it. Similar attitudes applied to other subjects have in the past been a drag on progress, and have long been discarded in respectable universities. They still hang on, however, even in the best institutions with respect to statistics. The spectacular advances of the last three decades in statistics should make it clear to anyone who has followed them that statistical method is far from static, that the best techniques of present-day statistics may tomorrow be replaced by something better, and that unsolved problems regarding the theory and methods of statistics are sticking out in every direction. A vast amount of research, mostly of a highly mathematical character, is needed and is in prospect. Anyone who does not keep in active touch with this research will after a short time not be a suitable teacher of statistics. Unfortunately, too many people like to do their statistical work as they say their prayers—merely substitute in a formula found in a highly respected book written a long time ago.

d. *The system of making appointments to teach statistics within particular departments that are devoted primarily to other subjects.* In effect, the teacher of statistical method is too often selected by economists or sociologists or engineers or psychologists or medical men because he is to teach in one of these departments. Thus the task of selection devolves upon people unacquainted with the subject, though realizing the need for it in connection with a very specific application. Under such conditions there is an inevitable tendency to emphasize the immediately practical and specific at the expense of the fundamental work of wider applicability and greater long-run importance. Confusion between a science and its applications is most pronounced with those who know little about it, and the distinction between statistical method and applied statistics is likely to be completely lost when a sociologist or an engineer is confronted with the problem of finding someone to teach statistics. If he does make the distinction at all he is likely to choose in favor of applied statistics.

Strangely, the actual teaching that ensues is bound to consist largely of statistical theory, because the students will ordinarily not have had statistical theory elsewhere, and they must have some in order to apply it. What often happens is that a sociologist or an engineer who has made some study of statistics embarks on what he thinks will be a career of teaching the application of statistical method to sociological or engineering problems, only to discover that because of the ignorance of the students he is compelled to teach the fundamentals of statistics, an entirely different subject for which he lacks preparation, talent, and interest.

An incident of this sort has been cited previously.[6] A prominent economist was asked to teach a course entitled "Price forecasting" in a leading university, and accepted. He found, however, that his lectures on this subject were over

[6] Harold Hotelling, "The teaching of statistics." *Annals of Math. Stat.*, vol. xi, 1940, pp. 457–470.

the heads of the students because he was using statistical concepts unfamiliar to them. He therefore went back over the ground covered so as to explain these particular statistical concepts along with their application. But in explaining them he found himself using other statistical concepts, which in turn called for explanation. At the end of the semester he found that he had not given the course in price forecasting which he had planned, and for which the large class had enrolled, but instead had taught a somewhat disordered course in elementary statistics, a subject in which he did not feel particularly competent, and for which the students had not come. When he was asked to teach price forecasting a year later he proposed that a prerequisite of a course in statistics be imposed, but this proposal was rejected by the chairman of the department, and the course was not repeated.

11. Appointments under the existing system are not all bad. More by accident than by design in the existing system, not all statistical appointments by departments of application are bad. Some professors in these departments make conscientious excursions into statistical theory, are well advised by competent specialists in statistics, and bring about the appointment of men of high quality well acquainted with statistical method and theory of the currently best sort. This may work out well if the man so appointed is an able and energetic scholar deeply devoted to his subject, if he is placed immediately in the highest professorial rank, and if he does not feel under obligation to devote himself too exclusively to the special interests of the department of which he finds himself a member. He is then free to pursue his specialty, to keep informed on the latest developments in statistical method and himself to add to the subject, while at the same time transmitting to students a well rounded and up-to-date selection of knowledge. It is in this way that some of the present leaders in statistics have developed. It is a wrong procedure, however, to depend on accidents of this kind.

