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LOCALLY HOMEOMORPHIC λ CONNECTED PLANE CONTINUA

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A continuum X is λ connected if each two of its points can be joined by a hereditarily decomposable subcontinuum of X. Suppose that X and Y are plane continua and that there is a local homeomorphism that sends X onto Y. It follows from Theorem 5 in [2] that Y is λ connected if X is λ connected. Here we prove that, conversely, if Y is λ connected, then X is λ connected.

A continuous function f of a topological space X to a topological space Y is a *local homeomorphism* if for each point x of X there exists an open set U in X containing x such that $f(U)$ is open in Y and f restricted to U is a homeomorphism of U onto $f(U)$.

A nondegenerate compact connected metric space is a *continuum*.

Let X be a plane continuum. A continuum L in X is said to be a *link* in X if L is either the boundary of a complementary domain of X or the limit of a convergent sequence of complementary domains of X.

It is known that a plane continuum is λ connected if and only if each of its links is hereditarily decomposable [3, Th. 2].

THEOREM. *Suppose that X and Y are plane continua and that f is a local homeomorphism that sends X onto Y. Then if one of the two continua X or Y is λ connected, so is the other.*

Proof. In [2] it is proved that every planar continuous image of a λ connected continuum is λ connected. Hence to establish this theorem it will be sufficient to show that Y being λ connected implies that X is λ connected.

Assume that Y is λ connected and X is not. By Theorem 2 of [3], there exists an indecomposable continuum I that is contained in a link in X. Since f is a local homeomorphism, $f(I)$ is a continuum in Y.

We first show that every subcontinuum of Y that contains a nonempty open subset of $f(I)$ contains $f(I)$. To accomplish this we suppose that there exists a continuum H in Y that contains a nonempty open subset G of $f(I)$ and does not contain $f(I)$. Define p to be a point of G. Let q be a point of $f(I) - H$. There exist points x and y of I and disjoint open sets U and V of X containing x and y respectively such that (1) $f(x) = p$ and $f(y) = q$, (2) $f(V) \cap$

$H = \varnothing$, and (3) $f(U \cap I)$ is a subset of G.

Since I is contained in a link in X, every continuum in X that contains a nonempty open subset of I contains I [2, Th. 1]. Hence infinitely many components of $X - V$ meet $U \cap I$. Since $f^{-1}(H)$ and V are disjoint in X and $f(U \cap I)$ is contained in G, it follows that $f^{-1}(H)$ has infinitely many components in X. According to Whyburn's theorem [6, Th. 7.5, p. 148], each component of $f^{-1}(H)$ must be mapped onto H by f. But since X is compact and f is a local homeomorphism, for each point z of Y, the set $f^{-1}(z)$ is finite. Hence we have a contradiction. It follows that every subcontinuum of Y that contains a nonempty open subset of $f(I)$ contains $f(I)$.

Note that $f(I)$ is indecomposable [4, Th. 9] and therefore a proper subcontinuum of Y. By Theorem 2 of [1], there exists a composant C of $f(I)$ such that each subcontinuum of Y that meets both C and $Y - f(I)$ contains $f(I)$. This contradicts the assumption that Y is λ connected. Hence X is λ connected.

Comment. We get a false statement when we substitute the word "arcwise" for "λ" in the preceding theorem. The so called "Warsaw circle" [5, Ex. 4, p. 230] is an arcwise connected plane continuum that is the image of a nonarcwise connected plane continuum under a local homeomorphism.

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