

Subject Index

- Aanderaa, S. [1974] 446
Absolute 431
Abstract 1-section 449
Abstraction term 435
Acceptable 451
—, almost 454
Ackermann, W. [1928] 37
Aczel, P. 104
— [1970] 334, 350
— [1975] 246
— and Hinman, P. G. [1974] 350, 363, 412
— and Richter, W. H. [1974] 446, 447
Addison, J. W., Jr. 89, 114, 180, 200
— [1954] 180
— [1955] 89, 180
— [1959] 214
— [1959a] 221
— [1965] 132
— and Moschovakis, Y. N. [1968] 214, 236
Adequate 455
Admissible 393, 432
— set with urelements 455
 \aleph_1 -(semi-) recursive 401
Almost acceptable 454
Almost all 220
Analytic sets 135
Analytical hierarchy 80, 84
— — over the reals 161
— — Theorem 85, 89
— —, relativized 86
— Indexing Theorem 84
— Substitution Theorem 85, 123
Anonymous [1961] 463, 466
Application 448
Arithmetic, axiomatic 118–124
—, language of 114–115
—, standard model for 114
Arithmetical forcing 124–132
— hierarchy 69, 73
— —, relativized 77
— Hierarchy Theorem 74, 80
— Indexing Theorem 73
— Substitution Theorem 76
Axiom of Choice (AC) 11, 214, 215,
222–223, 230, 341
— — Constructibility 221
Baire Category Theorem 17
Baire property 183–185, 188, 220, 244–246
Bar-Hillel, Y.
— [1965] 463, 466
— [1970] 464
—, Poznanski, E. I. J., Rabin, M. O., and
Robinson, A. [1961] 465
Barwise, K. J.
— [1975] 306, 383, 431, 432, 450, 452, 453,
455, 457
— [1977] 462, 466
Base 237
—, canonical 238
Basic open set 15
— — relation 16
Basis 106–114, 130, 155, 195, 207, 211, 218,
228, 229, 296
— Theorem 198
 β -model 428
Binary decimal 19
— irrational 19–20, 160–161
— sequence 17
Blackwell, D. [1967] 231, 236
Boldface 78, 87
Borel determinacy 222
— hierarchy 78, 156–163, 236
— —, effective 80, 163–172, 246
— — over the reals 160
— Hierarchy Theorem 159

- — —, effective 172
- Indexing Theorem 159
- sets (relations) 26, 88, 192–193
- Bounded quantifier 10, 31, 431
- Bounded search 30, 44, 373
- Boundedness Theorem 143, 152, 206, 211, 218, 228, 295, 353
- — —, effective 153, 209, 305, 311
- Bulloff, J. J., Hahn, S. W., and Holyoke, T. C. [1969] 466

- Calculable, mechanically 27, 28, 35, 38, 62, 260, 343, 371
- Canonical base 238
- Cantor's Theorem 13
- Cardinal (number) 13
 - , κ - 427
 - — —, regular 429
 - , regular 385, 419
- Cases, definition by 31
- — — positive 55
- Category 16, 180, 220
- Theorem, Baire 17
 - see also Meager
- Cenzer, D.
 - [1974] 447
 - [1974b] 447
- Characteristic functional 8
- Chinese Remainder Theorem 117
- Choice, Axiom of (AC) 11, 214, 215, 222–223, 230, 341
- axioms 123, 148, 154, 200, 428
- , dependent (DC) 11, 223, 230
- principles, effective 154, 212, 297, 354
- Church's Thesis 27
- Clarke, D. A. [1964] 250
- Closed downward 17
- Closed-open 65, 78, 89
- Closure ordinal 23, 91, 103, 105, 150, 155, 198
- Closure property 30
 - — —, effective 37, 166, 271
- Closure under finitary functions 24, 40
- Coding functions 10
- Cofinal 12
- Cohen, P. J. [1963/64] 124
- Collapsing map 216
- Collection, Δ_0 - and Σ - 432
 - Principle 341
 - , strong Δ_0 - 443
- Comeager 17
- Compact operator 26, 260
 - space 18
- Companion 453
- Complement 8

