ON DESIGNING SINGLE SAMPLING INSPECTION PLANS

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1. Summary. In designing single sampling inspection plans, a problem is to find the acceptance number, c, and the smallest sample size, n, such that if the fraction defective of the material inspected is equal to an acceptable value, p_1, a large percentage, say, 95% of such lots will be accepted under the sample criteria, whereas if the fraction defective of the material inspected is objectionable and equal to p_2 (where $p_1 < p_2$), then a large percentage, say, 90% of such lots will be rejected. A solution to this problem for the case where the lot size is large compared to the sample size is given in this paper and tables are provided for quick determination of the sample size n and acceptance number c.

2. Introduction. In sampling inspection of material one practice is to set an acceptable quality level $= p_1$, say, such that the consumer desires to accept practically all—95% or more—of lots of fraction defective p_1 or less (and hence desires to reject at most a maximum of about 5% of lots which are of quality p_1 or better) and to set also an objectionable fraction defective $= p_2$, say, which represents quality so poor that the consumer cannot afford to accept more than about 10% or less of lots of this quality or poorer.[1] From the standpoint of the producer, he should have very few rejections, 5% or less, for his submitted lots the fractions defective of which are equal to or better (less) than p_1, whereas he should be willing and also expect to suffer increasingly more rejections if his process average percent defective departs from the acceptable quality level p_1 toward poor or objectionable quality. In this connection, if we are given p_1 an acceptable quality level, p_2 an objectionable percent defective, the risk $\alpha = 5\%$ of rejecting a lot of fraction defective p_1, and the risk $\beta = 10\%$ of accepting a lot of the objectionable fraction defective p_2, a problem of importance in single sampling inspection is to find the smallest sample size n and the acceptance number c which will approximate closely the protection stated above. Due to the discrete nature of n and c, it is not usually possible to find n and c such that precisely the above protection is guaranteed; however, it is possible to pick that single sampling plan which, for all practical purposes, gives the desired protection, i.e. it is possible to select that single sampling plan which more nearly satisfies

[1] When this paper was first presented for publication, the percent defectives p_1 and p_2 were labeled "Acceptable Quality Level" and "Lot Tolerance Percent Defective," respectively. In view of the suggestions of H. G. Romig and H. F. Dodge, strict reference to these particular terms have been avoided in order that the percent defectives p_1 and p_2 would appear in a more generalized form. This recommendation is considered especially desirable in view of the fact that Table I and Table II of the paper are percentage points of the Binomial Distribution and hence are useful in problems other than that of designing single sampling inspection plans.

the above protection requirements than any other plan. The values of n and c can be found simply by looking for an entry in Table I below which is close to p_1 and an entry in Table II close to p_2 such that column heading c and row heading n in Table I correspond exactly with the respective column and row headings in Table II. For the sample sizes n, acceptance numbers c and quality levels p covered in Tables I and II, the above procedure makes unnecessary any computation of or any approximation to the sample size and acceptance number. It will be noticed, however, that usually the proper choice of c is clear whereas some slight judgment may be necessary in selecting n.

It is remarked also that Tables I and II solve the equivalent problem of finding n and c in connection with testing the hypothesis H_0 that the fraction defective of the Binomial population sampled is p_1 or less as against an alternative hypothesis H_1 which states that the fraction defective of the lot, population, process, etc., sampled is p_2 or greater ($p_2 > p_1$), where $\alpha = .05$ is the maximum risk of erroneously rejecting H_0 when it is true and $\beta = .10$ is the maximum risk of erroneously accepting H_0 when the alternative H_1 is true.

The solution to the problem of finding an appropriate single sampling plan in this paper is given by solving the infinite case, i.e. by assuming the lot to be an infinite Binomial population. In practice lots are of finite size. However, it is well known that Binomial probabilities (infinite universe) give excellent practical approximations to Hypergeometric probabilities (finite lot) provided the sample size is only a small percentage of the lot size. Hence, the reader is warned in using the tables for sampling inspection problems that the lot size should be at least 10 or 15 times the sample size.

3. Basis for construction of Table I and Table II. It is well known that if $P(c, n, p)$ represents the probability of obtaining c or less defectives in a random sample of size n from a Binomial Population of fraction defective p, then the relation between $P(c, n, p)$ and the Incomplete Beta Function Ratio is given by

$$(1)\quad P(c, n, p) = I_{1-p}(n - c, c + 1) = \frac{1}{\beta(n - c, c + 1)} \int_0^{1-p} x^{n-c-1}(1 - x)^c \, dx.$$

Consequently, using a table of percentage points for the Incomplete Beta Function (1), values of p_1 can be found for Table I such that

$$P(c, n, p_1) = .95,$$

and values of p_2 can be found for Table II presented at the end such that

$$P(c, n, p_2) = .10.$$

Also, Table I and Table II can be computed by using percentage points of the F-distribution (2). Upon making the transformation

$$x = \frac{2(n - c)}{2(n - c) + 2(c + 1)F}$$

in (1) above to the F-distribution, we obtain easily that

$$(2)\quad P(c, n, p) = \frac{1}{\beta(c+1, n-c)} \int_{(n-c)p/(c+1)q}^{\infty} [2(c+1)]^{c+1} [2(n-c)]^{n-c} F^{c} \cdot [2(n-c) + 2(c+1)F]^{-n-1}\, dF,$$

where $q = 1 - p$.