The system of departmental organization and of making appointments and recognizing proficiency in the teaching of statistics needs to be altered. The usual story is typified by the appointment of a promising young scholar in statistical method to a junior position in some department of application where he is expected to work on problems and to teach statistical methods with a sole eye to the work of the specific department. He is then under pressure to concentrate on a particular kind of applied statistics, for his advancement will depend, not on his statistical attainments at all, but on his study of the literature, terminology, techniques and theories of the application. His usual associates will be in the department in which he is teaching rather than others teaching statistics. The loss, although not total, is great, because the opportunity to make the most of the man's statistical ability is lost, and his ability as an economist, agricultural scientist, engineer, or something else that he is not particularly fitted for, is substituted.

A still less favorable circumstance, and unfortunately more common, is that in which the teacher of statistics is not even selected for scholarship in the theory of statistics. Studies in some other field, with some slight dabbling in the appli-

cation of statistical methods to it, plus a pleasing personality, have all too frequently been thought to comprise sufficient qualifications for teaching statistical methods and theory.

12. Unsatisfactory texts. The uncritical character of the teaching is reflected in the long line of textbooks written by teachers who have not made any genuinely fundamental study of statistics, but pass on to students in a magisterial fashion what was passed on to them. Authority takes the place of derivations and ultimate sources. It is no wonder that these textbooks, copied from each other, contain increasing accumulations of errors; or that long delays have intervened between the introduction of important new statistical methods and theories in the periodical literature and their appearance in the textbooks and courses put before students.

The latest discoveries in the theory of statistics affect what should be taught in elementary courses, and no syllabus can be expected to survive more than a few years of research. The development of new statistical methods and ideas of overwhelming importance must be allowed to compete with material already well established as true and useful. The new material is equally true and in some cases even more useful than matter usually incorporated in the best of current courses and textbooks.

13. Omission of probability theory from texts and teaching. One of the important weaknesses in much of the current teaching of statistics is a failure to make proper use of the theory of probability. Without probability theory, statistical methods are of only minor value, for although they may put data into forms from which intuitive inferences are easy, such inferences are very likely to be incorrect. The objective weighing of the degree of confidence to be placed in inductive conclusions is necessary to avoid fallacies. Indeed, the whole foundation of descriptive statistical methods, of inductive inference, and of the design of experiments, rests upon probability theory.

The relevance of probability to much statistical work was indeed questioned a quarter-century ago by a group of economists impressed by the lack of independence between consecutive observations, and this attitude, in conjunction with an exaggerated and belated remnant of nineteenth-century empiricism, has had a certain influence, particularly on the statistical methods in use by economists. This view is now rapidly giving way to a tendency to use the powerful new statistical methods discovered in the meantime. It is now perceived that efficient objective methods can be used over a much wider range of cases than was formerly supposed, because the independence assumed in their derivations refers not to observations but to residuals from the theoretical model used. Furthermore, research is under way, and has already achieved promising results, on the extension of accurate methods to still more extensive classes of problems.

C. PROPER QUALIFICATIONS OF TEACHERS OF STATISTICS

14. Statistics compared with other subjects. The qualifications appropriate for teachers of statistical method and theory are not essentially different in degree from those for teachers of other subjects in the same institutions; proficiency in

statistical method and theory is merely to be substituted for it in other subjects. This substitution is, however, vital. It must not be imagined that proficiency in some other subject in which statistical methods are used incidentally is equivalent to proficiency in statistical method itself. The error of such a supposition, if carried over into another field, might lead to the appointment of a man as professor of chemistry on the ground that he could cook.

The first requisite of the college or university professor of any subject is a profound and thorough knowledge of that subject. It is customary in the better institutions at least to restrict appointments to the rank of assistant professor to persons who have demonstrated scholarly qualifications by work equivalent to that leading to a Ph.D. degree, including an original contribution to the subject that the individual is to teach. Promotion to the higher ranks is conditioned upon a number of criteria, among which published research is by far the most important in those institutions.

