Théorie des Nombres. Tome second: Le Second Degrée Binaire, by E. Cahen, Paris, Hermann, 1924. 736 pp.

This volume is devoted to the development of that part of the theory of numbers which centers around the study of the binary quadratic forms. As the number of pages suggests, the treatment is very comprehensive and besides giving a complete account of all theory necessary in the development, problems in Diophantine analysis, which spring out of the theory of quadratic forms, are taken up for consideration when their interest is of sufficient generality to warrant it. To each chapter is appended a list of exercises in which problems of less general interest are left to the reader.

For any arithmetic theory relative to the field of rational numbers a comprehension of the classical theory of numbers is essential. The author devotes the first four chapters to the development of this theory concluding with the consideration of congruences of the second degree in one unknown. This development is followed by five chapters in which treat the idea of a number in general and various methods of approximation. Special attention is given to the theory of continued fractions and the development of numbers into continued fractions, and this again with special reference to quadratic irrationalities.

Chapter ten is a discussion of Diophantine equations in which one member is a quadratic polynomial in x and y. A very complete study is made of Fermat's equation of the first and second species

$$x^2-Dy^2 = \pm 1, \qquad x^2+xy-ky^2 = \pm 1.$$

The first of these is what is commonly called Pell's equation, an error of long standing.

The discussion of quadratic Diophantine equations in two unknowns is followed by a general study of binary quadratic forms in which both the algebraic and arithmetic theories are considered. The study includes the theory of the classification of binary quadratic forms and the separation of the classes into genera. The group theory with special reference to finite abelian groups is developed sufficiently for a good comprehension of the theory of composition of forms and the group of classes.

In the study of quadratic forms the author considers the general form $ax^2+bxy+cy^2$ in place of the form $ax^2+2bxy+cy^2$.

Chapter 25 is a study of the equation $x^p+y^p = z^p$ for $p = 2, 3, 4$. The study of these equations leads, as is well known, to quadratic irrationalities $i = \sqrt{-1}$ and $j = (-1+\sqrt{-3})/2$. The two fields $C(i)$ and $C(j)$ are studied in detail in chapters 26 and 27.

The remaining chapters contain the development of the general theory of quadratic number fields with the laws governing the factorization of rational primes in such fields and the classification of ideals.

G. E. Wahlin