With the aid of a table of percentage points of the F-distribution (2), we may determine for various combinations of $n - c$ and $c + 1$ those values of p such that

$$P(c, n, p_1) = .95 \quad \text{for Table I;}$$

and

$$P(c, n, p_2) = .10 \quad \text{for Table II.}$$

In fact, if $P(c, n, p) = \alpha$, then

$$\frac{(n-c)p}{(c+1)q} = F_{\alpha}\{2(c+1), 2(n-c)\},$$

or

$$p = \frac{(c+1)F_{\alpha}\{2(c+1), 2(n-c)\}}{(n-c) + (c+1)F_{\alpha}\{2(c+1), 2(n-c)\}},$$

for which relation values of p_1 for $\alpha = .95$ are given in Table I below and values of p_2 for $\alpha = .10$ are given in Table II.

Although the 95% points are not given directly in (2), they are easily obtainable from the relation

$$F_{.95}(\nu_1, \nu_2) = \frac{1}{F_{.05}(\nu_2, \nu_1)}.$$

Interpolation was required for the great majority of the entries in Tables I and II. The values given were obtained by harmonic or linear interpolation using References [1] and [2] and are believed accurate to within one unit in the last place.

It will be noticed that if the chosen acceptable quality level, p_1, is greater than the appropriate tabulated value in Table I for the single sampling plan (n, c), then the operating characteristic curve will pass below the point $(p_1, .95)$. That is, the risk of rejection under the sampling plan for lots of fraction defective p_1 will be somewhat more than 5%. On the other hand, if a selected acceptable quality level p_1 is less than the appropriate entry in Table I, the risk of rejection for a product of fraction defective p_1 will be less than 5%. Similar considerations apply also to the fractions defective, p_2, in Table II.

4. Single sampling plans based on the Poisson approximation to the binomial. Tables I and II are useful for determination of a single sampling plan when the desired percent defectives are listed and n does not exceed 150. Table III is particularly useful in designing a single sampling plan when we are interested in fractions defective not greater than about .10. A somewhat similar procedure has already been suggested by Peach and Littauer [3]. If we designate by $P(c, a)$ the sum of individual Poisson probabilities,

$$P(c, a) = \sum_{m=0}^{c} \frac{e^{-a} a^m}{m!},$$

then Table III gives values $a_1 = np_1$ of a for which

$$P(c, a_1) = .95$$

and values $a_2 = np_2$ of a for which

$$P(c, a_2) = .10.$$

Hence, to find the single sampling plan whose operating characteristic curve passes nearly through the points $(p_1, .95)$ and $(p_2, .10)$ one merely divides values of a_1 in Table III for various values of c by the acceptable quality level p_1 and divides values of a_2 in Table III by the objectionable percent defective p_2. Then the acceptance number c is picked for which a_1/p_1 most nearly equals a_2/p_2 and the approximate sample size n may be determined by rounding to an integer the average of the two approximately equal numbers a_1/p_1 and a_2/p_2.

5. Example on the use of Tables I, II, III. Given an acceptable percent defective or quality level of .01 and an objectionable quality level of .10, it is desired to find the single sampling plan which will accept 95% of product which is of quality $p_1 = .01$ and which will reject 90% (or accept only 10%) of product of quality $p_2 = .10$. Looking in Table I for entries p_1 which approximately equal .01 and in Table II for entries p_2 which approximately equal .10 such that the c and n of Tables I and II correspond, we see that c must be equal to 1 whereas n may take possibly any one of the values 35, 36, 37, 38. In this connection, we have to set up some criteria for the choice of n. Although any of several criteria may be used, a reasonable criterion appears to involve picking n such that the sum of the absolute departures of the Operating Characteristic Curve from the risks $\alpha = .05$ at p_1 and $\beta = .10$ at p_2 is a minimum. This may be determined by using appropriate tables of Binomial Probabilities or by computing at p_1 and p_2 the chance of obtaining c or less defectives in n for the various possible combinations of c and n. If the above criterion were applied to the present example, the combination $c = 1$ and $n = 37$ would be selected, i.e. the single sampling plan would be $c = 1, n = 37$. For this sampling plan, the probability of passing at $p_1 = .01$ is .9471 and the probability of passing at $p_2 = .10$ is .1036. For the sake of expediency, another proposal would be merely to select somewhat of a "middle" value of n especially when the variation in sample size is slight.

If we use Table III for the above example, we can select n and c with the aid of the following simple tabulation:

n	c			
	0	1	2	3
a_1/p_1	5.1	35.5	81.8	136.6
a_2/p_2	23.0	38.9	53.2	66.8

Since the sample sizes "cross" at $c = 1$, we would select $c = 1$ and $n = 1/2$ $(35.5 + 38.9) = 37.2$ or $n = 37$.

A use of Table I of some practical importance is in determining at a glance those values of p for which the probability of obtaining c or less defectives in a sample of n is equal to .95. As a matter of fact, a series of tables similar to Table I and Table II for which $P(c, n, p) = .99, .95, .90, .10, .05, .01$ etc. would be of considerable practical use.

Acknowledgment. The author is indebted to Miss Helen J. Coon for carrying out the computations for the tables.

