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ON MINIMAL FINITE QUOTIENTS OF MAPPING CLASS GROUPS

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ABSTRACT. We prove that the minimal nontrivial finite quotient group of the mapping class group $\mathcal{M}_g$ of a closed orientable surface of genus g is the symplectic group $\mathrm{PSp}_{2g}(\mathbf{Z}_2)$, for $g = 3$ and 4 (this might remain true, however, for arbitrary genus $g > 2$). We also discuss some results for arbitrary genus g.

1. Introduction. It is an interesting, but in general difficult, problem to classify the finite quotients (factor groups) of certain geometrically interesting infinite groups. This becomes particularly significant if the group in question is perfect (has trivial abelianization) since in this case each finite quotient projects onto a minimal quotient which is a nonabelian finite simple group, and there is the well-known list of finite simple groups (always understood to be nonabelian in the sequel).

As an example, the finite quotients of the smallest volume Fuchsian triangle group of type (2,3,7) (two generators of orders two and three whose product has order seven) are the so-called Hurwitz groups, the groups of orientation-preserving diffeomorphisms of maximal possible order $84(g-1)$ of closed orientable surfaces of genus g. There is a rich literature on the classification of Hurwitz groups, and in particular of simple Hurwitz groups; the smallest Hurwitz group is the projective linear or linear fractional group $\mathrm{PSL}_2(7)$ of order 168, acting on Klein's quartic of genus three.

One of the most interesting groups in topology is the mapping class group $\mathcal{M}_g$ of a closed orientable surface $\mathcal{F}_g$ of genus g which is the group of orientation-preserving homeomorphisms of $\mathcal{F}_g$ modulo the subgroup of homeomorphisms isotopic to the identity; alternatively, it is the orientation-preserving subgroup of index two of the outer automorphism group $\mathrm{Out}\,(\pi_1(\mathcal{F}_g))$ of the fundamental group. It is well known that $\mathcal{M}_g$ is a perfect group, for $g \geq 3$ [**13**]. By abelianizing the

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fundamental group $\pi_1(\mathcal{F})$ and reducing coefficients modulo a positive integer k, we get canonical projections

$$\mathcal{M}_g \longrightarrow \mathrm{Sp}_{2g}(\mathbf{Z}) \longrightarrow \mathrm{Sp}_{2g}(\mathbf{Z}_k) \longrightarrow \mathrm{PSp}_{2g}(\mathbf{Z}_k)$$

of the mapping class group $\mathcal{M}_g$ onto the symplectic group $\mathrm{Sp}_{2g}(\mathbf{Z})$ and the finite projective symplectic groups $\mathrm{PSp}_{2g}(\mathbf{Z}_k)$ (see [**11**]); we note that, for p prime and $g \geq 2$, $\mathrm{PSp}_{2g}(\mathbf{Z}_p)$ is a simple group, with the only exception of $\mathrm{PSp}_4(\mathbf{Z}_2)$ isomorphic to the symmetric group Σ_6. The kernel of the surjection $\mathcal{M}_g \to \mathrm{Sp}_{2g}(\mathbf{Z})$ is the Torelli group $\mathcal{T}_g$ of all mapping classes which act trivially on the first homology of the surface.

It is well known that the symplectic groups $\mathrm{Sp}_{2g}(\mathbf{Z})$ and the linear groups $\mathrm{SL}_n(\mathbf{Z})$ are perfect, for $g \geq 3$, respectively $n \geq 3$. As a consequence of the congruence subgroup property for these groups, the following holds (p denotes a prime number):

Theorem 1. i) *For $n \geq 3$, the finite simple quotients of the linear group $\mathrm{SL}_n(\mathbf{Z})$ are the linear groups $\mathrm{PSL}_n(\mathbf{Z}_p)$.*

ii) *For $g \geq 3$, the finite simple quotients of the symplectic group $\mathrm{Sp}_{2g}(\mathbf{Z})$ are the symplectic groups $\mathrm{PSp}_{2g}(\mathbf{Z}_p)$.*

Theorem 1 will be proved in Section 4. For the case of mapping class groups, the following is the main result of the present paper.