- Complete 75, 102
 - theory 120
- Completeness Theorem 122
- Component 8
- Composition 9
 - , functional 28
 - , relational 31
- Comprehension axioms 122, 123, 148, 200, 221, 235, 306, 403, 428
 - see also Separation
- Computation tree 35, 262, 345
- Consistency 108, 214, 222, 230, 235, 341
- Constructibility, Axiom of 221
 - , Hypothesis of 201, 214–221, 229, 341, 419, 443, 444, 446, 447
- Constructible 82, 230, 442
 - sets, hierarchy of 215
- Constructive ordinal 143
- Continuous 16, 28
 - image 188–194, 198–199, 229
- Continuum Hypothesis 11, 13, 185, 188, 219, 443
 - — —, Generalized 11, 214
- Co-semi-recursive 52, 70
 - — — in I 261
- Countable 13
 - choice (AC_ω) 11
- Course-of-values recursion 33, 43
- Crossley, J. N. [1967] 461, 465

- Davis, M. [1964] 229, 234, 236
- Decomposable operator 25, 99–102
- Definable 214
 - , Δ - 432
 - , Σ - 432
- Degree 63, 435
 - , hyper- 86, 149, 221
 - , Δ_r^1 - 221, 235
- Dekker, J. C. E. [1962] 465
- Δ -definable 432
 - -Separation 432
- Δ_0 -Collection 432
 - — —, strong 443
 - formula 430
 - -Separation 432
- Dense 16
- Dependent Choice (DC) 11, 223, 230
- Descriptive Set Theory 18, 58, 64, 156, 163
- Determinacy 222, 230, 241
 - , projective 201, 221–236, 353, 446
- Determined 222
- Deterministic 448
- Devlin, K. J. [1973] 214, 221
- Diagonal argument 37
- Discrete topology 15

- Domain 7
 Dual 238, 266
- Effective 409
 — Borel hierarchy 80, 163–173, 246
 — Hierarchy Theorem 172
 — boundedness principle 153, 209, 305, 311
 — choice principles 154, 212, 297, 354
 — closure property 37, 166, 271
 —, κ - 407
 — Recursion Theorem 45
 — transfinite recursion 166
- Effectively Borel 80, 163–172
 — closed 52
 — closed-open 50
 — continuous 50
 — discontinuous 284
 — open 52
- Empty sequence 8
- Enderton, H. B.
 — [1964] 250
 — [1972] 114
- Envelope 448
- Epsilon (\in)-structure 214
- Expansion 30
- Extended functional 317–334
- Family 236
- Fan 240
- Feferman, S. [1964/65] 124, 132
- Fenstad, J. E.
 — [1971] 461
 — [1971a] 236
 — [1974] 457
 — [1975] 457
 — [1980?] 457
 — and Hinman, P. G. [1974] 459, 460, 462, 464, 465, 466
 — and Normann, D. [1974] 246
- Field 13
- Finite 371, 382
 —, κ - 392
 — sequence 8
 — —, coding of 10–11, 32, 374, 382
- First category 16
 — Recursion Theorem 61, 283
- Fixed point, smallest 23
 — —, largest 25
- Flow diagram 286
- Forcing 124–132
- Formula 115, 214
 —, Δ_0 430
 —, \exists' and \forall' 116, 443
 —, first-order positive 450
 —, Σ and Π 431
- Friedberg–Mučnik Theorem 64, 149, 398
- Friedman, H. M. [1971] 222, 230, 236
- Fubini's Theorem 220
- Function 7
 — of rank k 8
 — quantification 57, 75, 82, 345, 355
- Functional 8
 —, characteristic 8
 —, initial 28
 — quantification 337, 365, 366
 — quantifier hierarchy 337
 — substitution 50, 60, 266, 279
- Gale, D. and Stewart, F. M. [1953] 231, 236
- Game 103, 222, 240, 330
 —, open 231, 241
 — quantifier 103, 240, 330, 331, 359, 452
- Gandy, R. O. 156, 212, 350
 — [1960] 156
 — [1967] 271
 — [1967a] 271, 291, 350
 — and Yates, C. E. M. 462–466
- Generalized continuum hypothesis 11, 214
- Generic 126
- Gödel, K. 82, 118
 — Incompleteness Theorem 121
 — numbering 120, 216, 438
 — [1931] 15, 37, 124
 — [1939] 214
- Gordon, C. [1970] 456
- Graph 9
- Grilliott Selection Theorem 366, 370, 457
 — [1969] 370
 — [1971] 284
 — [1971a] 447, 457
- Harrington, L. A.
 — [1973] 450
 — [1974] 423
 — [1978] 230
 — and MacQueen, D. B. [1976] 370
 — and Kechris, A. S. [1975] 448
 — — — [1976] 455
- Harrison, J. 187
- Height 216
- Hierarchy 69, 80, 144, 153, 156, 163, 207, 214, 218, 228, 243, 295, 305, 314, 337, 353
- Hinman, P. G.
 — [1964] 350
 — [1966] 250, 251, 284, 315, 350
 — [1969] 244, 250, 251, 315, 334
 — [1969a] 132
 — [1973] 334
 — and Moschovakis, Y. N. [1971] 359, 447
- Hyperanalytical 356