TABLE I

Values of $p = p_1$ such that $P(c, n, p_1) = .95$

n	c = 0	1	2	3	4	5	6	7	8	9	n
1	.0500										1
2	.0253	.224									2
3	.0170	.135	.368								3
4	.0127	.0976	.249	.473							4
5	.0102	.0764	.189	.343	.549						5
6	.00851	.0628	.153	.271	.418	.607					6
7	.00730	.0534	.129	.225	.341	.479	.652				7
8	.00639	.0464	.111	.193	.289	.400	.529	.688			8
9	.00568	.0410	.0978	.169	.251	.345	.450	.571	.717		9
10	.00512	.0368	.0873	.150	.222	.304	.393	.493	.606	.741	10
11	.00465	.0333	.0788	.135	.200	.271	.350	.436	.530	.636	11
12	.00427	.0305	.0719	.123	.181	.245	.315	.391	.473	.562	12
13	.00394	.0281	.0660	.113	.166	.224	.287	.355	.427	.505	13
14	.00366	.0260	.0611	.104	.153	.206	.264	.325	.390	.460	14
15	.00341	.0242	.0568	.0967	.142	.191	.244	.300	.360	.423	15
16	.00320	.0227	.0531	.0903	.132	.178	.227	.279	.333	.391	16
17	.00301	.0213	.0499	.0846	.124	.166	.212	.260	.311	.364	17
18	.00285	.0201	.0470	.0797	.116	.156	.199	.244	.291	.341	18
19	.00270	.0190	.0445	.0753	.110	.147	.188	.230	.274	.320	19
20	.00256	.0181	.0422	.0714	.104	.140	.177	.217	.259	.302	20
21	.00244	.0172	.0401	.0678	.0988	.132	.168	.206	.245	.286	21
22	.00233	.0164	.0382	.0646	.0941	.126	.160	.196	.233	.271	22
23	.00223	.0157	.0365	.0617	.0898	.120	.152	.186	.222	.258	23
24	.00213	.0150	.0350	.0590	.0859	.115	.146	.178	.212	.246	24
25	.00205	.0144	.0335	.0566	.0823	.110	.139	.170	.202	.236	25
26	.00197	.0138	.0322	.0543	.0790	.106	.134	.163	.194	.226	26
27	.00190	.0133	.0310	.0522	.0759	.101	.129	.157	.186	.217	27
28	.00183	.0128	.0298	.0503	.0731	.0977	.124	.151	.179	.208	28
29	.00177	.0124	.0288	.0485	.0705	.0942	.119	.145	.172	.200	29
30	.00171	.0120	.0278	.0469	.0681	.0909	.115	.140	.167	.193	30
31	.00165	.0116	.0269	.0453	.0658	.0878	.111	.135	.161	.187	31
32	.00160	.0112	.0260	.0438	.0637	.0850	.107	.131	.155	.180	32
33	.00155	.0109	.0252	.0425	.0617	.0823	.104	.127	.150	.175	33
34	.00151	.0106	.0245	.0412	.0598	.0798	.101	.123	.146	.169	34
35	.00146	.0102	.0238	.0400	.0580	.0774	.0978	.119	.141	.164	35

TABLE I—*Continued*

n	c = 0	1	2	3	4	5	6	7	8	9	n
36	.00142	.00996	.0231	.0389	.0564	.0752	.0950	.116	.137	.159	36
37	.00139	.00969	.0225	0378	.0548	.0731	.0923	.112	.133	.155	37
38	.00135	.00943	.0219	.0368	.0533	.0711	.0898	.109	.130	.150	38
39	.00131	.00919	.0213	.0358	.0519	.0692	.0874	.106	.126	.146	39
40	.00128	.00896	.0208	.0349	.0506	.0674	.0851	.104	.123	.142	40
41	.00125	.00874	.0202	.0340	.0493	.0657	.0830	.101	.120	.139	41
42	.00122	.00853	.0198	.0332	.0481	.0641	.0809	.0985	.117	.135	42
43	.00119	.00833	.0193	.0324	.0470	.0626	.0790	.0961	.114	.132	43
44	.00117	.00814	.0188	.0317	.0459	.0611	.0771	.0938	.111	.129	44
45	.00114	.00795	.0184	.0309	.0448	.0597	.0754	.0917	.109	.126	45
46	.00111	.00778	.0180	.0302	.0438	.0584	.0737	.0896	.106	.123	46
47	.00109	.00761	.0176	.0296	.0429	.0571	.0720	.0876	.104	.120	47
48	.00107	.00745	.0172	.0290	.0420	.0559	.0705	.0857	.101	.118	48
49	.00105	.00730	.0169	.0284	.0411	.0547	.0690	.0839	.0993	.115	49
50	.00103	.00715	.0166	.0278	.0402	.0536	.0676	.0822	.0972	.113	50
51	.00101	.00701	.0162	.0272	.0394	.0525	.0662	.0805	.0953	.110	51
52	.000986	.00688	.0159	.0267	.0387	.0515	.0649	.0789	.0934	.108	52
53	.000967	.00675	.0156	.0262	.0379	.0505	.0637	.0774	.0916	.106	53
54	.000949	.00662	.0153	.0257	.0372	.0495	.0625	.0759	.0898	.104	54
55	.000932	.00650	.0150	.0252	.0365	.0486	.0613	.0745	.0881	.102	55
56	.000916	.00638	.0148	.0248	.0358	.0477	.0602	.0731	.0865	.100	56
57	.000899	.00627	.0145	.0243	.0352	.0468	.0591	.0718	.0849	.0984	57
58	.000884	.00616	.0142	.0239	.0346	.0460	.0580	.0705	.0834	.0966	58
59	.000869	.00606	.0140	.0235	.0340	.0452	.0570	.0693	.0820	.0949	59
60	.000855	.00595	.0138	.0231	.0334	.0445	.0561	.0681	.0806	.0933	60
61	.000841	.00586	.0135	.0227	.0329	.0437	.0551	.0670	.0792	.0917	61
62	.000827	.00576	.0133	.0223	.0323	.0430	.0542	.0659	.0779	.0902	62
63	.000814	.00567	.0131	.0220	.0318	.0423	.0533	.0648	.0766	.0887	63
64	.000801	.00558	.0129	.0216	.0313	.0416	.0525	.0637	.0754	.0873	64
65	.000789	.00549	.0127	.0213	.0308	.0410	.0516	.0627	.0742	.0859	65
66	.000777	.00541	.0125	.0210	.0303	.0403	.0508	.0618	.0730	.0846	66
67	.000765	.00533	.0123	.0206	.0299	.0397	.0501	.0608	.0719	.0833	67
68	.000754	.00525	.0121	.0203	.0294	.0391	.0493	.0599	.0708	.0820	68
69	.000743	.00517	.0120	.0200	.0290	.0385	.0486	.0590	.0698	.0808	69
70	.000733	.00510	.0118	.0198	.0286	.0380	.0479	.0582	.0687	.0796	70