Theorem 2. *For $g = 3$ and 4, the minimal nontrivial finite quotient group of the mapping class group $\mathcal{M}_g$ of genus g is the symplectic group $\mathrm{PSp}_{2g}(\mathbf{Z}_2) = \mathrm{Sp}_{2g}(\mathbf{Z}_2)$.*

Theorem 2 raises more questions than it answers, for instance:

- which are the finite simple quotients of $\mathcal{M}_g$?
- what is the minimal index of any subgroup of $\mathcal{M}_g$?

Nevertheless, the proof of the theorem appears nontrivial and interesting: considering for $g = 3$ and 4 the list of finite simple groups of order less than that of $\mathrm{PSp}_{2g}(\mathbf{Z}_2)$, we are able to exclude all of them by considering certain finite subgroups of $\mathcal{M}_g$ which have to inject.

For the construction of finite quotient groups of mapping class groups, see also [**15, 16**]: most of these groups are again closely related to the symplectic groups $\mathrm{PSp}_{2g}(\mathbf{Z}_k)$.

In Section 3, we prove Theorem 2 for $g = 3$. In Section 4, we discuss some results for arbitrary genus g and then deduce Theorem 2 for $g = 4$; we also prove the following Theorem.

Theorem 3. *For $g \geq 3$, let $\phi : \mathcal{M}_g \to G$ be a surjection of $\mathcal{M}_g$ onto a finite simple group G. Then either G is isomorphic to a symplectic group $\mathrm{PSp}_{2g}(\mathbf{Z}_p)$, or G has an element of order $4g + 2$.*

Remarks. The case of genus two is special. The abelianization of the (hyperelliptic) mapping class group $\mathcal{M}_2$ is a cyclic group of order ten ([**13**]); in particular, $\mathcal{M}_2$ is not a perfect group and has abelian quotients of orders two, five and ten. However, the methods of the present paper may also be applied to the case $g = 2$; we note that the symplectic group $\mathrm{PSp}_4(\mathbf{Z}_2)$ is isomorphic to the symmetric group Σ_6 and hence not simple. See [**16**] for the subgroups of $\mathcal{M}_2$ of low index, and [**9**] for comments on the congruence subgroup property for the mapping class group of genus two and in general.

Finally we note that there is a classical link of mapping class groups of low genus and the corresponding symplectic groups $\mathrm{PSp}_{2g}(\mathbf{Z}_2)$ to classical algebraic geometry (dating back as far as to Jordan); see, for example, the books [**4, 7**].

2. Proof of Theorem 2 for $g = 3$. Let G be a finite group of orientation-preserving diffeomorphisms of a closed surface $\mathcal{F}_g$ of genus $g > 1$. Then the quotient $\mathcal{F}_g/G$ is a closed 2-orbifold; its underlying topological space is again a closed surface of some genus $\overline{g}$, and there are finitely many branch points of orders $n_1, \dots, n_k$. We will say that the G-action is of type $(\overline{g}; n_1, \dots, n_k)$.

One can give the surface $\mathcal{F}_g$ a hyperbolic (respectively, complex) structure such that G acts by isometries (respectively, conformal maps) of the Riemann surface, by just uniformizing the quotient orbifold $\mathcal{F}_g/G$ by a Fuchsian group of signature $(\overline{g}; n_1, \dots, n_k)$ (see, e.g., [**17**]). Then this Fuchsian group is obtained as the group of all lifts of elements of

G to the universal covering of $\mathcal{F}_g$ (which is the hyperbolic plane), and there is a surjection of this Fuchsian group onto G whose kernel is the universal covering group of the surface. We will say in the following that the finite G-action is given by a surjection of a Fuchsian group $\Gamma(\overline{g}; n_1, \dots, n_k)$ of type or signature $(\overline{g}; n_1, \dots, n_k)$ onto G.

By [**5**, Theorem V.3.3], every conformal map of a closed Riemann surface of genus $g > 1$ which induces the identity on the first homology is the identity. In particular, every finite group of orientation-preserving diffeomorphisms of a closed surface of genus $g > 1$ injects into the mapping class group $\mathcal{M}_g$ and its quotient, the symplectic group $\mathrm{PSp}_{2n}(\mathbf{Z})$, and we will speak in the following of the finite group of mapping classes G of $\mathcal{F}_g$, of type $(\overline{g}; n_1, \dots, n_k)$, determined by a surjection

$$\Gamma(\overline{g}; n_1, \dots, n_k) \longrightarrow G$$

of a Fuchsian group of type $(\overline{g}; n_1, \dots, n_k)$ onto G.

