

UFFE HAAGERUP – HIS LIFE AND MATHEMATICS

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Communicated by F. H. Szafraniec

ABSTRACT. In remembrance of Professor Uffe Valentin Haagerup (1949–2015), as a brilliant mathematician, we review some aspects of his life, and his outstanding mathematical accomplishments.



Figure 1. Uffe Haagerup – 2012

Date: Received: Aug. 10, 2017; Accepted: Sep. 8, 2017.

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2010 *Mathematics Subject Classification.* Primary 01A99; Secondary 01A60, 01A61, 43-03, 46-03, 47-03.

Key words and phrases. Uffe Haagerup, history of mathematics, operator algebras.

1. A BIOGRAPHY OF UFFE HAAGERUP

Uffe Valentin Haagerup was born on 19 December 1949 in Kolding, a mid-size city in the South-West of Denmark, but grew up in Faaborg (near Odense). Since his early age he was interested in mathematics. At the age of 10, Uffe started to help a local surveyor in his work of measuring land. Soon the work also involved mathematical calculations with sine and cosine, long before he studied these at school.

At age 14, Uffe got the opportunity to develop a plan for a new summer house area close to Faaborg. Due to Uffe's young age, this was recognized by both local and nationwide media. A plan had previously been made by a Copenhagen-based engineering company, but their plan was flawed and eventually had to be discarded.



Figure 2. A young Uffe Haagerup

Throughout his childhood, Uffe developed a strong interest in mathematics, and his skills were several years ahead of those of his peers. During primary school, he borrowed text books from his four-year older brother, and thus he came to know mathematics at high school level. This continued throughout high school, where his knowledge about mathematics was supplemented by university text books.

He graduated from high school at Svendborg Gymnasium in 1968. In the same year, he entered the University of Copenhagen to study mathematics and physics. He was fascinated by the physical theories of the 20th century, including Einstein's theory of relativity and quantum mechanics. His love for the exact language of mathematics led him to mathematical analysis and in particular the field of operator algebras, which originally aimed at providing a mathematically exact formulation of quantum mechanics.

Uffe got his international breakthrough already as a student at the University of Copenhagen, as he developed an exciting new view on a mathematical theory developed only a few years before by two Japanese mathematicians Tomita and Takesaki. From then on, Uffe's name was acknowledged throughout the international community of operator algebraists and beyond.

Uffe received his cand. scient (masters) degree in 1973 from the University of Copenhagen. By the time of his graduation, job opportunities at Danish universities were very limited. Initially Uffe taught a semester at a high school in Copenhagen, but fortunately his talent was recognized by the mathematics department of the newly founded University of Odense (renamed to University of Southern Denmark in 1998), where he was employed from 1974 until his death.

From 1974 to 1977 he served as Adjunkt (Assistant Professor) at the University of Odense. During 1977–79 he had a research fellow position at the same university, which enabled him to devote all of his working hours to scientific work instead of teaching. From 1979 to 1981 he served as Lektor (Associate Professor) and in 1981 the University of Odense promoted him to full Professor at the age of 31, making him the youngest full professor in Denmark.

In 2010–2014, he was on leave from his position at the University of Southern Denmark, to work as a professor at the University of Copenhagen while he held an ERC Advanced grant. In 2015 he returned to his position in Odense. He supervised the following 14 Ph.D. students:

Marianne Terp (1981), John Kehlet Schou (1991), Steen Thorbjørnsen (1998), Flemming Larsen (1999), Jacob v. B. Hjelmberg (2000, co-advisor Mikael Rørdam), Lars Aagaard (2004), Agata Przybyszewska (2006), Hanne Schultz (2006), Troels Steenstrup (2009), Søren Møller (2013), Tim de Laat (2013, co-advisor Magdalena Elena Musat), Søren Knudby (2014), Kang Li (2015, co-advisor Ryszard Nest), Kristian Knudsen Olesen (2016, co-advisor Magdalena Elena Musat).

His research area mainly falls within operator theory, operator algebras, random matrices, free probability and applications to mathematical physics. Several mathematical concepts and structures carry his name:

THE HAAGERUP PROPERTY (a second countable locally compact group G is said to have the Haagerup (approximation) property if there is a sequence of normalized continuous positive-definite functions φ which vanish at infinity on G and converge to 1 uniformly on compact subsets of G ; see [11]), THE HAAGERUP

SUBFACTOR and THE ASAEDA–HAAGERUP SUBFACTOR (Exotic subfactors of finite depth with Jones indices $(5 + \sqrt{13})/2$ and $(5 + \sqrt{17})/2$; see [1]), and THE HAAGERUP LIST (a list of the only pairs of graphs as candidates for (dual) principal graphs of irreducible subfactors with small index above 4 and less than $3 + \sqrt{3}$; cf. [13]); see also [7].

He spent sabbatical leaves at the Mittag–Leffler Institute in Stockholm, the University of Pennsylvania, the Field Institute for Research in Mathematical Sciences in Toronto and the Mathematical Science Research Institute at Berkeley.

He served as editor-in-chief for *Acta Mathematica* from 2000 to 2006. He was one of the editors of the Proceedings of the sixth international conference on Probability in Banach spaces, Sandbjerg, Denmark, June 16-21, 1986 published by Birkhäuser in 1990.