15. Current research in statistical method is essential for teachers. Research is even more essential in the teacher of statistics than in teachers of most other subjects, because so much remains to be worked out that is of immediate importance. Some college teachers do no research. This is usually regarded as deplorable. The evil is, however, of quite different magnitude according to the nature of what is taught by such teachers. In a new subject in which sharp differences of opinion exist or have recently existed on fundamental questions, in which current discoveries have an important bearing, and in which there have not yet been the time and consensus necessary for the preparation of an adequate and virtually error-free textbook, teaching without research may have calamitous effects. The effective teacher must, of course, have teaching ability, but no skill in pedagogy, no lustre of personality, can atone for teaching errors instead of truth. Errors are very likely to be taught by those who do no research, and then the more skillful the pedagogic indoctrination, the greater the harm. Sound educational policy calls for devotion to research of a large fraction of the time and energies of the teaching staff in a subject like statistical theory. Students also are in particular need of encouragement to do original and critical work in relatively new areas of this kind. They must be taught to shun the use of formulae and methods given merely on authority without full and convincing reasons, and to insist on looking closely and critically at assertions.

Even in the teaching of elementary statistical methods for direct practical use by specific occupational groups, where it might be thought that the teaching would most predominate over the research element, the teacher must face difficult questions whose answers call for research in statistical theory. Let us illustrate this by one example out of the many possible. In teaching the analysis of variance for use in agricultural experimentation, questions arising out of the possible non-normality of the underlying distributions must be dealt with in some way. The formulae, even those in the best textbooks, are accurate only if the distribution is normal, and neither this fact nor the non-normality of many distributions should be concealed from the students. Obviously something more

needs to be said on the subject at this point. What the teacher can say depends on how deep he has gone into a whole series of perplexing questions, on some of which the views of scholars are not yet stabilized, and on which a tremendous amount of research is needed before the maximum practical value can be attained for a technique whose usefulness is already amazing.

16. Minimum requirements in mathematics for the training of teachers and research men in statistical theory. Because research in the theory of statistics requires advanced mathematics, and is indeed largely mathematical in character, a mastery of a substantial amount of higher mathematics must be an essential part of the training of prospective teachers of statistics. To specify exactly what or how much mathematics is necessary would be a difficult task. Something of the algebra of matrices and of the theory of functions are minimum necessities, and a good deal of additional knowledge of algebra, geometry, and analysis add richness and power to the work of the statistical theorist, the inventor of new statistical methods. On the other hand, the time of the graduate student in statistics is much occupied with the theory of statistics itself; and some of his time should also go into the study of applied statistics. If the students entering a graduate school for advanced work in statistics went there equipped with a knowledge of matrix algebra and theory of functions and some additional higher mathematics, as is obtainable by undergraduates at some institutions, they would have time for applied statistics and could do some real work on applications.

There is a cruel dilemma here, resulting from the delay in learning mathematics imposed by the elementary curricula which have become customary in this country. The weakness of the mathematical element in the prevailing curricula affects both teachers and students of statistics to an extent justifying some attention from those interested in the improvement of statistics. In American universities elementary calculus is not often taught before the sophomore year, and the more advanced parts of algebra come still later, if at all.

If calculus could be pushed down into the high schools and assumed as a prerequisite for college courses in mathematics, statistics, economics, physics and several other subjects, the efficiency of instruction in all these departments could be increased. For example the difficulties experienced by students of economics with ideas of marginal cost, marginal revenue and the like correspond closely with the difficulties experienced by mathematicians for centuries in trying to define infinitesimals and derivatives, but now successfully overcome. The student who really knows differential calculus need not have the slightest difficulty with the marginal ideas of economics. Similarly in physics, the fundamental concepts of speed, acceleration, potential theory, conductivity, thermal capacity and radiation, are all mathematical and easier to grasp once and for all as such than to be learned afresh with each new application from textbooks in physics sometimes not clearly written and taught by teachers who must for one reason or another avoid a mathematical approach.

The possibilities of teaching quite advanced mathematics to young children

have scarcely begun to be explored. Children of kindergarten age are fascinated and thrilled by the wonders of topology. Groups and number theory can be tremendous sensations in the fifth grade, though all these subjects are ordinarily reserved for graduate students specializing in mathematics. What is lacking is teachers who know mathematics and its applications and who possess enough freedom to teach what they know instead of the long, dull and relatively useless drill on problems of wallpaper hanging and the like, problems turning on mere conventions which are quickly forgotten, painful repetitious work which makes children resolve to quit mathematics as soon as possible.