TABLE I—*Continued*

n	c = 0	1	2	3	4	5	6	7	8	9	n
71	.000722	.00503	.0116	.0195	.0282	.0374	.0472	.0573	.0678	.0785	71
72	.000712	.00496	.0115	.0192	.0278	.0369	.0465	.0565	.0668	.0773	72
73	.000702	.00489	.0113	.0189	.0274	.0364	.0459	.0557	.0658	.0762	73
74	.000693	.00482	.0111	.0187	.0270	.0359	.0452	.0549	.0649	.0752	74
75	.000684	.00476	.0110	.0184	.0266	.0354	.0446	.0542	.0641	.0742	75
76	.000675	.00470	.0108	.0182	.0263	.0349	.0440	.0535	.0632	.0732	76
77	.000666	.00463	.0107	.0179	.0259	.0345	.0434	.0528	.0623	.0722	77
78	.000657	.00457	.0106	.0177	.0256	.0340	.0429	.0521	.0615	.0712	78
79	.000649	.00452	.0104	.0175	.0253	.0336	.0423	.0514	.0607	.0703	79
80	.000641	.00446	.0103	.0173	.0249	.0332	.0418	.0507	.0600	.0694	80
81	.000633	.00440	.0102	.0170	.0246	.0328	.0413	.0501	.0592	.0685	81
82	.000625	.00435	.0100	.0168	.0243	.0323	.0408	.0495	.0585	.0677	82
83	.000618	.00430	.00992	.0166	.0240	.0319	.0403	.0489	.0577	.0668	83
84	.000610	.00425	.00980	.0164	.0237	.0316	.0398	.0483	.0570	.0660	84
85	.000603	.00420	.00969	.0162	.0235	.0312	.0393	.0477	.0564	.0652	85
86	.000596	.00415	.00957	.0160	.0232	.0308	.0388	.0471	.0557	.0645	86
87	.000589	.00410	.00946	.0159	.0229	.0305	.0384	.0466	.0550	.0637	87
88	.000583	.00405	.00936	.0157	.0227	.0301	.0379	.0460	.0544	.0630	88
89	.000576	.00401	.00925	.0155	.0224	.0298	.0375	.0455	.0538	.0622	89
90	.000570	.00396	.00915	.0153	.0221	.0294	.0371	.0450	.0532	.0615	90
91	.000564	.00392	.00904	.0152	.0219	.0291	.0367	.0445	.0526	.0608	91
92	.000557	.00388	.00895	.0150	.0217	.0288	.0363	.0440	.0520	.0602	92
93	.000551	.00383	.00885	.0148	.0214	.0285	.0359	.0435	.0514	.0595	93
94	.000546	.00379	.00875	.0147	.0212	.0282	.0355	.0431	.0509	.0589	94
95	.000540	.00375	.00866	.0145	.0210	.0279	.0351	.0426	.0503	.0582	95
96	.000534	.00371	.00857	.0144	.0207	.0276	.0347	.0421	.0498	.0576	96
97	.000529	.00368	.00848	.0142	.0205	.0273	.0344	.0417	.0493	.0570	97
98	.000523	.00364	.00840	.0141	.0203	.0270	.0340	.0413	.0487	.0564	98
99	.000518	.00360	.00831	.0139	.0201	.0267	.0337	.0408	.0482	.0558	99
100	.000513	.00357	.00823	.0138	.0199	.0265	.0333	.0404	.0478	.0553	100
101	.000508	.00353	.00814	.0136	.0197	.0262	.0330	.0400	.0473	.0547	101
102	.000503	.00350	.00806	.0135	.0195	.0259	.0327	.0396	.0468	.0542	102
103	.000498	.00346	.00799	.0134	.0193	.0257	.0323	.0392	.0463	.0536	103
104	.000493	.00343	.00791	.0132	.0191	.0254	.0320	.0389	.0459	.0531	104
105	.000488	.00339	.00783	.0131	.0189	.0252	.0317	.0385	.0454	.0526	105