He was a member of the “Royal Danish Academy of Sciences and Letters” and the “Norwegian Academy of Sciences and Letters” and received the following prestigious awards, prizes and honors ([34, 29]):

- The Samuel Friedman Award (UCLA and Copenhagen - 1985) for his solution to the so-called “Champagne Problem” posed by Alain Connes.



Figure 3. Uffe receives the Samuel Friedman Award – 1985

- Invited speaker at ICM1986 (Berkeley - 1986).
- The Danish Ole Rømer Prize (Copenhagen - 1989).
- A plenary speaker at ICM2002 (Beijing - 2002).

- Distinguished lecturer at the Fields Institute of Mathematical Research (Toronto - 2007).
- The German Humboldt Research Award (Münster - 2008).
- The European Research Council Advanced Grant (2010–2014).
- A plenary speaker at the International Congress on Mathematical Physics ICMP12 (Aalborg - 2012).
- The 14th European Latsis Prize from the European Science Foundation (Brussels - 2012) for his ground-breaking and important contributions to operator algebra.
- A Honorary Doctorate from East China Normal University (Shanghai - 2013).

He also attended numerous conferences and workshops as an invited speaker, such as the 1986 International Congress of Mathematicians in Berkeley, the 2012 International Congress of Mathematicians in Beijing, the 2012 International Congress on Mathematical Physics in Aalborg, and the Conference on Operator Algebras and Applications in Cheongpung.



Figure 4. Conference in Cheongpung, Korea, 2014

Uffe had two sons, Peter and Søren. He tragically drowned on the 5th of July 2015 while swimming in the sea near Faaborg.



Figure 5. Uffe and his family: (From left) Pia, Peter, Søren, Uffe – 2002

In the next two sections, we present some highlights of Uffe Haagerup’s mathematical career and works.

2. UFFE HAAGERUP’S WORK BEFORE 1990

As mentioned before, Uffe began his studies at the University of Copenhagen in 1968. At first his main interest was mathematical physics, and in particular quantum physics. He got interested in operator algebras via a seminar where papers by the mathematical physicist Irving Segal, who showed how parts of the physical theory could be described by means of operator algebras, were studied.

Let us say a few words on the field operator algebras. This branch of mathematics was initiated in the years around 1930, when one wished to develop a mathematical theory for quantum mechanics, which had been developed a few years earlier by among others Niels Bohr. The theory was then developed by mathematicians, in particular John von Neumann, but there were few active participants in the field until the 1960’s. Then physicists got interested, and the theory of operator algebras became a popular field.

At the time when Uffe started to learn about operator algebras there was a major breakthrough in the subject. The Japanese mathematician Tomita solved one of the main open problems in von Neumann algebras, and Takesaki wrote an issue of the Springer Lecture Notes series, which contains the proof plus further developments. Uffe and a fellow student studied these notes in detail. Then Uffe asked Gert Kjærgaard Pedersen if he could write his master thesis on the subject with Pedersen as thesis advisor. This wish was well received by Pedersen. Uffe wrote his master thesis in the winter 1972-1973, while Pedersen was abroad. It was closely related to Tomita–Takesaki theory and the main results were eventually published in the paper “The standard form of von Neumann algebras” [9],

which appeared in 1975. It is to this day one of his most cited papers and gave him immediately international recognition. His masters thesis also contained another major result, equally published in 1975 [10]: every normal weight on a von Neumann algebra is a supremum of normal states. This solved a problem first formulated by Dixmier.

After this it was unnecessary for Uffe to take a doctoral degree.

In the second half of the 1970's, Uffe produced a number of other important results related to Tomita-Takesaki theory, such as the construction of the L^p -spaces associated to an arbitrary von Neumann algebra, and a sequence of papers on operator valued weights. One can say that throughout his career, he kept a special affection for (and constantly produced new results in) the area of von Neumann algebras. But he also began, very quickly, to contribute to other fields.

At the University of Odense, Uffe was for a long time the only operator algebraist. However, his colleagues in other fields occasionally told him about famous problems which he was then able to solve. An early example of this was triggered by a problem mentioned to him by his colleague in Banach space theory, Niels J. Nielsen. This gave rise to Uffe's 1978 proof of the best constants in Kinchin's inequality, which consists of 50 pages with difficult classical analysis all the way. Another example is the characterization of simplices of maximal volume in hyperbolic n -space, in a joint work with his Odense colleague in algebraic topology, Hans J. Munkholm.

Uffe also contributed to other areas of operator algebra theory, in particular on C^* -algebras, from the late 1970's onwards, and found new applications in immediately adjacent areas. There is a close relationship between von Neumann algebras and groups. Groups are central in mathematics and are algebraic structures where the elements can be multiplied and have inverses. Many constructions of operator algebras involve groups, and the algebras often inherit properties from the underlying group. But the converse is uncommon. Uffe discovered an example of how properties of groups follow from operator algebras. He found an example of a so-called non-nuclear C^* -algebra with the metric approximation property. To do that he started to study a hard analysis problem, and as it often happened when he solved a problem, he introduced new ideas which were fruitful for further research. This time he found a new property of groups, which plays an important role in geometric group theory. The property is now called

the “Haagerup property” or a-T-menable, in Gromov’s terminology, as a strong negation of Kazhdan’s property (T); see [3].

