# BIRKHOFF ON RELATIVITY

*Relativity and Modern Physics.* By G. D. Birkhoff. Cambridge, Harvard University Press, 1923. xi + 283 pp.

Birkhoff's general plan for his book on relativity is excellent. He begins with a rapid survey of those branches of classical physics which are to be affected most seriously by the new theory. This survey shows that the old theories have been fundamentally limited by the underlying concept of a rigid body in empty space, and leads to the inquiry as to what sort of a theory can be built up by using for our physical model "a number of very small material particles in motion at comparatively great distances from each other in otherwise empty space".

For simplicity he limits attention at first to a one-dimensional case. The position of a particle $B$ with respect to a particle $A$ is determined by the time $t_1$ of emission and the time $t_2$ of return of a flash of light from $A$ which is reflected by $B$.* Thus the events of a one-dimensional universe are seen to constitute a two-dimensional manifold. It is shown how the coordinates $t_1$ and $t_2$ may be replaced by the more conventional $x$, $t$, but the early part of the discussion is carried out in terms of the physically primordial $t_1$ and $t_2$.

By introducing certain assumptions (isometric, etc.) as to the character of space-time, attention is limited to two special cases which Birkhoff calls "aeolotropic" and "isotropic" respectively. The first leads to the classical and the second to the special relativity metric. The consequences of the two assumptions are developed far enough by the end of chapter four to give a clear general idea of the differences between the two cases. From this point on attention is restricted to the isotropic case.

Three chapters are now devoted to the development of the special relativity in a two-dimensional space-time, the main topics being the dynamics of a particle and of a system of particles and one-dimensional hydrodynamics. From this discussion there emerges that complex of ideas which Einstein has bound together in the energy-momentum tensor. Then comes a chapter on tensor analysis and Riemann geometry, followed by a chapter on gravitational theory in two dimensions.

On page 150 (there are 270 pages of text) we turn for the first time to physical problems of more than one space dimension. In two chapters

* This method of arriving physically at a coordinate system had previously been suggested by J. L. Synge, NATURE, vol. 108, p. 275 (October 27, 1921) and worked out to a certain extent for the three-dimensional case.

the previous discussion of special relativity is generalized to four dimensions, so that on arriving at page 186 we have the kinematics, dynamics, and hydrodynamics of the special theory behind us. Reversing the historical order, the author now closes his discussion of the special relativity with an account of the electromagnetic equations.

The remaining sixty odd pages of the book are devoted to the general relativity. They contain first the determination of the possible types of linear homogeneous tensor equations of the second order. Then follows an account of Einstein's partial differential equations for a gravitational field with or without matter. Then the Schwarzschild form for the solar field is deduced and followed by an account of the three famous astronomical applications.

As intimated above, the arrangement of the book seems to the reviewer to be a very happy one. It responds very well to the fact that although the relativity theory arose from a study of the electromagnetic equations it was obtained by revising our views of kinematics and matter. Also, the prolonged attention to the two-dimensional case accustoms the reader to the machinery of tensor analysis and to a long series of abstract conceptions while he is dealing with figures that are easily visualized. The reader is thus prepared to meet the difficulties due to the complicated set of relations in the four-dimensional case, with the most important general concepts already in mind. It is true, on the other hand, as pointed out by Eisenhart in a careful review of this book in *Science* for December 28, 1923, that the methods which suggest themselves first in the two-dimensional case cannot always be used in the four-dimensional one. But such cases are exceptional in this book.

The book is intended as an introduction to the subject which may be used as a text in an undergraduate course. From this point of view, the first four chapters seem very difficult reading. There are other books, as, for instance, that of Kopff, with which it would be easier to start. It seems likely that the book will find its place not as a textbook in the ordinary sense but as one of two or three texts to which the student may refer for different points of view.

The reviewer has heard the opinions of various people who have read Birkhoff's book, or parts of it, without being previously well acquainted with the Einstein theory. In general, the accounts were more favorable from those who were not professional mathematicians than from those who were. The distinction seems to be due to the fact that readers of the latter class were disturbed by what they regarded as lapses from mathematical elegance or clarity of statement. In many cases they were handicapped by their own ignorance of physics. No one in either class regarded the book as easy reading, but all were impressed with the author's originality in point of view and with his grasp of the subject.

The reviewer's own opinion is that the real merit of the book is that it will convey new ideas and stimulating points of view to a reader who has already devoted a good deal of thought to the subject. The derivation of the Schwarzschild form without the hypothesis that the field is static and irreversible is obviously an important contribution. So is the chapter on linear tensor equations. But there are many less prominent matters which show great originality of method or content. The reviewer has dipped into the book at several places and always found something worth thinking about. See, for example, the treatment of statistical mass on page 72, or the form for the equations for gravitation on page 229, or the account of the tensor $\Phi_{ijkl}$ (which is in fact the second extension of $g_{ij}$) on page 123.

The postulational treatment to which the author calls special attention in the preface would better, in the opinion of the reviewer, be characterized as prolegomena to a set of postulates. The term "postulational treatment" has come in recent years to have a very precise significance. It implies an explicit list of undefined terms and a sharp statement of the logical form of the unproved propositions. It usually implies also an investigation of independence proofs. Such a study would doubtless have been impracticable in this book, but it would be interesting to see it carried out on some other occasion along the lines indicated by Birkhoff.

In various places Birkhoff brings out either explicitly or implicitly the great role played by considerations of symmetry and simplicity in the derivation of the equations of physics. One is tempted to feel that many of the other reasons advanced for adopting the expressions actually used in physical theories merely cover up the fact that these expressions are the simplest and most symmetrical ones which do not flagrantly contradict known facts. This reflection is also induced frequently on reading Einstein's book, *The Meaning of Relativity*, and it is doubtless capable and worthy of much more explicit development.

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