As an example, the Hurwitz action of the linear fractional group $\mathrm{PSL}_2(7)$ on the surface $\mathcal{F}_3$ of genus three (or Klein's quartic) is determined by a surjection (unique up to conjugation in $\mathrm{PGL}_2(7)$)

$$\Gamma(2, 3, 7) \longrightarrow \mathrm{PSL}_2(7)$$

of the triangle group $\Gamma(0; 2, 3, 7) = \Gamma(2, 3, 7)$ onto the linear fractional group $\mathrm{PSL}_2(7)$, so this defines a subgroup $\mathrm{PSL}_2(7)$ of the mapping class group $\mathcal{M}_3$.

We will consider in the following some finite subgroups of $\mathcal{M}_3$, represented by finite groups of diffeomorphisms of a surface of genus three, or equivalently by surjections from Fuchsian groups. For a convenient list and a classification of the finite groups acting on a surface of genus three, see [**2**].

Up to conjugation, $\mathcal{F}_3$ has three orientation-preserving involutions which are of types $(1; 2, 2, 2, 2) = (1; 2^4)$, $(0; 2^8)$ (a "hyperelliptic involution") and $(2; -)$ (a free involution). For general genus g, the following is proved in [**8**].

Proposition 1 ([**8**]). i) *If $g \geq 3$ is odd, every involution of type $((g-1)/2; 2, 2, 2, 2)$ normally generates $\mathcal{M}_g$.*

ii) *If* $g \geq 4$ *is even, every involution of type* $(g/2; 2, 2)$ *normally generates* $\mathcal{M}_g$.

In particular, the cyclic group $\mathbf{Z}_2$ of order two of $\mathcal{M}_3$ generated by an involution of type $(1; 2^4)$ normally generates $\mathcal{M}_3$ and hence maps nontrivially under any nontrivial homomorphism $\phi : \mathcal{M}_3 \to G$. On the other hand, we note that the mapping class represented by an involution of type $(2^8) = (0; 2^8)$ lies in the kernel of the canonical surjection

$$\mathcal{M}_3 \longrightarrow \mathrm{PSp}_6(\mathbf{Z}) \longrightarrow \mathrm{PSp}_6(\mathbf{Z}_2).$$

We consider now the Hurwitz action of $\mathrm{PSL}_2(7)$ on $\mathcal{F}_3$ defined by a surjection $\pi : \Gamma(2,3,7) \to \mathrm{PSL}_2(7)$ and realizing $\mathrm{PSL}_2(7)$ as a subgroup of $\mathcal{M}_3$. Up to conjugation, $\mathrm{PSL}_2(7)$ contains a unique subgroup $\mathbf{Z}_2$, and the preimage $\pi^{-1}(\mathbf{Z}_2)$ in the triangle group $\Gamma(2,3,7)$ is a Fuchsian group of signature $(1; 2^4)$ (since subgroup $\mathbf{Z}_2$ has index four in its normalizer in $\mathrm{PSL}_2(7)$ which is a dihedral group of order eight). Since $\mathrm{PSL}_2(7)$ is a simple group and an involution of type $(1; 2^4)$ normally generates $\mathcal{M}_3$, we have:

Lemma 1. *Every nontrivial group homomorphism* ϕ *from* $\mathcal{M}_3$ *to a group* G *is injective on the subgroup* $\mathrm{PSL}_2(7)$.

Remark. The preimage $\pi^{-1}(\mathbf{Z}_3)$ of the subgroup of order three of $\mathrm{PSL}_2(7)$ (unique up to conjugation) is a Fuchsian group of type (1;3,3), and $\pi^{-1}(\mathbf{Z}_7)$ is a triangle group of type (7,7,7) (the normalizer of $\mathbf{Z}_3$ is dihedral of order 6, that of $\mathbf{Z}_7$ is the subgroup of $\mathrm{PSL}_2(7)$ represented by all upper triangular matrices which has order 21). Since $\mathrm{PSL}_2(7)$ is simple, the corresponding subgroups $\mathbf{Z}_3$ and $\mathbf{Z}_7$ of $\mathcal{M}_3$ inject under any nontrivial ϕ.