Uffe didn’t forget his background in physics either. A joint work from 1986 with Peter Sigmund, a professor of physics at the University of Odense, shows Uffe’s strong analytic powers at work with Bethe’s model of energy loss of charged particles as they penetrate matter [26]. In the fall of 2010, a semester on quantum information theory was held at Institute Mittag-Leffler near Stockholm. There, Musat came to give a lecture on some joint work with Uffe. We quote from the report which was written on the program the following year:

“One of the highlights was a pair of visits and talks by Musat, who spoke on her work with Uffe on factorizable maps, and its implications for the so-called ‘quantum Birkhoff conjecture’, which they showed was false. The first talk generated so much excitement that questioning went on for more than an hour, with enthusiastic longer discussions for the rest of Musat’s stay.”

$\text{Re } \lambda > 0$

$$\frac{d}{d\lambda} \left(\frac{\lambda - \sqrt{\lambda^2 - 4}}{2} \right) = \frac{1}{2} - \frac{1}{2} \frac{2\lambda}{2\sqrt{\lambda^2 - 4}}$$

$$= \frac{1}{2} \frac{\lambda^2 - 4 - \lambda}{\sqrt{\lambda^2 - 4}}$$

$$\text{So } (4 - \lambda^2) G'_0(\lambda) = -\frac{1}{2} \lambda^2 + 2 + \frac{1}{2} \sqrt{\lambda^2 - 4}$$

When $\lambda H_0(\lambda) = + \frac{\lambda^2 - 1}{2} \sqrt{\lambda^2 - 4}$

$$\text{So } (4 - \lambda^2) G'_0(\lambda) + \lambda H_0(\lambda) = 2 \quad \checkmark$$

We know by ... that $G_u(\lambda)$ has an asymptotic expansion with $H_0(\lambda) = G(\lambda)$

$$G_u(\lambda) \sim H_0(\lambda) + \frac{1}{u^2} H_2(\lambda) + \frac{1}{u^4} H_4(\lambda) + \dots$$

Inserting this in ... we find that

$$\frac{d}{d\lambda} \left(\frac{1}{u^2} H_0'(\lambda) + \frac{1}{u^4} H_2'(\lambda) + \dots \right) + (4 - \lambda^2) \left(H_0'(\lambda) + \frac{1}{u^2} H_2'(\lambda) + \dots \right) + \lambda H_0'(\lambda) = 2$$

$$(4 - \lambda^2) H_k(\lambda) + \lambda H_k(\lambda) = H_{k-1}(\lambda)$$

$$(4 - \lambda^2) H_k(\lambda) + \lambda H_k(\lambda) = H_{k-1}(\lambda)$$

$$u = 1, 2, \dots$$

Beach, at første del de hange e lign
 $(\lambda^2 - 4) p'(\lambda) - \lambda p(\lambda) = 0 \Leftrightarrow p(\lambda) = c \cdot \sqrt{\lambda^2 - 4}$
 $\lambda \in [-2, 2]$

Figure 6. A handwritten note by Uffe

Those of us who have had the pleasure of writing joint papers with Uffe will recognize the following pattern: we had struggled with a problem without success. Then we got into contact with Uffe and told him about the difficulties, where upon he sat down and solved them.

Erling Størmer recalls one example from a conference in Romania in 1983:

I was going to give a lecture about a formula for the diameter of a set constructed from the states on a von Neumann algebra. But when I came to the conference I discovered that there were two possible formulas for the diameter, and I was unable to show which one is the correct one. Fortunately Uffe was there, so I asked him the first day we were there. “It must be that one”, Uffe said and pointed at one of them. In the evening he sat down at his desk, and the next morning he gave me a 6 page proof showing that the formula he had pointed at, was the right one. Then I could give my lecture with a good feeling.

A couple of years later, in 1985, Størmer and Haagerup shared an apartment in Berkeley in California, for a month. They followed up their work with the diameter formula and ended up with an 80 pages long paper. In addition to learning much mathematics from this collaboration, Størmer learned one more thing, namely patience. He wrote a draft, which he sent to Uffe early in the fall of 1987. But Uffe got ill that fall, so he was delayed in the work of finishing the manuscript, but it took a long time for other reasons too, because Uffe was a very patient mathematician, who could keep a manuscript in his drawer for a long time before he had them typed and published. Some he never published but sent copies of them to his colleagues. So it was far into the winter before he gave the final manuscript to the secretary who was going to type it for them. But that also took a long time, so it was only sent to a journal late in the following summer, and then it took at least another two years before it was finally published.

