argument may be simplified by deducing therefrom weaker propositions, either by deducing one of the factors from a product, or by deducing from a proposition a sum (alternative) of which it is a summand.

Formulas (2) and (4) are called the *principle of composition*, because by means of them two inclusions of the same antecedent or the same consequent may be combined (*composed*). In the first case we have the product of the consequents, in the second, the sum of the antecedents.

The formulas of the principle of composition can be transformed into equalities by means of the principles of the syllogism and of simplification. Thus we have

1 (Syll.) $(x < ab)\ (ab < a) < (x < a),$

(Syll.) $(x < ab)\ (ab < b) < (x < b).$

Therefore

(Comp.) $(x < ab) < (x < a) \cdot (x < b).$

2 (Syll.) $(a < a + b)\ (a + b < x) < (a < x),$

(Syll.) $(b < a + b)\ (a + b < x) < (b < x).$

Therefore

(Comp.) $(a + b < x) < (a < x)\ (b < x).$

If we compare the new formulas with those preceding, which are their converse propositions, we may write

$$(x < ab) = (x < a)\ (x < b),$$

$$(a + b < x) = (a < x)\ (b < x).$$

Thus, to say that x is contained in ab is equivalent to saying that it is contained at the same time in both a and b; and to say that x contains $a + b$ is equivalent to saying that it contains at the same time both a and b.

9. The Laws of Tautology and of Absorption.—Since the definitions of the logical sum and product do not imply any order among the terms added or multiplied, logical addition and multiplication evidently possess commutative and associative properties which may be expressed in the formulas

$ab = ba,$	$a + b = b + a,$
$(ab)\ c = a\ (bc),$	$(a + b) + c = a + (b + c).$

Moreover they possess a special property which is expressed in the *law of tautology:*

$$a = aa, \quad \Big| \quad a = a + a.$$

Demonstration:

1 (Simpl.) $aa < a,$

(Comp.) $(a < a)\ (a < a) = (a < aa)$

whence, by the definition of equality,

$$(aa < a)\ (a < aa) = (a = aa).$$

In the same way:

2 (Simpl.) $a < a + a,$

(Comp.) $(a < a)\ (a < a) = (a + a < a),$

whence

$$(a < a + a)\ (a + a < a) = (a = a + a).$$

From this law it follows that the sum or product of any number whatever of equal (identical) terms is equal to one single term. Therefore in the algebra of logic there are neither multiples nor powers, in which respect it is very much simpler than numerical algebra.

Finally, logical addition and multiplication possess a remarkable property which also serves greatly to simplify calculations, and which is expressed by the *law of absorption:*

$$a + ab = a, \quad \Big| \quad a\ (a + b) = a.$$

Demonstration:

1 (Comp.) $(a < a)\ (ab < a) < (a + ab < a),$

(Simpl.) $a < a + ab,$

whence, by the definition of equality,

$$(a + ab < a)\ (a < a + ab) = (a + ab = a).$$

In the same way:

2 (Comp.) $(a < a)\ (a < a + b) < [a < a\ (a + b)],$

(Simpl.) $a\ (a + b) < a,$

whence

$$[a < a\ (a + b)]\ [a\ (a + b) < a] = [a\ (a + b) = a].$$

Thus a term (a) *absorbs* a summand (ab) of which it is a factor, or a factor $(a + b)$ of which it is a summand.