Now, for the proof of Theorem 2, suppose that $\phi : \mathcal{M}_3 \to G$ is a surjection onto a nontrivial finite group G of order less than that of $\mathrm{PSp}_6(\mathbf{Z}_2)$; since $\mathcal{M}_3$ is perfect, we can assume that G is a finite nonabelian simple group. By Lemma 1, G has a subgroup $\mathrm{PSL}_2(7)$. The nonabelian simple groups smaller than $\mathrm{PSp}_6(\mathbf{Z}_2)$ and having a subgroup $\mathrm{PSL}_2(7)$ are the following (in the notation of [**3**, page 239 ff.] to which we refer for the simple groups of small order as well as their subgroups):

$$\mathrm{L}_2(7),\ \mathbf{A}_7,\ \mathrm{U}_3(3),\ \mathbf{A}_8,\ \mathrm{L}_3(4),\ \mathrm{L}_2(49),\ \mathrm{U}_3(5),\ \mathbf{A}_9,\ \mathrm{M}_{22},\ \mathrm{J}_2,$$

where $\mathrm{L}_n(p^r) = \mathrm{PSL}_n(p^r)$ denotes a linear group over the finite field with p^r elements, $\mathrm{U}_n(p) = \mathrm{PSU}_n(p)$ a unitary and $\mathbf{A}_n$ an alternating group; M_{22} is a Mathieu group and J_2 the second Janko or Hall-Janko group.

Since none of these groups has three elements of orders 8, 9 and 12 (see [**3**]), the proof of Theorem 2 follows from the following:

Lemma 2. *There are cyclic subgroups of orders* 8, 9 *and* 12 *of* $\mathcal{M}_3$ *on which every nontrivial homomorphism* ϕ *from* $\mathcal{M}_3$ *to a group* G *is injective.*

Proof. i) A subgroup $\mathbf{Z}_8$ of $\mathcal{M}_3$ is defined by a surjection $\pi : \Gamma(4,8,8) \to \mathbf{Z}_8$. The preimage $\pi^{-1}(\mathbf{Z}_2)$ is a Fuchsian group of type $(1;2^4)$ which hence defines a subgroup $\mathbf{Z}_2$ of $\mathcal{M}_3$ which normally generates $\mathcal{M}_3$. (See also [**14**] for the determination of the signature of a subgroup of a Fuchsian group.)

ii) A subgroup $\mathbf{Z}_9$ of $\mathcal{M}_3$ is defined by a surjection $\pi : \Gamma(3,9,9) \to \mathbf{Z}_9$, and the preimage $\pi^{-1}(\mathbf{Z}_3)$ gives a subgroup $\mathbf{Z}_3$ of $\mathcal{M}_3$ of type (3^5) (we note that, up to conjugation, there are exactly two periodic diffeomorphisms of order three of $\mathcal{F}_3$, of types (3^5) and (1;3,3)). Suppose, by contradiction, that ϕ is trivial on subgroup $\mathbf{Z}_3$.

We consider a subgroup $\mathrm{SL}_2(3)$ of $\mathcal{M}_3$ defined by a surjection $\pi : \Gamma(3,3,6)) \to \mathrm{SL}_2(3)$; the linear group $\mathrm{SL}_2(3)$ of order 24 is isomorphic to the binary tetrahedral group $\mathbf{A}_4^*$ and is a semi-direct product $Q_8 \ltimes \mathbf{Z}_3$. The preimage $\pi^{-1}(\mathbf{Z}_3)$ defines a subgroup $\mathbf{Z}_3$ of $\mathcal{M}_3$ of type (3^5) which, by hypothesis, is mapped trivially by ϕ. Now it follows easily that ϕ has to be trivial on the whole subgroup $Q_8 \ltimes \mathbf{Z}_3$ and, in particular, on the unique cyclic subgroup $\mathbf{Z}_2$ of order two of Q_8 which is of type $(1;2^4)$. Since $\mathbf{Z}_2$ normally generates $\mathcal{M}_3$, the homomophism ϕ is trivial.

iii) A subgroup $\mathbf{Z}_{12}$ of $\mathcal{M}_3$ is defined by a surjection $\pi : \Gamma(3,4,12) \to \mathbf{Z}_{12}$; now $\pi^{-1}(\mathbf{Z}_2)$ is of type $(1;2^4)$, and $\pi^{-1}(\mathbf{Z}_3)$ of type (3^5). Since ϕ is nontrivial, it cannot be trivial on the subgroup $\mathbf{Z}_2$ of $\mathcal{M}_3$ of type

$(1; 2^4)$. On the other hand, if ϕ is trivial on the subgroup $\mathbf{Z}_3$ of type (3^5), then one concludes as in ii) that ϕ is trivial.