Uffe spent the academic years 1982-83 in the USA, first at UCLA, then at the University of Pennsylvania, both places simultaneously with the 3 years younger Vaughan Jones. At that time Jones showed some very important results on von Neumann algebras. They were about factors, which in a way are the building blocks in the theory, and have the property in common with the $n \times n$ matrices that their center, i.e. the operators in the algebra which commutes with all the others, consists only of the scalar multiples of the identity operator. For an inclusion $A \supseteq B$ of factors, Jones introduced an index, which in a way measures the difference of the sizes of the two factors. For some factors he found a formula for the index, which turned out to be very important for the theory of knots. This was a sensation, as it was an application of the infinite dimensional theory of von Neumann algebras to the finite dimensional knot theory. At the world congress in mathematics in 1990 Jones was rewarded the Field’s Medal, which is

the most prestigious award a mathematician younger than 40 years can get. We return to some of Uffe's contributions to subfactor theory in the next section.

We have now arrived at Uffe's most famous result. He himself also considered this to be his best result ever. When we indicated what von Neumann algebras are, we started with the $n \times n$ matrices. Consider an infinite long increasing sequence of matrix algebras, where each matrix algebra contains the previous ones. From this infinite sequence one can generate many different von Neumann algebras, and in particular factors by use of states. They are called hyperfinite or injective factors, and have been central in the theory since Murray and von Neumann started the development of the theory in the 1930's. Factors are divided into classes of types I, II and III, and each of these has several subclasses. In this connection type III is most important, and this class is further divided into the types III_λ , where λ moves through the interval from 0 to 1. These are the most "infinite" von Neumann algebras, and were considered almost untractable before Connes' groundbreaking work, based on Tomita-Takesaki theory.

Alain Connes received the Field's Medal in 1982 for his seminal work on von Neumann algebras, and especially for his classification of hyperfinite factors, published in 1976. In particular, he obtained a complete classification of hyperfinite factors of type III_λ , where $0 \leq \lambda < 1$. But there was one problem he did not succeed to solve, namely whether there is one or more hyperfinite type III_1 -factors. Uffe visited Connes in 1978 at his country house in Normandie, where they discussed the problem. Hjelmberg, while preparing an interview with Uffe in 2002, got the following description from Connes on these discussions:

"We had long and intense discussions in my country house ending up when both of us got a terrible migraine [22]".



Figure 7. Uffe Haagerup (left) and Alain Connes (right), about 40 years ago

Uffe thought much about the problem later on, but didn't get the opportunity to work seriously on it before the years 1982-83. Based on Connes' work, Uffe finally solved the problem in the fall of 1984, by showing that there is only one hyperfinite type III₁-factor. The proof was published in Acta Mathematica in 1987 and was over 50 pages long (see [12]). It demonstrated convincingly how exceptionally good Uffe was in analysis. The problem was known as the "Champagne Problem", as Connes had promised a fine bottle of Champagne to the person who could solve it. Uffe received the announced Champagne from Connes for the result, as well as the Samuel Friedman Award in 1985. In an obituary written shortly after Uffe's passing [6], Connes expressed his admiration as follows:

Uffe Haagerup was a wonderful man, with a perfect kindness and openness of mind, and a mathematician of incredible power and insight. His whole career is a succession of amazing achievements and of decisive and extremely influential contributions to the field of operator algebras, C*-algebras and von Neumann algebras. (...) From a certain perspective, an analyst is characterized by the ability of having "direct access to the infinite" and Uffe Haagerup possessed that quality to perfection. His disappearance is a great loss for all of us.

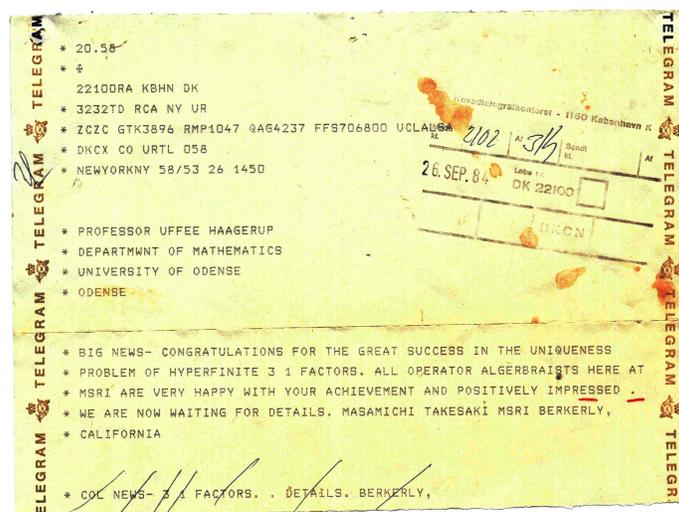


Figure 8. Congratulation telegram from Masamichi Takesaki, for the solution of the Champagne Problem



Figure 9. Uffe and his wife Pia in 1985

3. UFFE HAAGERUP'S WORK AFTER 1990

As appears from the preceding section, most of Uffe's research was centered around the theory of von Neumann's algebras, which he mastered to the highest international level, and in particular he became known for using von Neumann algebra techniques in order to solve C^* -algebra problems and more generally for using methods from analysis to prove results that had been established previously by other methods. A couple of examples of this are the following:

- (a) In the paper “Random Matrices with Complex Gaussian Entries” (ref. [20]) new proofs were given for the limiting behavior of the empirical spectral distribution and the smallest and largest eigenvalues of certain Gaussian random matrix ensembles. In particular these results include the celebrated semi-circle law of Wigner (see [28]). Where previous proofs of the mentioned results involved a substantial amount of combinatorial work, Uffe took the point of view of studying the “moment generating function” $s \mapsto \mathbb{E}[\text{Tr}(\exp(sA))]$, where A is the random matrix under consideration, \mathbb{E} denotes expectation and Tr denotes the trace. Expanding this function as a power series, Uffe and his co-author could identify it explicitly in terms of certain hypergeometric functions. This approach resembles methods from analytic number theory, which Uffe was actually quite interested in and taught several courses on.
- (b) Another example is the paper “On Voiculescu's R - and S -transforms for free non-commuting random variables” [14] in which (among other results)

Uffe provided a new and completely analytical proof of the additivity (with respect to free convolution) of Voiculescu’s R -transform (see [27]). Voiculescu’s original proof was based on the Helton–Howe formula from representation theory, and other proofs (e.g. by Nica and Speicher; see [24]) are based on the development of some rather heavy combinatorial machinery. Uffe’s proof is based on Banach-algebra techniques, which he used e.g. to express the R -transform explicitly as an analytic function in a neighborhood of zero. Voiculescu recently used Uffe’s approach to establish a key formula for the analog of the R -transform in Voiculescu’s recent theory of bi-free probability. As it happens, neither Voiculescu’s original approach, nor the combinatorial approach work in the bi-free setting. In his talk at the celebration of Uffe’s 60’th birthday, Voiculescu gave the following general characterization of Uffe’s papers (quoted freely from memory): “Everything is very clear and looks very easy. Then suddenly a ‘miracle’ occurs, from which everything falls out and is again clear and easy”.

In the rest of this section we outline a few highlights from the second half of Uffe’s career. They are listed in chronological order and should be viewed mainly as examples of his impressive achievements. Many other of Uffe’s results from the last 25 years equally deserve to be highlighted, but time and space limitations prevents a thorough encyclopedic approach.

- Uffe was inspired by the time he spent with Jones in 1983 (cf. previous section) and started to work on subfactor theory, as it is called. He eventually ended up by solving a central problem in the theory, which Jones had left open, namely to find a finite depth, irreducible subfactor of the hyperfinite factor of type II_1 with index strictly between 4 and $3 + \sqrt{2}$. Haagerup proved that no such subfactor can have index smaller than $(5 + \sqrt{13})/2$ ([13]), and subsequently, with Asaeda, ([1]), he proved the existence and uniqueness of a (finite depth, irreducible) subfactor of precisely this index. This subfactor, called the “Asaeda–Haagerup subfactor”, has a very complicated construction and cannot be constructed by standard methods. Although this result is less spectacular than the uniqueness of the hyperfinite type III_1 -factor, it is certainly a second major problem left open by a Field-medalist and solved by Uffe.

- From around the late 1990's Uffe (and collaborators) made important contributions to Voiculescu's free probability theory. In ref. [21] he proved (jointly with Thorbjørnsen) that the operator norm of a non-commutative polynomial in several independent GUE-random matrices converges almost surely, as the dimension goes to infinity, to the limit anticipated by free probability theory. This further led to the settlement (in the positive) of the conjecture on the existence of non-invertible elements in the extension semi-groups of the reduced C^* -algebras associated to the free groups. Jointly with Schultz, he also made huge progress on the invariant subspace problem. Specifically they proved in [19] that any operator T in a II_1 -factor has a non-trivial invariant subspace affiliated with the von Neumann algebra generated by T , provided that the Brown measure of T is non-trivial.
- In 2008 Uffe and Musat solved in [18] a long standing conjecture by Effros–Ruan and Blecher by establishing the following Grothendieck type inequality: For any C^* -algebras A and B and any jointly completely bounded bilinear form $u: A \times B \rightarrow \mathbb{C}$ there exist states f_1, f_2 on A and g_1, g_2 on B , such that

$$|u(a, b)| \leq \|u\|_{\text{jcb}} (f_1(aa^*)^{1/2} g_1(b^*b)^{1/2} + f_2(a^*a)^{1/2} g_2(bb^*)^{1/2}),$$

for any a in A and b in B . The *jointly completely bounded norm* $\|u\|_{\text{jcb}}$ may be defined as the completely bounded norm of the mapping $A \rightarrow B^*$ associated to u . The work of Uffe and Musat extended previous work by Pisier and Shlyakhtenko (see [25]).

- In a series of two papers ([16],[17]) Uffe and de Laat showed recently that all connected, simple Lie groups with real rank greater than or equal to 2 do not have the Approximation Property (AP) (see e.g. [16] for the definition of this property). Since connected, simple Lie groups with real rank 0 (resp. 1) are known to be amenable (resp. weakly amenable), and since amenability implies weak amenability, which again implies (AP), Uffe and de Laat's result shows that connected simple Lie groups have (AP), if and only if their real rank is at most 1. Specifically Uffe and de Laat proved that the symplectic group $\text{Sp}(2, \mathbb{R})$ and its universal covering group $\widetilde{\text{Sp}}(2, \mathbb{R})$ do not have the (AP). A few years before it had been established by Lafforgue and de la Salle that $\text{SL}(3, \mathbb{R})$ does not have the

approximation property (see [23]). Furthermore it is well-known that any connected simple Lie Group with real rank greater than or equal to 2 has a closed connected subgroup, which is *locally* isomorphic to either $\mathrm{Sp}(2, \mathbb{R})$ or $\mathrm{SL}(3, \mathbb{R})$, and hence isomorphic to a quotient of one of the universal covering groups $\widetilde{\mathrm{Sp}}(2, \mathbb{R})$ or $\widetilde{\mathrm{SL}}(3, \mathbb{R})$ by a discrete normal central subgroup. Combining the results mentioned above, Uffe and de Laat’s result may then be deduced from the fact that (AP) passes from a group to its closed subgroups.