D. NEED FOR RELATING THEORY WITH APPLIED STATISTICS

17. An example of the interaction between theory and practice. A professor of psychology working with mental tests might enlist the assistance of a young statistical theorist with mutual benefit. The young man might for a short time do some of the drudgery of scoring tests and computing, passing on soon to the problems of test construction and the distribution of various functions of correlation coefficients. This last is on a new and exciting frontier of statistical theory. The advancement of this frontier, which is really the main business of the young man in his capacity as prospective statistical theorist, would in this way come to him naturally as a problem or series of problems having a tangible meaning additional to its mathematical content. The empirical context is in such cases often of great value in suggesting suitable approaches, for example, suitable approximations in the study of functions not susceptible to simple mathematical representation in terms of elementary functions.

If the young theorist succeeds in extending the boundaries of multivariate statistical analysis by discovering the distribution of some new function of correlation coefficients, the chances are that this discovery will also have applications in anthropology, medicine, banking, and other pursuits which in the aggregate will greatly outweigh the application originally in view.

The discovery should be regarded primarily as a contribution to the general theory of statistics, and published in a journal devoted to mathematical statistics. It will then become available to a wide circle of teachers of statistics, who may incorporate it into their courses, and its methods and results will be studied by other investigators from the standpoint of possible generalizations and analogs. The importance of the discovery would be much more limited if it were thought of as a development in psychology and published only in a psychological journal. Perhaps dual or multiple publication ought to be permitted in such cases, but the first publication should be in a journal of mathematical statistics. Far too many good statistical ideas have been buried in connexion with obscure special applications.

18. Supplying opportunities for applications in graduate studies of statistics. The statistician who does any work in applications must know statistics as an art as well as a science. The theoretical statistician, if he wishes to be of the utmost use to his colleagues in other disciplines, needs to know by personal

experience something of their lives and collateral problems. Indeed, experience with applications, and the challenge of problems arising out of applications, have played a most important part in the development of statistical theory. It follows that the graduate student in statistics needs contact with applied statistics which the institution should undertake to provide, or at least facilitate. This need is next in importance after the needs for theoretical statistics and for pure mathematics. The distribution of time among the three—theoretical statistics, mathematics, and applied statistics—is hard to specify exactly, and must in any case depend on the nature of the student's previous work. If his mathematical preparation has been full and rich, more time should be spent on applied statistics in his graduate years than if he has already had substantial contact with applied statistics in some other way but is deficient in higher mathematics.

Applied statistics entails a somewhat detailed acquaintance with the field of application. Such a field might be life insurance, or mental testing, or industrial quality control, or sampling in the work of the Bureau of the Census or some other government agency; it might be agricultural economics, or business cycles. Proficiency in any such field calls for rather prolonged study, and it would be too much to expect the embryo statistical theorist to reach this stage of advancement in all subjects. He should, however, make more than a superficial study of some chosen field of application. This study might or might not be at the university. The requisite familiarity with applied statistics might in some cases be acquired by work in a government bureau, or in a research organization studying business cycles or something else involving applied statistics. What is most desirable is that the work should have brought the student to the point both of applying statistical methods in a reasonably effective way, and of perceiving the limitations of existing statistical methods. Perception of existing limitations has frequently been the germ of progress in the subject.

One satisfactory arrangement is an internship in statistical research, as is currently provided by some institutions. By this arrangement, interns work under competent leadership in various government or private agencies that are engaged in large-scale statistical studies. The interns do research in theory, adapt the physical circumstances to theory and vice versa, and have actual practice in the design of experiments, construction of questionnaires, writing of instructions and tabulation plans, analysis of the results and appraisal of sampling variances.