- Hyperarithmetical 147
 — hierarchy 178, 180
 Hyperdegree 86, 149, 221, 253, 255
 Hyperelementary 445, 451
 Hyperjump 155, 249, 255
 Hyperprojective 356
 Hypothesis of Constructibility ($V = L$) 201,
 214–221, 229, 341, 419, 443, 444, 446, 447
 — — Projective Determinacy (PD) 201,
 221–236, 353, 446
- Idealized computer 27, 260, 343, 371
 Image 7, 57, 188–194, 296, 390, 397
 Implicitly 79, 106, 113–114, 132, 153, 155,
 251–255
 Inaccessible cardinal 235
 —, recursively 403, 420, 427
 Incompleteness Theorem 121
 Index 34, 40, 62, 261
 —, natural 272
 Indexable 59
 Indexing Theorem 73, 84, 159
 Induction, course-of-values 26
 — over an inductively defined set 23
 —, transfinite 12
 Inductive operator 22–26
 — —, arithmetical 82, 103–105, 155, 419
 — —, compact 26
 — —, decomposable 25, 99–102
 — —, effective Φ -positive 248, 332
 — —, Δ^1_1 94, 96, 101, 102, 105, 198, 359
 — —, Δ^2_1 341
 — —, monotone 22
 — —, Σ^0_1 91–94, 101, 217
 — —, Σ^1_1 326–330
 — —, Σ^1_2 97–99, 101, 102, 104, 198
 — —, Σ^2_1 341
 — —, over κ^ω 99
 — —, over κ^{κ^ω} 99–102, 339–342, 359, 447
 — —, Π^0_1 90, 94, 100, 103, 150–152, 155
 — —, Π^1_1 89, 97–98, 100, 102, 150
 — —, Π^2_1 339–340
 — —, positive semi-recursive in \mathbb{I} 284
 — —, universal 104
 Infimum (inf) 11
 Infinitary language 382
 Infinite branch 17
 (∞, λ) -partial recursive 377
 ∞ -partial recursive 377
 Infinity Lemma 17
 Initial functional 28, 39
 Interval 16
 Intuitively calculable
 see Mechanically calculable
 Irrational numbers 18
- , binary 19, 20, 160, 161
 Is defined 7
 Isolated 17
- Jump operator 249, 307–315
 — —, Δ^1_1 250
 —, ordinary 65, 249
- Kalmar sets 21
 Kantorovitch, L. and Livenson, E.
 [1932/33] 245
 κ -cardinal 427, 429
 — -effective 407
 — -finite 392
 — -partial recursive 377
 — -projectible 427
 Kechris, A. S.
 [1973] 114
 [1973a] 449
 — and Moschovakis, Y. N. [1977] 450
 Kino, A. and Takeuti, G.
 [1962] 382, 403
 [1962a] 382, 403
 Kleene, S. C. 124, 271, 383
 [1935/36] 46, 51
 [1943] 89
 [1950] 89
 [1952] 28, 46
 [1955] 26
 [1955a] 26, 105, 143
 [1955b] 46, 68, 89
 [1959] 46, 270, 350
 [1963] 250, 284, 350
 Kleene Basis Theorem 108, 213
 Kolmogorov, A. N. 245
 Kondô, M. [1938] 200
 Kreisel, G.
 [1961] 382
 [1962] 123, 154
 — and Sacks, G. E. [1965] 383, 403
 Kreisel Basis Theorem 108
 Kripke, S. 412
 [1964] 383, 393, 419, 444
 [1964a] 383, 403
 Kuratowski, K. [1966] 220
 Kuratowski-Ulam Theorem 220
- Language, infinitary 382
 — of arithmetic 114–115
 — of set theory 214
 λ -search 374
 — -stable 412
 Least non- Δ^1_1 97
 — nonprojectible 424