- In recent years Uffe became interested in the famous problem on the possible amenability of the smallest of the Thompson groups, here denoted by F . In 2015 he published the joint paper ref. [8] with Ramirez–Solano and his youngest son, Søren, in which they give precise lower bounds for the norms of two operators associated to the generators of F . By work of Kesten, the amenability of F is equivalent to the statement that these norms equal 3 and 4, respectively. Extensive computer calculations, performed by Uffe and his co-authors, suggest that the norms are approximately around 2.95 and 3.87, respectively, but their upper bounds are not precise enough to establish non-amenability. In the paper [15], Uffe and Knudsen Olesen established that if the reduced C^* -algebra of the larger Thompson group, T , is simple, then F is non-amenable. Very recently Le Boudec and Matte Bon proved that non-amenability of F is in fact equivalent to simplicity of $C_r^*(T)$ (see [2]).

4. UFFE HAAGERUP AS TEACHER AND SUPERVISOR

Many of the numerous students who were taught by Uffe over the years at the University of Southern Denmark, mainly saw him as someone who was able to write incredibly fast (while still producing readable text) on a blackboard. Little did they realize that they were enjoying the privilege of being lectured to by one of the greatest and most influential Danish mathematicians of all times. Their ignorance is (partly) excused by Uffe’s general attitude and appearance, to which the word “modest” immediately springs to the mind of anyone who have met him. Of course the students who took more advanced courses with Uffe, and in particular those who wrote their masters or Ph.D-thesis under his supervision, eventually realized that there was full concordance between the pace of his handwriting and that of his mathematical mind. One of Uffe’s students (Carl

Winsløw) at the University of Southern Denmark remembers Uffe's marvelous teaching and supervision as follows:

My first memories of Uffe Haagerup date back to a linear algebra course in the late 1980's, at the University of Odense. The lectures were astonishing, superior to all other I have attended. While his teaching was spontaneous (no manuscript) and very lively, leaving the audience in no doubt on the rationale for the current details, he filled the blackboards with crystal clear proofs and simple examples – always more elegant and illuminating than those in the textbook we had. He repeated the same act in later courses I had the chance to take with him, on functional analysis, von Neumann algebras and so on.

Later, at weekly meetings with him as my master thesis supervisor, the blackboard was replaced with his favorite working instrument: blank sheets of paper and a classical pencil, which was frequently sharpened while the sheets were filled, and the sheets were eventually stapled when some proof was done. My thesis was to be an exposition of the details of Connes' 1973 paper [4]. Of course Uffe knew this monumental work intimately; in fact one of his most famous achievements was to complete the classification in question by proving the uniqueness of the injective type III_1 factor, in 1984. At the supervision meetings, the following often happened: I had struggled with some elegant but very short proof from Connes' paper, and asked Uffe about it. He would take a look at the French text, mainly to get the result to be proved, then provide an elaborate and crystal clear proof on white sheets, out of his head, which I suspect was quite independent from the explanation in the paper. It also happened, sometimes, that I brought up other questions for which I could not find an answer in the literature. Usually, he would go: "Yes, I once thought about that", take a stack of white paper, and begin writing a sequence of lemmas and so on - often quite technical with subtle inequalities that were stated without hesitation and then proved quickly, with occasional corrections done by simply barring a line or two (I don't recall him having an eraser). On seldom, happy occasions, he would reach for one of his endless folders of stapled, handwritten manuscripts, which filled the shelves in his office - but even then, he usually ended up writing a new one from scratch.

This little anecdote is communicated here because we think any of his students (graduate or undergraduate) would recognize the point: Uffe incarnated mathematical creativity in a way that is shared by few (if any) they have met. For him there was a perfect continuity between "teaching" and "research" - it was about producing and sharing mathematical ideas. Even in his lectures on linear algebra (where, of course, no results were new) one got an experience how reasoning and connections are built "in vivo".

In the literature on the modes and effects of mathematics teaching, the activities in which mathematicians build new knowledge have sometimes been used as

an ideal model for the activity of the student; the teacher, then, should arrange situations in which the student could learn by solving and posing problems. Uffe certainly practiced this art in many ways. But his acts of “direct teaching” (allured to above) were also very far from the caricature image that is sometimes presented as the “opposite” of that ideal: lectures which leave the students completely passive. Indeed, many lectures fail to help students to go beyond the role of spectator. But, as his students would say, Uffe’s did not.