This concludes the proof of Lemma 2 and also of the case $g = 3$ of Theorem 2. □

3. Some results for arbitrary genus: proof of Theorem 2 for $g = 4$. The following is the main result of [**12**].

Proposition 2 ([**12**]). *For $g \geq 3$, the index of any proper subgroup of $\mathcal{M}_g$ is larger than $4g + 4$; equivalently, there are no surjections of $\mathcal{M}_g$ onto an alternating group $\mathbf{A}_n$ or onto any transitive subgroup of $\mathbf{A}_n$, if $3 \leq n \leq 4g + 4$.*

It would be interesting to know if there exists any surjection $\phi : \mathcal{M}_g \to \mathbf{A}_n$, for $n > 4g + 4$.

The maximal order of a cyclic subgroup of $\mathcal{M}_g$ is $4g+2$, for any $g > 1$, and such a maximal subgroup $\mathbf{Z}_{4g+2}$ is generated by a diffeomorphism of type $(2, 2g + 1, 4g + 2)$; the subgroup $\mathbf{Z}_2$ of $\mathbf{Z}_{4g+2}$ is generated by a hyperelliptic involution of type $(0; 2^{2g+2})$. The following result is proved in [**6**].

Proposition 3 ([**6**]). *Let $g \geq 3$.*

i) *Let h be an orientation-preserving diffeomorphism of (maximal possible) order $4g+2$ of $\mathcal{F}_g$. If $1 \leq k \leq 2g$, then h^k normally generates $\mathcal{M}_g$.*

ii) *The normal subgroup of $\mathcal{M}_g$ generated by the hyperelliptic involution h^{2g+1} contains the Torelli group $\mathcal{T}_g$ as a subgroup of index two and is equal to the kernel of canonical projection $\mathcal{M}_g \to \mathrm{Sp}_{2g}(\mathbf{Z}) \to \mathrm{PSp}_{2g}(\mathbf{Z}) = \mathrm{Sp}_{2g}(\mathbf{Z})/\{\pm I\}$.*

iii) *Let G be a group without an element of order $g - 1$, g or $2g + 1$. Then any homomorphism $\phi : \mathcal{M}_g \to G$ is trivial.*

Note that i) and ii) of Proposition 3 combined with Theorem 1 imply Theorem 3.

We consider the case $g = 4$ now.

Lemma 3. *Let $\phi : \mathcal{M}_4 \to G$ be a surjection onto a finite simple group G.*

i) *The symmetric group Σ_5 is a subgroup of G.*

ii) *Either G has elements of orders 10, 16 and 18, or G is isomorphic to a symplectic group $\mathrm{PSp}_8(\mathbf{Z}_p)$.*

Proof. The mapping class group $\mathcal{M}_4$ has a subgroup $\mathbf{A}_5$ of type (2,5,5) and a subgroup Σ_5 of type (2,4,5). An involution in a subgroup $\mathbf{A}_5$ of $\mathcal{M}_4$ defines a subgroup $\mathbf{Z}_2$ of type (2;2,2); by Proposition 1, such a subgroup $\mathbf{Z}_2$ normally generates $\mathcal{M}_4$, and hence ϕ injects $\mathbf{A}_5$, Σ_5 and also their subgroups $\mathbf{Z}_5$ which are of type $(0;5^4)$. Now $\mathcal{M}_4$ has a subgroup $\mathbf{Z}_{10}$ of type (5,10,10), and since its subgroups $\mathbf{Z}_5$ and $\mathbf{Z}_2$ are of type $(0;5^4)$ and $(2;2,2)$ and hence inject, $\mathbf{Z}_{10}$ also injects.

Also, $\mathcal{M}_4$ has a subgroup $\mathbf{Z}_{16}$ of type $(2,16,16)$ whose subgroup $\mathbf{Z}_2$ is of hyperelliptic type $(0;2^{10})$, and a maximal cyclic subgroup $\mathbf{Z}_{18}$ of type $(2,9,18)$. Lemma 3 ii) is now a consequence of Proposition 3 and Theorem 1.

