pretations the relation $<$ may be translated approximately by "therefore".

Remark.—Such a relation as "$a < b$" is a proposition, whatever may be the interpretation of the terms a and b. Consequently, whenever a $<$ relation has two like relations (or even only one) for its members, it can receive only the propositional interpretation, that is to say, it can only denote an implication.

A relation whose members are simple terms (letters) is called a *primary* proposition; a relation whose members are primary propositions is called a *secondary* proposition, and so on.

From this it may be seen at once that the propositional interpretation is more homogeneous than the conceptual, since it alone makes it possible to give the same meaning to the copula $<$ in both primary and secondary propositions.

4. Definition of Equality.—There is a second copula that may be defined by means of the first; this is the copula $=$ ("equal to"). By definition we have

$$a = b,$$

whenever

$$a < b \text{ and } b < a$$

are true at the same time, and then only. In other words, the single relation $a = b$ is equivalent to the two simultaneous relations $a < b$ and $b < a$.

In both interpretations the meaning of the copula $=$ is determined by its formal definition:

C. I.: $a = b$ means, "All a's are b's and all b's are a's"; in other words, that the classes a and b coincide, that they are identical.[1]

P. I.: $a = b$ means that a implies b and b implies a; in

[1] This does not mean that the concepts a and b have the same meaning. Examples: "triangle" and "trilateral", "equiangular triangle" and "equilateral triangle".

other words, that the propositions a and b are equivalent, that is to say, either true or false at the same time.[1]

Remark.—The relation of equality is symmetrical by very reason of its definition: $a = b$ is equivalent to $b = a$. But the relation of inclusion is not symmetrical: $a < b$ is not equivalent to $b < a$, nor does it imply it. We might agree to consider the expression $a > b$ equivalent to $b < a$, but we prefer for the sake of clearness to preserve always the same sense for the copula $<$. However, we might translate verbally the same inclusion $a < b$ sometimes by "a is contained in b" and sometimes by "b contains a".

In order not to favor either interpretation, we will call the first member of this relation the *antecedent* and the second the *consequent.*

C. I.: The antecedent is the *subject* and the consequent is the *predicate* of a universal affirmative proposition.

P. I.: The antecedent is the *premise* or the *cause*, and the consequent is the *consequence.* When an implication is translated by a *hypothetical* (or *conditional*) judgment the antecedent is called the *hypothesis* (or the *condition*) and the consequent is called the *thesis.*

When we shall have to demonstrate an equality we shall usually analyze it into two converse inclusions and demonstrate them separately. This analysis is sometimes made also when the equality is a datum (a *premise*).

When both members of the equality are propositions, it can be separated into two implications, of which one is called a *theorem* and the other its *reciprocal.* Thus whenever a theorem and its reciprocal are true we have an equality. A simple theorem gives rise to an implication whose antecedent is the *hypothesis* and whose consequent is the *thesis* of the theorem.

It is often said that the hypothesis is the *sufficient condition* of the thesis, and the thesis the *necessary condition* of the hy-

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[1] This does not mean that they have the same meaning. Example: "The triangle ABC has two equal sides", and "The triangle ABC has two equal angles".

pothesis; that is to say, it is sufficient that the hypothesis be true for the thesis to be true; while it is necessary that the thesis be true for the hypothesis to be true also. When a theorem and its reciprocal are true we say that its hypothesis is the necessary and sufficient condition of the thesis; that is to say, that it is at the same time both cause and consequence.

5. Principle of Identity.—The first principle or axiom of the algebra of logic is the *principle of identity*, which is formulated thus:

(Ax. I) $$a < a,$$

whatever the term a may be.

C. I.: "All a's are a's", i. e., any class whatsoever is contained in itself.

P. I.: "a implies a", i. e., any proposition whatsoever implies itself.

This is the primitive formula of the principle of identity. By means of the definition of equality, we may deduce from it another formula which is often wrongly taken as the expression of this principle:

$$a = a,$$

whatever a may be; for when we have

$$a < a, \quad a < a,$$

we have as a direct result,

$$a = a.$$

C. I.: The class a is identical with itself.

P. I.: The proposition a is equivalent to itself.

6. Principle of the Syllogism.—Another principle of the algebra of logic is the principle of the *syllogism*, which may be formulated as follows:

(Ax. II) $$(a < b) \quad (b < c) < (a < c).$$

C. I.: "If all a's are b's, and if all b's are c's, then all a's are c's". This is the principle of the *categorical syllogism.*

P. I.: "If a implies b, and if b implies c, a implies c." This is the principle of the *hypothetical syllogism.*