E. RECOMMENDATIONS ON THE ORGANIZATION OF STATISTICAL TEACHING AND RESEARCH IN INSTITUTIONS OF HIGHER LEARNING

19. Research should be encouraged; teaching schedules should not be overloaded. Colleges and universities usually expect the members of their faculties to engage in research as well as in teaching, the relative emphasis on these two functions varying greatly from institution to institution and to a lesser extent among departments within the same institution. Reasons why teachers of

statistics must do current research in order to teach the subject have already been given in Art. 15. In the organization of statistical teaching it is thus of extraordinary importance that colleges and universities emphasize research in the theory of statistics as a leading part of the work of the teaching staff in this field. Hours of teaching and other duties must be kept within such bounds as to make research possible, the initial selection of teachers must be of persons capable of research in statistics, and there must be provision of needed secretarial, computational and other assistance. The library must be adequate, not only in publications containing statistical theory, but in the larger field of pure mathematics as well.

20. Organizing statistical service in the university. In addition to the customary duties of teaching and research, faculty members expert in statistical methods find that they cannot escape a third, viz., advice to their colleagues and others regarding the statistical aspects of their problems. This often takes a good deal of time. Clearly it is in the interest of the academic enterprise that such services be provided. Scholars in many departments are finding that their work is greatly improved by competent statistical advice not only in the interpretation of their data but also in the design of their experiments and other investigations. The provision of competent advice frequently requires extended consideration of the general content of the problem as well as special analysis of its statistical features. And initial advice often needs to be supplemented by further service. The statistician, like the physician, often finds that one interview at which a prescription is dispensed does not end the matter satisfactorily.

Teaching hours must be distinctly limited if statisticians are to be able to render this service to the rest of the institution as well as maintain a high level of research in their own field.

One way to handle the problem of statistical service, especially in a large institution, is through a special organization devoted to this purpose. Such an organization, whether called a Statistical Institute, a Department of Applied Statistics, Statistical Laboratory, or something else, might supply not only advice but a more active kind of assistance, including computational and chart-drawing services.

A statistical service organization should be removed from the teaching of statistics only to the extent necessary to gain the advantages of some degree of specialization and to prevent undue interruption of the teacher's other work of teaching and of research in theory. There are distinct advantages for all parties in a fairly close connexion between practical statistical work, research in statistical theory, and statistical teaching. Each of these activities benefits the others, provided only that it does not take away from it too much time. Research in statistical theory, like medical research, needs frequent revitalizing injections of specific practical problems. It also needs the stimulus of contact with students. The teaching of statistical method is made more vigorous both by research in the subject and by the presence of applications with which students can be confronted. And the needs of applications are better met if through an organiza-

tion such as is here envisaged they can be brought to the attention of appropriate specialists, and if also students can be enlisted when needed for their treatment.

A university organization dealing with statistics may properly comprise two parts with overlapping personnel, one devoted chiefly to applied statistics, the other to theoretical statistics. The teaching might be done by both, but at least at the more advanced levels would be primarily the concern of the theoretical part. Migration between the two ought to be easy and frequent, though some individuals are so definitely adapted to one kind of work or the other as to make it undesirable to have fixed rules calling for periodic transfers.

In smaller institutions it may not be practicable to have statistical organizations sufficiently well staffed to provide adequate consulting service. To meet the needs in some of these cases regional centres for advice and service in applied statistics might be established at large universities throughout the country, with access made readily available for sister institutions. These centres might also carry on work in applied statistics in behalf of government agencies and other organizations, much as various agricultural colleges have for years been carrying on cooperative work with the federal Department of Agriculture.

The question how far, if at all, such a university centre of applied statistics should go into the market place and engage commercially in service to business concerns is a debatable one. While there may be favorable reactions upon scientific work, there are grave dangers to the intellectual integrity of the institution which need serious consideration.

21. Organization for teaching. Passing from questions of personnel and the research and service functions of academic statisticians to teaching itself, we have to consider problems of departmental organization, of course contents, of systems of prerequisites, and of methods of teaching. All these we consider secondary problems, not in the sense of being unimportant, but because we believe that proper solutions of them will be reached with reasonable promptness when personnel of the kind described in Sec. C of this report are at work in some such general setting as has just been described. The ideas recorded below are general in character and are to be regarded as a starting-point for developing a program in a particular institution, once suitable faculty members have been obtained.