- non-recursive 97
- recursively inaccessible 420–421
- — Mahlo 423
- — regular 395
- stable 417
- weakly stable 418
- Lebesgue, H. 143
- Length of a computation 261, 351, 457
- — — norm 212
- — — finite sequence 8
- Lerman, M. [198?] 64
- Lévy, A.
- [1963] 382
- [1965] 444
- [1965a] 108
- [1978] 11, 15
- Limit 12, 162
- Ljapunov, A. A. [1953] 246
- , Stscheolkow, E. A., and Arsenin, W. J. [1955] 194
- Local countability 449
- Locally correct 302, 357
- Logic, first order 24, 70, 82, 102, 114–121
- , second order 121–123
- Logical notation 9
- Lorenzen, P. and Myhill, J. [1959] 105
- Löwenheim–Skolem Theorem 413
- Lower Classification 146, 208, 219, 228, 297, 359
- Luzin, N. N. [1930] 143, 187
-
- \mathfrak{M} -positive-Inductive 451
- — — Hyperelementary 451
- -Spector class 451
- Machover, M. [1961] 382
- Mahlo 448
- , recursively 422–423
- Many-one reducible 65, 75, 90, 136
- Martin, D. A. 213, 235
- [1968] 231, 236
- [1970] 222
- [1975] 222
- [1977] 230
- [1979?] 236
- Meager 16, 63, 110, 114, 130, 183–185, 220
- , co- 17
- Measurable 182, 188, 220, 229, 244–246
- cardinal 188, 222
- Measure 20, 110–113
- Mechanically calculable 27, 28, 35, 38, 62, 260, 343, 371
- semi-calculable 52
- Metafinite 371
- Moldestad, J. [1977] 457
- Monomorphic 24
-
- Monotone operator 22
- see also Inductive operator
- Moschovakis, Y. N. 156, 211, 213, 229
- [1967] 214, 291, 334, 359, 363
- [1969] 214, 456, 457
- [1970] 214, 341
- [1971] 457
- [1971a] 229
- [1972] 102, 334
- [1973] 236
- [1974] 26, 334, 448, 450, 457
- [1974a] 448–454
- [1974b] 447, 454
- [1979?] 236
- Mostowski, A. M. 156
- [1946] 89
- [1969] 214, 221
- Müller, G. H., Oberschelp, A., and Potthoff, K. [1975] 461, 465
- Mycielski, J.
- [1964] 231, 235
- and Steinhaus, H. [1962] 236
- and Swierkowski, S. 229
-
- Natural index 272
- number 8
- Neumann, J. von 236
- Nonprojectible 423
- Norm 14, 202, 212
- Normal 244, 448
- Normal form 48, 49, 51, 388–389
- —, absence of 259, 262
- Notations for ordinals 173
- Novikov, P. S. 214
- Nowhere dense 16
-
- ω (omega) 8
- -rule 121
- -structure 121
- ω_1 (omega one) 97
- -computable 398
- —, weakly 398
- -(semi-)recursive 396, 402
- Open 16, 64, 78
- , effectively 52
- game 231, 241
- Operation 237, 307
- \mathcal{A} 88, 135, 237, 240, 245
- , Δ^1_1 239
- , Δ^1_2 247
- , normal 244
- Operator
- see Inductive operator, Jump operator
- Order 215
- Ordering 13