TABLE I—*Continued*

n	c: 0	1	2	3	4	5	6	7	8	9	*n*
106	.000484	.00336	.00776	.0130	.0188	.0249	.0314	.0381	.0450	.0521	106
107	.000479	.00333	.00768	.0129	.0186	.0247	.0311	.0378	.0446	.0516	107
108	.000475	.00330	.00761	.0127	.0184	.0245	.0308	.0374	.0442	.0511	108
109	.000470	.00327	.00754	.0126	.0182	.0242	.0305	.0370	.0438	.0506	109
110	.000466	.00324	.00747	.0125	.0181	.0240	.0302	.0367	.0433	.0502	110
111	.000462	.00321	.00741	.0124	.0179	.0238	.0300	.0364	.0430	.0497	111
112	.000458	.00318	.00734	.0123	.0178	.0236	.0297	.0360	.0426	.0492	112
113	.000454	.00315	.00727	.0122	.0176	.0234	.0294	.0357	.0422	.0488	113
114	.000450	.00313	.00721	.0121	.0174	.0232	.0292	.0354	.0418	.0484	114
115	.000446	.00310	.00715	.0120	.0173	.0230	.0289	.0351	.0414	.0479	115
116	.000442	.00307	.00709	.0119	.0171	.0228	.0287	.0348	.0411	.0475	116
117	.000438	.00305	.00702	.0118	.0170	.0226	.0284	.0345	.0407	.0471	117
118	.000435	.00302	.00696	.0117	.0168	.0224	.0282	.0342	.0404	.0467	118
119	.000431	.00299	.00691	.0116	.0167	.0222	.0279	.0339	.0400	.0463	119
120	.000427	.00297	.00685	.0115	.0166	.0220	.0277	.0336	.0397	.0459	120
121	.000424	.00294	.00679	.0114	.0164	.0218	.0275	.0333	.0394	.0455	121
122	.000420	.00292	.00674	.0113	.0163	.0216	.0272	.0330	.0390	.0451	122
123	.000417	.00290	.00668	.0112	.0162	.0215	.0270	.0328	.0387	.0448	123
124	.000414	.00287	.00663	.0111	.0160	.0213	.0268	.0325	.0384	.0444	124
125	.000410	.00285	.00657	.0110	.0159	.0211	.0266	.0322	.0381	.0440	125
126	.000407	.00283	.00652	.0109	.0158	.0209	.0264	.0320	.0378	.0437	126
127	.000404	.00281	.00647	.0108	.0156	.0208	.0262	.0317	.0375	.0433	127
128	.000401	.00278	.00642	.0107	.0155	.0206	.0259	.0315	.0372	.0430	128
129	.000398	.00276	.00637	.0107	.0154	.0204	.0257	.0312	.0369	.0427	129
130	.000394	.00274	.00632	.0106	.0153	.0203	.0255	.0310	.0366	.0423	130
131	.000391	.00272	.00627	.0105	.0152	.0201	.0253	.0308	.0363	.0420	131
132	.000389	.00270	.00622	.0104	.0150	.0200	.0252	.0305	.0360	.0417	132
133	.000386	.00268	.00618	.0103	.0149	.0198	.0250	.0303	.0358	.0414	133
134	.000383	.00266	.00613	.0103	.0148	.0197	.0248	.0301	.0355	.0410	134
135	.000380	.00264	.00608	.0102	.0147	.0195	.0246	.0298	.0352	.0407	135
136	.000377	.00262	.00604	.0101	.0146	.0194	.0244	.0296	.0350	.0404	136
137	.000374	.00260	.00599	.0100	.0145	.0192	.0242	.0294	.0347	.0401	137
138	.000372	.00258	.00595	.00996	.0144	.0191	.0240	.0292	.0344	.0398	138
139	.000369	.00256	.00591	.00989	.0143	.0190	.0239	.0290	.0342	.0395	139
140	.000366	.00254	.00587	.00982	.0142	.0188	.0237	.0288	.0339	.0393	140

TABLE I—*Concluded*

n	c										n
	0	1	2	3	4	5	6	7	8	9	
141	.000364	.00253	.00582	.00975	.0141	.0187	.0235	.0285	.0337	.0390	141
142	.000361	.00251	.00578	.00968	.0140	.0186	.0234	.0283	.0335	.0387	142
143	.000359	.00249	.00574	.00961	.0139	.0184	.0232	.0281	.0332	.0384	143
144	.000356	.00247	.00570	.00954	.0138	.0183	.0230	.0279	.0330	.0382	144
145	.000354	.00246	.00566	.00948	.0137	.0182	.0229	.0278	.0328	.0379	145
146	.000351	.00244	.00562	.00941	.0136	.0180	.0227	.0276	.0325	.0376	146
147	.000349	.00242	.00559	.00935	.0135	.0179	.0226	.0274	.0323	.0374	147
148	.000346	.00241	.00555	.00928	.0134	.0178	.0224	.0272	.0321	.0371	148
149	.000344	.00239	.00551	.00922	.0133	.0177	.0223	.0270	.0319	.0369	149
150	.000342	.00237	.00547	.00916	.0132	.0176	.0221	.0268	.0317	.0366	150