The teaching of statistics may be organized in any of the following ways:

a. In a department of theory and a department of applied statistics, both forming an Institute of Statistics.
b. In a single Department of Statistics.
c. Under an inter-departmental committee.
d. Under the exclusive jurisdiction of the Department of Mathematics.
e. It may be scattered among heterogeneous departments of application, without formal coordination.

Only a few large institutions will be in position to adopt the first plan. It is likely that the second will be most suitable for the majority. The third should probably be regarded as a makeshift for the transitional period until a proper department of statistics can be organized, a step that will not at the moment be

reasonably possible for most institutions because the right kind of scholarly personnel does not exist in adequate numbers. It is of course possible that some vestige of an inter-departmental committee, perhaps in the form of an Advisory Board, might be a useful adjunct of a department of statistics in order to keep it informed of the needs of applications. It is also possible that something of the sort might function with respect to a department of mathematics, or any other department. On the other hand, the desired consultations and adjustments might be accomplished in less formal ways.

To make statistics a subdivision of a mathematics department is a solution that will appeal to administrators desirous of keeping down the number of departments. The subject-matter of statistics is to a sufficient extent mathematical to give some apparent weight to this plan, and some mathematicians have the unsound idea that any mathematician can teach statistics without specialized study or experience in application. On the other hand, statistics has some features uncongenial to traditional mathematics, arising partly from the urgency of practical needs which go beyond what can immediately be provided by rigorous mathematical theory. Again we may cite the problem in the teaching of the analysis of variance of what to do about possible non-normality of the underlying distribution (Art. 15). The user of this technique has the responsibility of verifying that the situation conforms to the assumptions, including that of normality, underlying the tabulated probability criteria. But he is in a very poor position to do this in a large proportion of the applications actually made of the analysis of variance. Yet the analysis of variance in some form—possibly through the use of rank-order numbers or through a transformation or some other auxiliary device—remains the one powerful means of attacking a very large and important class of practical situations. The practicing statistician needs to do some highly educated guessing on such matters—guessing that will be assisted but not made determinate by knowledge of a considerable range of mathematical truths regarding approaches to the normal distribution, moments of the variance-ratio in samples from non-normal populations, asymptotic large-sample theory, and other such topics. His mathematical insight needs to be supplemented by consideration of the particular subject-matter of application. Moreover, it is desirable that students of statistics have some practice with actual empirical data designed to develop the art of guessing in such ways.

Another example of non-rigorous mathematics used extensively in statistics is the whole business of asymptotic standard errors found by the differential method. It is desirable that good mathematics replace bad in such connexions, but something is to be said for the position into which so many practical statisticians have been driven, that even bad mathematics may be better than none at all. The requisite good mathematics along these lines can come only through those who have made really serious studies of statistics, though a sufficiently interested pure mathematician might eventually be led by such a student of statistics to undertake and complete the necessary research. Practical needs make approximations necessary; the goodness of a particular approximation can

often be judged adequately by a statistician familiar with the particular application long before the heavy artillery of advanced mathematical analysis can be brought up.

The teacher of statistics must have a genuine sympathy and understanding for applications, and these are not possessed by many pure mathematicians, at least in the opinion of some of those concerned with the applications; and it is this opinion rather than the possible fact that is of interest at the moment. For so long as such an opinion is maintained, for example by psychologists and economists, these specialists will be suspicious that courses in statistics given by a department consisting largely of pure mathematicians is unsuitable for their purposes. The result is likely to be a sabotaging of attempts at centralization, the different departments reverting to the old and ultimately objectionable system of teaching their own separate courses in statistical methods.

These difficulties are not necessarily insuperable, and it is to be expected that many medium-sized and small institutions will make their mathematical departments responsible for statistical teaching. But this ought not be be done without a consideration of the possible dangers.