- Order-type 12, 14
 Ordinal (number) 11
 — arithmetic 373
 —, closure 23, 91, 103, 105, 150, 155, 198
 —, constructive 143
 —, Δ_1^1 97, 208, 230
 —, least see Least
 — notation 173
 —, recursive 97, 104, 246
 —, Σ_1^1 and Π_1^1 148
 Ordinal Comparison Theorem 104–105,
 266, 285, 291, 323, 351, 365
 Oxtoby, J. C. [1971] 187, 220
- Pair 432
 Parameter 100, 258, 347, 377
 Parametrizable 67, 79
 Paris, J. [1972] 236
 Partial continuous 16, 48, 49, 64, 67
 Partial function 7
 Partial recursive 40, 48, 335
 —— in I 261
 —— in \mathbb{I} 344
 —— in \mathcal{I} 364
 ——, κ --, (∞, λ) --, or ∞ - 377
 ——, —, in parameters 377
 PD (Hypothesis of Projective Determinacy)
 201, 221–236, 353, 446
 Peaches 306
 Perfect set 17, 26, 219, 229, 234
 Péter, R.
 — [1935] 37
 — [1967] 37
 Π -definable 432
 — formula 431
 Platek, R. A. 412
 — [1966] 383, 392, 419, 444
 Plus-One Theorem 450, 457
 — Two Theorem 450, 457
 Positive analytic 237
 Post's Theorem 76, 87
 Power set 13
 Predecessor function 29, 43, 380
 Pre-wellordering 13
 —, Δ_1^1 208, 230
 —, norm associated with 14, 202
 — property 201–214, 218, 221, 223, 227,
 230, 294, 341
 Prime computable 457
 Primitive recursion 29, 43, 373
 — Recursion Theorem 41
 Primitive recursive functional 29
 —— in β 37
 —— in I 269
 —— index 34
- — ordinal 142
 — — relation 30, 56
 Product measure 21
 — topology 15
 Projectible 423, 429, 453
 —, κ - 427
 Projection 87, 89, 135
 Projective determinacy 201, 221–236, 353,
 446
 — hierarchy 87, 89
 — ordinals 230
 Projectum 423
 Property of Baire 183–185, 188, 220,
 244–246
 Provably Δ_2^1 246
 Putnam, H. [1964] 105
- Quantification, function 57, 75, 82, 345, 355
 —, functional 337, 365, 366
 —, number 54, 71, 82, 116, 262, 266, 294,
 325
 — over κ 390
 —, set 88
 Quantifier 88, 104, 238, 315–318
 —, game 103, 240, 330, 331, 359, 452
- Rank 8
 Raspberries 306
 Real numbers 18, 160–161, 163, 187
 Recursion, course-of-values 33, 43
 —, definition by 12, 24, 433
 —, primitive 29, 43, 373
 —, transfinite 12, 433
 —, —, effective 166
 Recursion Theorem 41, 45, 261, 335, 344,
 380
 —, effective 45
 —, —, First 61, 283
 —, —, Primitive 41
 Recursive 27, 40, 56, 70, 335
 — in 63, 282, 348
 — — a functional 261, 307, 318, 344, 364
 — — β 62
 —, κ --, (∞, λ) --, or ∞ - 377
 —, primitive see Primitive recursive
 —, semi- see Semi-recursive
 — union 80
 Recursively axiomatizable 120
 — enumerable 58, 62
 — hyper-inaccessible 421
 — inaccessible 403, 420–421, 427
 — inseparable 60
 — Mahlo 422–423
 — regular 383ff., 395
 Reduce 58