TABLE II

Values of $p = p_2$ such that $P(c, n, p_2) = .10$

n	c = 0	1	2	3	4	5	6	7	8	9	n
1	.900										1
2	.684	.949									2
3	.536	.804	.965								3
4	.438	.680	.857	.974							4
5	.369	.584	.753	.888	.979						5
6	.319	.510	.667	.799	.907	.983					6
7	.280	.453	.596	.721	.830	.921	.985				7
8	.250	.406	.538	.655	.760	.853	.931	.987			8
9	.226	.368	.490	.599	.699	.790	.871	.939	.988		9
10	.206	.337	.450	.552	.646	.733	.812	.884	.945	.990	10
11	.189	.310	.415	.511	.599	.682	.759	.831	.895	.951	11
12	.175	.288	.386	.475	.559	.638	.712	.781	.846	.904	12
13	.162	.268	.360	.444	.523	.598	.669	.736	.799	.858	13
14	.152	.251	.337	.417	.492	.563	.631	.695	.757	.815	14
15	.142	.236	.317	.393	.464	.532	.596	.658	.718	.774	15
16	.134	.222	.300	.371	.439	.504	.565	.625	.682	.737	16
17	.127	.210	.284	.352	.416	.478	.537	.594	.650	.703	17
18	.120	.199	.269	.334	.396	.455	.512	.567	.620	.671	18
19	.114	.190	.257	.319	.378	.434	.489	.541	.592	.642	19
20	.109	.181	.245	.304	.361	.415	.467	.518	.567	.615	20
21	.104	.173	.234	.291	.345	.397	.448	.497	.544	.590	21
22	.0994	.166	.224	.279	.331	.381	.430	.477	.523	.568	22
23	.0953	.159	.215	.268	.318	.366	.413	.459	.503	.546	23
24	.0915	.153	.207	.258	.306	.352	.398	.442	.485	.526	24
25	.0880	.147	.199	.248	.295	.340	.383	.426	.467	.508	25
26	.0847	.142	.192	.239	.284	.328	.370	.411	.451	.491	26
27	.0817	.137	.185	.231	.275	.317	.358	.397	.436	.475	27
28	.0789	.132	.179	.223	.265	.306	.346	.385	.422	.459	28
29	.0763	.128	.173	.216	.257	.297	.335	.372	.409	.445	29
30	.0739	.124	.168	.209	.249	.288	.325	.361	.397	.432	30
31	.0716	.120	.163	.203	.241	.279	.315	.350	.385	.419	31
32	.0694	.116	.158	.197	.234	.271	.306	.340	.374	.407	32
33	.0674	.113	.153	.191	.228	.263	.297	.331	.364	.396	33
34	.0655	.110	.149	.186	.221	.256	.289	.322	.354	.385	34
35	.0637	.107	.145	.181	.216	.249	.282	.313	.345	.375	35

TABLE II—*Continued*

n	c: 0	1	2	3	4	5	6	7	8	9	*n*
36	.0620	.104	.141	.176	.210	.242	.274	.305	.336	.366	36
37	.0603	.101	.138	.172	.205	.236	.267	.298	.327	.357	37
38	.0588	.0985	.134	.167	.199	.230	.261	.290	.319	.348	38
39	.0573	.0961	.131	.163	.195	.225	.254	.283	.312	.340	39
40	.0559	.0938	.128	.159	.190	.220	.248	.277	.305	.332	40
41	.0546	.0916	.125	.156	.186	.215	.242	.270	.298	.324	41
42	.0533	.0895	.122	.152	.181	.210	.237	.264	.291	.317	42
43	.0521	.0875	.119	.149	.177	.205	.232	.259	.285	.310	43
44	.0510	.0856	.116	.146	.174	.201	.227	.253	.279	.304	44
45	.0499	.0837	.114	.142	.170	.196	.222	.248	.273	.297	45
46	.0488	.0819	.112	.140	.166	.192	.218	.243	.268	.291	46
47	.0478	.0803	.109	.137	.163	.188	.213	.238	.262	.285	47
48	.0468	.0786	.107	.134	.160	.185	.209	.233	.257	.280	48
49	.0459	.0771	.105	.131	.157	.181	.205	.229	.252	.274	49
50	.0450	.0756	.103	.130	.154	.178	.201	.224	.248	.269	50
51	.0441	.0741	.101	.126	.151	.174	.197	.220	.243	.264	51
52	.0433	.0728	.0991	.124	.148	.171	.194	.216	.239	.259	52
53	.0425	.0714	.0973	.122	.145	.168	.190	.212	.235	.255	53
54	.0417	.0701	.0956	.120	.143	.165	.187	.208	.230	.250	54
55	.0410	.0689	.0939	.117	.140	.162	.184	.205	.227	.246	55
56	.0403	.0677	.0923	.115	.138	.159	.180	.201	.223	.242	56
57	.0396	.0665	.0907	.113	.135	.157	.177	.198	.219	.238	57
58	.0389	.0654	.0892	.112	.133	.154	.175	.195	.216	.234	58
59	.0383	.0643	.0877	.110	.131	.152	.172	.191	.212	.230	59
60	.0376	.0633	.0863	.108	.129	.149	.169	.188	.209	.226	60
61	.0370	.0623	.0849	.106	.127	.147	.166	.185	.206	.223	61
62	.0365	.0613	.0836	.105	.125	.145	.164	.183	.203	.219	62
63	.0359	.0603	.0823	.103	.123	.142	.161	.180	.200	.216	63
64	.0353	.0594	.0810	.101	.121	.140	.159	.177	.197	.213	64
65	.0348	.0585	.0798	.0999	.119	.138	.156	.174	.194	.210	65
66	.0343	.0577	.0786	.0984	.117	.136	.154	.172	.191	.207	66
67	.0338	.0568	.0775	.0970	.116	.134	.152	.169	.188	.204	67
68	.0333	.0560	.0764	.0956	.114	.132	.150	.167	.185	.201	68
69	.0328	.0552	.0753	.0943	.113	.130	.148	.165	.182	.198	69
70	.0324	.0544	.0743	.0930	.111	.128	.146	.162	.179	.195	70