22. The statistical curriculum. We next consider curricular problems. These may be divided into those of the graduate school and those of the undergraduate college. Those of the graduate school may in turn be divided into those of specialization in statistics and of auxiliary teaching of statistics to students in other departments, such as sociology, who need to use statistical methods, have not studied them sufficiently as undergraduates, and cannot afford to put much time on them. Of these two subdivisions the number of students at present is greater in the second and the ultimate importance is greater in the first, because the whole future of statistics depends on improvement and enlargement of this graduate teaching.

The incidental teaching of elementary statistical methods to graduate students in other subjects, without any prerequisite in mathematics or statistics, cannot equip these students with a command of the subject at all comparable to that which could be obtained by a better integration of undergraduate with graduate work. A prospective sociologist, economist, psychologist, or physicist ought to study elementary statistical methods and concepts while still an undergraduate, and without special reference to his ultimate field of specialization.

The features of statistical methods peculiar in their applications, beyond what is taught through illustrations and exercises in an elementary course, may be fit material for a course, graduate or undergraduate, in a department of the application. Such a course should require as a prerequisite an elementary course in a department of statistics, or at least one taught by specialists in statistical method and theory.

For the undergraduate college, in place of the sporadic offerings now current in different departments, we recommend a combination of two general fundamental courses with a number of advanced courses. Of the latter some will be specialized to the work of particular departments or groups of departments.

Of the two fundamental courses one will require calculus as a prerequisite, the other only a knowledge of first-year algebra. It is to be hoped that the less mathematical of these two general statistical courses, instead of being elected by a majority of students, will gradually approach extinction, while the course based on calculus will become the vital point of contact of the student body with the concepts of statistics. The chief reason for insisting upon the importance of calculus as a prerequisite is simply the possibility of covering important statistical theory that is inaccessible to those who do not have it.

Modern statistical methods are based on the theory of probability. The general courses in statistics may therefore well begin with elementary probability. The duality between probability and statistical concepts,[7] for example between probability and relative frequency, between mathematical expectation and a sample mean, between parameter and statistic, should be explained. Derivations and the place of the normal distribution should be sketched, and the Student distribution should be derived and applied to a variety of problems in the first course based on calculus. Later courses given by the department of statistics, or whoever specializes in statistical theory, will naturally cover other statistical methods and theories. At the same time useful courses can be offered in economic statistics, mental testing, and other fields using statistical methods by specialists, regardless of departmental affiliation. There might be departmental cooperation; for example, the department of statistics might offer elementary and advanced courses in correlation and multivariate analysis, and the department of psychology might require these as prerequisites for some of its work in mental testing.

The teaching of statistics should be accompanied by considerable work in applied statistical problems, as well as exercises in mathematical theory, on the part of the students. A large part of this work in applied statistics is best conducted in a laboratory equipped with calculating machines, mathematical tables, drafting instruments, and other appurtenances.

Statistical laboratories require supervision, administration and maintenance. They are needed not only for the purpose of teaching statistics, pure and applied, at all levels, but also by research workers in many fields. There are possible gains of efficiency and economy in a centralized administration of them. One suggestion is that they be under the supervision of the university library. Another is that responsibility for them be lodged in a central department of statistics, or in a two-department statistical institute. Centralization can be carried too far, and it is likely that some units in a large organization will find it advantageous to have machines which are exclusively their own. The conflicting claims regarding machines and laboratories will require careful weighing.

23. Statistical method as a part of a liberal education. A question may also be raised as to whether some work in the statistical method should not be required of all college students as a part of a liberal education. This would be

[7] Cf. the article "Frequency distribution," *Encycl. of the Social Sciences* (1931).

a novel step, but has much to be said for it in view of the widespread use of statistics and growing interest in statistics. Another point is that the student who can't make up his mind as to his ultimate field of specialization or vocation will do well to study those things that can be used in many fields. Of such things, mathematics and statistics are leading examples. There are more or less sound objections to systems of required studies; but if we are to have them, the claim of statistics should not be rejected merely on grounds of novelty.