Proof of Theorem 2 for $g = 4$. Let $\phi : \mathcal{M}_4 \to G$ be a surjection onto a finite simple group G. Suppose that the order of G is less than the order 47.377.612.800 of $\mathrm{PSp}_8(\mathbf{Z}_2)$; see [**3**, page 239 ff.] for a list of these groups. The alternating groups of such orders are excluded by Proposition 2 since they have subgroups of index ≤ 20. The linear groups $\mathrm{PSL}_2(p^r)$ in dimension two are excluded by Lemma 3 since, with the exceptions of $\mathrm{PSL}_2(5^2)$ and $\mathrm{PSL}_2(5^4)$, they have no subgroups $\mathbf{S}_5 \cong \mathrm{PGL}_2(5)$. All remaining groups in the list can be excluded case by case by considering the possible orders of elements in each of these groups (see [**3**] for the character tables of most of these groups; the group theory package GAP can also be used to create the conjugacy classes and the orders of the elements of these group). It is easy to see then that none of these groups simultaneously has elements of orders 10, 16 and 18 (in some cases it may also be helpful to consider a Sylow 2-subgroup of G). Applying Lemma 3 again now completes the proof that the smallest (simple) quotient group of $\mathcal{M}_4$ is indeed the symplectic group $\mathrm{PSp}_8(\mathbf{Z}_2)$.

4. Proof of Theorem 1. i) Let $\phi : \mathrm{SL}_n(\mathbf{Z}) \to G$ be a surjection onto a finite simple group G. By the congruence subgroup property for linear groups in dimensions $n > 2$ (which holds also for symplectic groups, see [**1, 10**]), the kernel of ϕ contains a congruence subgroup, i.e., the kernel of a canonical projection $\mathrm{SL}_n(\mathbf{Z}) \to \mathrm{SL}_n(\mathbf{Z}_k)$, for some positive integer k, and hence ϕ induces a surjection $\psi : \mathrm{SL}_n(\mathbf{Z}_k) \to G$ (see [**11**, II.21]).

If $k = p_1^{r_1} \dots p_s^{r_s}$ is the prime decomposition,

$$\mathrm{SL}_n(\mathbf{Z}_k) \cong \mathrm{SL}_n(\mathbf{Z}_{p_1^{r_1}}) \times \cdots \times \mathrm{SL}_n(\mathbf{Z}_{p_s^{r_s}})$$

(see [**11**, Theorem VII.11]). Now the restriction of $\psi : \mathrm{SL}_n(\mathbf{Z}_k) \to G$ to some factor $\mathrm{SL}_n(\mathbf{Z}_{p_i^{r_i}})$ has to be nontrivial; since G is simple, this gives some surjection $\psi : \mathrm{SL}_n(\mathbf{Z}_{p^r}) \to G$.

Let K denote the kernel of canonical surjection $\mathrm{SL}_n(\mathbf{Z}_{p^r}) \to \mathrm{SL}_n(\mathbf{Z}_p)$, so K consists of all matrices in $\mathrm{SL}_n(\mathbf{Z}_{p^r})$ which are congruent to identity matrix I when entries are taken modulo p. By performing the binomial expansion of $(I + pA)^{p^{r-1}}$, one easily checks that K is a p-group; in particular, K is solvable. Then also kernel K_0 of the canonical surjection from $\mathrm{SL}_n(\mathbf{Z}_{p^r})$ to the central quotient $\mathrm{PSL}_n(\mathbf{Z}_p)$ of $\mathrm{SL}_n(\mathbf{Z}_p)$ is solvable. Since G is simple, ψ maps K_0 trivially and induces a surjection from $\mathrm{PSL}_n(\mathbf{Z}_p)$ onto G; since $n > 2$, $\mathrm{PSL}_n(\mathbf{Z}_p)$ is simple, and this surjection is an isomorphism.

ii) By [**11**, Theorem VII.26],

$$\mathrm{Sp}_{2g}(\mathbf{Z}_k) \cong \mathrm{Sp}_{2g}(\mathbf{Z}_{p_1^{r_1}}) \times \cdots \times \mathrm{Sp}_{2g}(\mathbf{Z}_{p_s^{r_s}}),$$

and the proof is then analogous to the first case.

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