In the first decades of his academic career, Uffe only took on a single Ph.D-student: Marianne Terp. From around the mid 1990’s he changed his policies on this matter, partly influenced by general tendencies at the Danish Universities, and until his death he acted as supervisor on at least another 13 Ph.D-theses. He never obtained a Ph.D-degree himself; a fact that was used as a friendly (and absurd) tease among students and colleagues. In 2013 he could, however, put an end to the teasing, as he was awarded an honorary doctoral degree from East China Normal University.



Figure 10. Uffe Haagerup and ECNU President Qun Chen at the award ceremony – 2013

In the minds of all his students and post docs, Uffe will always stand out as a true master of mathematical thinking and a great source of inspiration. Collaborating with Uffe was an immense privilege, and his modest and kind personality neutralized the feeling of mathematical inferiority one could easily get stung by in his presence. Arrogance was simply not a part of his character. A very precise description of Uffe as teacher and supervisor can be expressed with the Japanese term, *sensei*. It can be used to translate a variety of English terms: teacher, master, professor, expert, senior. Literally, it means “the one who proceeds” (or walks ahead of) you.

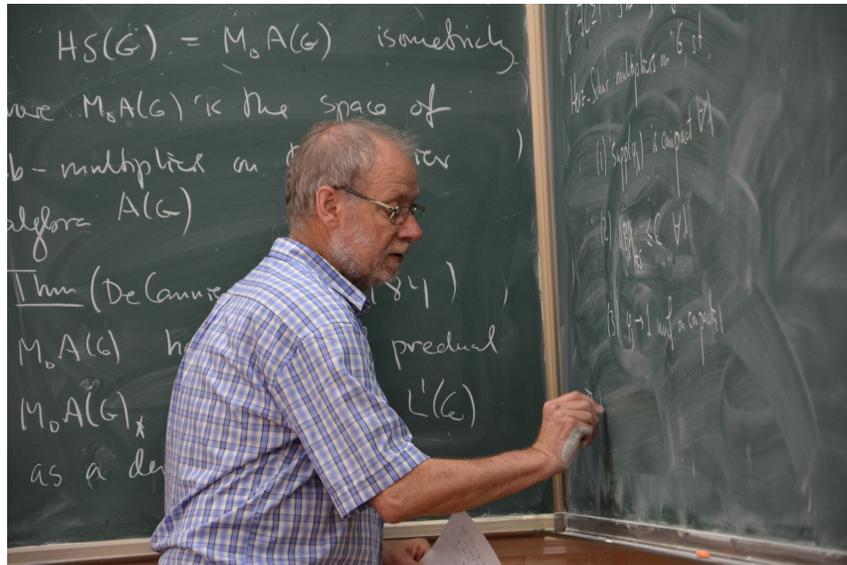


Figure 11. Uffe Haagerup in action

Uffe was a sensei in all the meanings of the word. A sensei badly missed, but whose memory is gladly honored.

5. BIBLIOMETRICS

Utilizing MathSciNet (MR)[30], Zentralblatt MATH (Zbl) [31], Scopus [33] and Web of Science (WOS) [32], we present some quantitative analysis of Uffe's publications until.

The first three most cited publications of Uffe in MR are:

- U. Haagerup, An example of a nonnuclear C^* -algebra, which has the metric approximation property. *Invent. Math.* 50 (1978/79), no. 3, 279-293. (206 citations)
- Michael Cowling and U. Haagerup, Completely bounded multipliers of the Fourier algebra of a simple Lie group of real rank one. *Invent. Math.* 96 (1989), no. 3, 507-549. (114 citations)
- Jean De Cannière and U. Haagerup, Multipliers of the Fourier algebras of some simple Lie groups and their discrete subgroups. *Amer. J. Math.* 107 (1985), no. 2, 455-500. (111 citations)

The first three most cited publications of Uffe in ZbMath are:

- U. Haagerup, An example of a non nuclear C^* -algebra, which has the metric approximation property, *Invent. Math.* 50, 279-293 (1979). Zbl 0408.46046 (138 citations)
- U. Haagerup, The standard form of von Neumann algebras, *Math. Scandinav.* 37(1975), 271-283 (1976). Zbl 0304.46044 (102 citations)
- Michael Cowling and U. Haagerup, Completely bounded multipliers of the Fourier algebra of a simple Lie group of real rank one, *Invent. Math.* 96, No.3, 507-549 (1989). Zbl 0681.43012 (81 citations)

The first three most cited publications of Uffe in WOS are:

- U. Haagerup, An example of a nonnuclear C^* -algebra, which has the metric approximation property. *Invent. Math.* 50 (1978/79), no. 3, 279-293. (297 citations)
- U. Haagerup, The standard form of von Neumann algebras, *Math. Scandinav.* 37(1975), 271-283. (232 citations)
- Michael Cowling and U. Haagerup, Completely bounded multipliers of the Fourier algebra of a simple Lie group of real rank one. *Invent. Math.* 96 (1989), no. 3, 507-549. (143 citations)

The first three most cited publications of Uffe in Scopus are:

- U. Haagerup, An example of a nonnuclear C^* -algebra, which has the metric approximation property. *Invent. Math.* 50 (1978/79), no. 3, 279-293. (268 citations)
- Michael Cowling and U. Haagerup, Completely bounded multipliers of the Fourier algebra of a simple Lie group of real rank one. *Invent. Math.* 96 (1989), no. 3, 507-549. (136 citations)

- U. Haagerup, All nuclear C^* -algebras are amenable. *Invent. Math.* 74 (1983), no. 2, 305–319. (118 citations)

The number of Uffe's publications recorded in MR and Zbl are 106 and 109, respectively. According to MR, they are cited 2407 times by 1250 authors. Functional analysis is the subject where Uffe has published most of his articles and where there are most citations to Uffe's works.