TABLE II—*Continued*

n	c: 0	1	2	3	4	5	6	7	8	9	n
71	.0319	.0537	.0732	.0917	.109	.127	.144	.160	.177	.193	71
72	.0315	.0530	.0722	.0904	.108	.125	.142	.158	.174	.190	72
73	.0310	.0522	.0713	.0892	.107	.123	.140	.156	.172	.188	73
74	.0306	.0516	.0703	.0881	.105	.122	.138	.154	.170	.185	74
75	.0302	.0509	.0694	.0869	.104	.120	.136	.152	.167	.183	75
76	.0298	.0502	.0685	.0858	.102	.119	.134	.150	.165	.180	76
77	.0295	.0496	.0676	.0847	.101	.117	.133	.148	.163	.178	77
78	.0291	.0490	.0668	.0836	.0999	.116	.131	.146	.161	.176	78
79	.0287	.0483	.0660	.0826	.0987	.114	.130	.145	.159	.174	79
80	.0284	.0478	.0652	.0816	.0974	.113	.128	.143	.157	.172	80
81	.0280	.0472	.0644	.0806	.0963	.111	.126	.141	.155	.170	81
82	.0277	.0466	.0636	.0797	.0951	.110	.125	.139	.154	.168	82
83	.0274	.0461	.0629	.0787	.0940	.109	.123	.138	.152	.166	83
84	.0270	.0455	.0621	.0778	.0929	.108	.122	.136	.150	.164	84
85	.0267	.0450	.0614	.0769	.0918	.106	.121	.135	.148	.162	85
86	.0264	.0445	.0607	.0760	.0908	.105	.119	.133	.147	.160	86
87	.0261	.0440	.0600	.0752	.0898	.104	.118	.132	.145	.158	87
88	.0258	.0435	.0594	.0743	.0888	.103	.117	.130	.143	.157	88
89	.0255	.0430	.0587	.0735	.0878	.102	.115	.129	.142	.155	89
90	.0253	.0425	.0581	.0727	.0869	.101	.114	.127	.140	.153	90
91	.0250	.0421	.0574	.0719	.0859	.0995	.113	.126	.139	.152	91
92	.0247	.0416	.0568	.0712	.0850	.0985	.112	.125	.137	.150	92
93	.0245	.0412	.0562	.0704	.0841	.0974	.110	.123	.136	.148	93
94	.0242	.0408	.0556	.0697	.0832	.0964	.109	.122	.135	.147	94
95	.0239	.0403	.0551	.0690	.0824	.0954	.108	.121	.133	.145	95
96	.0237	.0399	.0545	.0683	.0815	.0945	.107	.120	.132	.144	96
97	.0235	.0395	.0539	.0676	.0807	.0935	.106	.118	.131	.143	97
98	.0232	.0391	.0534	.0669	.0799	.0926	.105	.117	.129	.141	98
99	.0230	.0387	.0529	.0662	.0791	.0917	.104	.116	.128	.140	99
100	.0228	.0383	.0524	.0656	.0784	.0908	.103	.115	.127	.138	100
101	.0225	.0380	.0518	.0650	.0776	.0899	.102	.114	.125	.137	101
102	.0223	.0376	.0513	.0643	.0768	.0890	.101	.113	.124	.136	102
103	.0221	.0372	.0508	.0637	.0761	.0882	.100	.112	.123	.134	103
104	.0219	.0369	.0504	.0631	.0754	.0874	.0991	.111	.122	.133	104
105	.0217	.0365	.0499	.0625	.0747	.0865	.0981	.110	.121	.132	105