- Reducible, many-one 65, 75, 90, 136
 —, truth-table 66
- Reduction property 58, 67, 75, 80, 105, 138, 139, 141, 200, 205, 211, 218, 228, 230, 231, 236, 295, 390
- Reflecting 444
- Regular cardinal 385, 419
 —, recursively 383ff.
- Relation 8
 — on numbers 8
 —, ordering 13
- Relational substitution 50
- Relativized version 62
- Replacement axiom 236, 390
- R*-sets 245–246
- Residual 17
- Resolvable 453
- Restriction 7
- Richter, W. H. 103
 — [1971] 156
 — [1975] 446
- Rogers, H., Jr.
 — [1959] 167
 — [1967] 68, 200, 213
- Rosser, J. B. 124
- Rum 306
- Sacks, G. E.
 — [1969] 114
 — [1971] 398
 — [1974] 449
 — [1978] 449
 — and Simpson, S. G. [1972] 399
- Scale property 232–234
- Scott, D. S. [1971] 462, 464, 466
- Search, bounded 30, 44, 373
 — computable 456
 —, λ - 374
 —, unbounded 42, 64, 374
- Second category 16
- Section 448
 —, abstract 1- 449
- Selection Theorem 53, 145, 208, 211, 218, 228, 292, 304–305, 325, 352, 366, 390
 — —, Grilliot 366, 370, 457
- Self-determination principle 38, 64
- Semi-index 52, 62
- Semi-recursive 52, 62, 70, 119, 335
 —— in I 261
 ——, κ -, (∞, λ) -, or ∞ - 377
 —— set 57
- Semi-Spector class 448
- Separate 58
- Separation, Δ_0 - and Δ - 432
 — property 58, 67, 75, 80, 89, 139, 141, 161, 205, 211, 218, 228, 230, 295, 390
 — —, second 61
 —, Σ - 443
- Sequence closed 425
- Sequence coding 10–11, 32, 374, 382
- Set theory 11
 — —, language of 214
- Shoenfield, J. R.
 — [1958] 114
 — [1961] 444
 — [1962] 89, 230
 — [1967] 70, 114, 118, 122, 200
 — [1968] 250, 315
 — [1971] 132
 — [1971a] 68
 — [1971b] 188
- Shore, R. A. [1977] 398
- Sierpiński, W. 143
- Σ -Collection 432
 — -definable 432
 — formula 431
 — -Recursion 433
 — -substructure 443
- Simpson, S. G. 448
 — [1974] 403
 — [1984?] 398
- Solovay, R. M. 213, 230, 234, 246
 — [1969] 188
- Spector, C.
 — [1955] 143, 156, 180
 — [1958] 221
 — [1959] 155, 156
 — [1961] 26
- Spector class 447, 448
 — —, \mathfrak{M} - 451
 — —, semi- 448
 — computation theory 457
 — —Gandy Theorem 147, 154, 156, 199, 213, 229, 300, 333, 356
- Stable ordinal 412
 — —, least 417
- Stage 22
 — comparison see Ordinal comparison
- Standard model 114, 451
- Strategy 222
- Strawberries 306
- Strong, H. R. [1968] 457
- Subcomputation 36, 46, 274, 457
 —, immediate 276
- Subinterval 16
- Substitution, functional 50, 60, 266, 279, 336, 347–348, 365
 —, relational 50
- Substructure, Σ - 443
- Superjump 269, 282, 345, 348–349, 409–411, 423

- Suppes, P., Henkin, L., Joja, A., and Moisil,
Gr. C. [1973] 464
- Supremum (sup) 12
- Suslin, M. 143, 157, 245
- operation (quantifier) \mathcal{A} 88, 104, 135,
237, 240, 245
- 's Theorem 163
- Suzuki, Y. [1964] 256
- Takeuti, G. [1960] 382, 419
- Tanaka, H.
— [1967] 114
— [1970a] 114
- Tarski, A. 124
- Total function 7
- Transfinite induction 12
— recursion 12
— —, effective 166
- Transitive 11
- closure 433
- Tree 141
—, computation 35, 262, 345
- Tugué, T. [1960] 342
- Turing, A. M. [1936] 46
- Type 9
- Unbounded search 42, 64, 374
- Uniformizable 194
- Uniformization property 194, 199, 200, 218,
229, 232
- Theorem 196
- Uniform(ly) 100, 143
- Union 432
- , recursive 80
- Universal relation 59, 67, 73, 84, 93, 158,
171, 337, 338, 351
- inductive operator 104
- Upper Classification 146, 207, 219, 228,
296, 354
- Urelements 455
- $V = L$ (Hypothesis of Constructibility) 201,
214–221, 229, 341, 419, 444, 446, 447
- Wadge, W. 235
— games 235–236
- Wagner, E. G. [1969] 457
- Weakly stable 412
- Well founded 14, 216, 355
- Well-ordering 14
—, Δ_i^1 97, 208
—, κ -recursive 393
—, pre- see Pre-wellordering
— of ω 340
—, recursive 97
—, — in I 283
—, Σ_i^1 and Π_i^1 148
- Winning strategy 222
- Wolfe, P. 236
- Zermelo–Fraenkel set theory (ZF) 11, 214,
222, 230, 341
- Zermelo set theory 236
- Zero-one Law 21
- Zuckerman, M. M. [1974] 15