According to Zbl, the first three journals with most of Uffe's publications are *Journal of Functional Analysis* (13 papers), *Duke Mathematical Journal* (6 papers) and *Mathematica Scandinavica* (6 papers). He has had 52 collaborators; among them E. Størmer, S. Thorbjørnsen and K. J. Dykema with 9, 7 and 5 papers, respectively, have most joint papers with him.

Web of Science records 86 publications by Uffe. The sum of the times his papers are cited is 2775 and without self-citations is 2631. The average citation per publication is 32.27, and Uffe's h-index is 28.

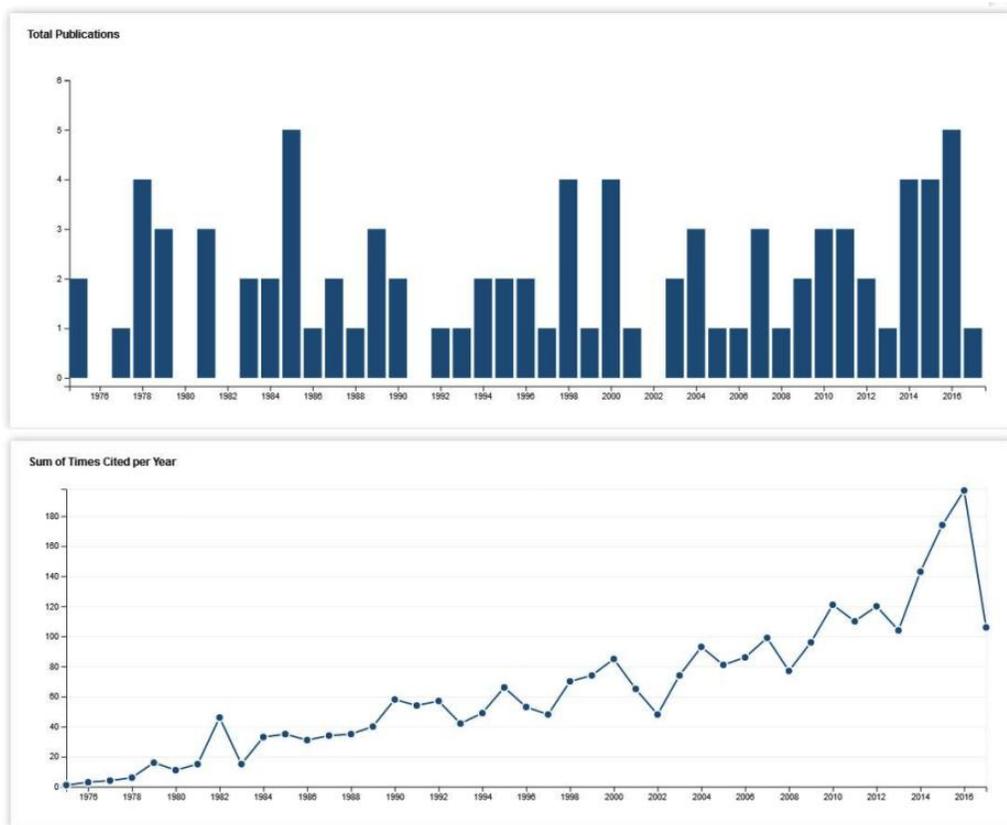


Figure 12. Bibliometrics - Ref. Web of Science - Sep 8, 2017

Scopus presents 69 document for Uffe Haagerup. Records show 1877 total citations by 1431 documents for him. His scopus h-index is 22.

His first paper appearing in MathSciNet is

- Haagerup, Uffe Normal weights on W^* -algebras. *J. Functional Analysis* 19 (1975), 302–317.

and his last sole author paper is

- Haagerup, Uffe On the uniqueness of the injective III_1 factor. *Doc. Math.* 21 (2016), 1193–1226.

which is typed by Hiroshi Ando and completed (due to some missed pages of the original handwritten note) by Cyril Houdayer and Reiji Tomatsu after Uffe passed away.

6. PUBLICATIONS BY UFFE HAAGERUP

His papers listed in MathSciNet are as follows:

- Haagerup, Uffe; Olesen, Kristian Knudsen Non-inner amenability of the Thompson groups T and V . *J. Funct. Anal.* 272 (2017), no. 11, 4838–4852.
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Acknowledgements. The authors would like to sincerely thank the sons of Uffe, Peter and Søren, for providing us several memorable photos of Uffe and for their valuable suggestions improving the biography of Uffe. The authors also wish to express their gratitude to Professor Alain Connes for supplying the photograph shown in Figure 7.

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