TABLE II—*Continued*

n	c = 0	1	2	3	4	5	6	7	8	9	n
106	.0215	.0362	.0494	.0619	.0740	.0857	.0972	.109	.120	.131	106
107	.0213	.0359	.0490	.0614	.0733	.0850	.0964	.108	.119	.130	107
108	.0211	.0355	.0485	.0608	.0727	.0842	.0955	.107	.118	.128	108
109	.0209	.0352	.0481	.0603	.0720	.0834	.0946	.106	.116	.127	109
110	.0207	.0349	.0477	.0597	.0714	.0827	.0938	.105	.115	.126	110
111	.0205	.0346	.0472	.0592	.0707	.0820	.0930	.104	.114	.125	111
112	.0204	.0343	.0468	.0587	.0701	.0812	.0921	.103	.113	.124	112
113	.0202	.0340	.0464	.0582	.0695	.0805	.0913	.102	.112	.123	113
114	.0200	.0337	.0460	.0577	.0689	.0798	.0906	.101	.111	.122	114
115	.0198	.0334	.0456	.0572	.0683	.0792	.0898	.100	.111	.121	115
116	.0197	.0331	.0452	.0567	.0677	.0785	.0890	.0994	.110	.120	116
117	.0195	.0328	.0449	.0562	.0672	.0778	.0883	.0986	.109	.119	117
118	.0193	.0326	.0445	.0557	.0666	.0772	.0875	.0977	.108	.118	118
119	.0192	.0323	.0441	.0553	.0661	.0765	.0868	.0969	.107	.117	119
120	.0190	.0320	.0437	.0548	.0655	.0759	.0861	.0961	.106	.116	120
121	.0189	.0318	.0434	.0544	.0650	.0753	.0854	.0954	.105	.115	121
122	.0187	.0315	.0430	.0539	.0645	.0747	.0847	.0946	.104	.114	122
123	.0185	.0313	.0427	.0535	.0639	.0741	.0841	.0938	.104	.113	123
124	.0184	.0310	.0424	.0531	.0634	.0735	.0834	.0931	.103	.112	124
125	.0183	.0308	.0420	.0527	.0629	.0729	.0827	.0924	.102	.111	125
126	.0181	.0305	.0417	.0523	.0624	.0724	.0821	.0917	.101	.110	126
127	.0180	.0303	.0414	.0519	.0620	.0718	.0815	.0909	.100	.110	127
128	.0178	.0301	.0410	.0515	.0615	.0713	.0808	.0902	.0995	.109	128
129	.0177	.0298	.0407	.0511	.0610	.0707	.0802	.0896	.0988	.108	129
130	.0176	.0296	.0404	.0507	.0606	.0702	.0796	.0889	.0980	.107	130
131	.0174	.0294	.0401	.0503	.0601	.0696	.0790	.0882	.0973	.106	131
132	.0173	.0291	.0398	.0499	.0596	.0691	.0784	.0876	.0966	.105	132
133	.0172	.0289	.0395	.0495	.0592	.0686	.0778	.0869	.0959	.105	133
134	.0170	.0287	.0392	.0492	.0588	.0681	.0773	.0863	.0952	.104	134
135	.0169	.0285	.0389	.0488	.0583	.0676	.0767	.0857	.0945	.103	135
136	.0168	.0283	.0387	.0485	.0579	.0671	.0762	.0850	.0938	.102	136
137	.0167	.0281	.0384	.0481	.0575	.0666	.0756	.0844	.0931	.102	137
138	.0165	.0279	.0381	.0478	.0571	.0662	.0751	.0838	.0925	.101	138
139	.0164	.0277	.0378	.0474	.0567	.0657	.0745	.0832	.0918	.100	139
140	.0163	.0275	.0376	.0471	.0563	.0652	.0740	.0826	.0912	.0996	140

TABLE II—*Concluded*

n	c 0	1	2	3	4	5	6	7	8	9	n
141	.0162	.0273	.0373	.0468	.0559	.0648	.0735	.0821	.0905	.0989	141
142	.0161	.0271	.0370	.0464	.0555	.0643	.0730	.0815	.0899	.0982	142
143	.0160	.0269	.0368	.0461	.0551	.0639	.0725	.0809	.0893	.0975	143
144	.0159	.0267	.0365	.0458	.0547	.0635	.0720	.0804	.0887	.0969	144
145	.0158	.0266	.0363	.0455	.0544	.0630	.0715	.0798	.0881	.0962	145
146	.0156	.0264	.0360	.0452	.0540	.0626	.0710	.0793	.0875	.0956	146
147	.0155	.0262	.0358	.0449	.0536	.0622	.0705	.0788	.0869	.0949	147
148	.0154	.0260	.0356	.0446	.0533	.0618	.0701	.0783	.0863	.0943	148
149	.0153	.0259	.0353	.0443	.0529	.0614	.0696	.0777	.0858	.0937	149
150	.0152	.0257	.0351	.0440	.0526	.0610	.0692	.0772	.0852	.0931	150

TABLE III

(Based on Poisson approximation to the binomial distribution)

Acceptance Number	Values of $a_1 = np_1$ for which $P(c, a_1) = .95$	Values of $a_2 = np_2$ for which $P(c, a_2) = .10$
0	.05129	2.303
1	.3554	3.890
2	.8177	5.322
3	1.366	6.681
4	1.970	7.994
5	2.613	9.275
6	3.285	10.53
7	3.981	11.77
8	4.695	12.99
9	5.425	14.21
10	6.169	15.41
11	6.924	16.60
12	7.690	17.78
13	8.464	18.96
14	9.246	20.13
15	10.04	21.29

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