implies

$$V(A \to \bar{B}, a) = T, V(\bar{B}, a \cdot c) = F$$

implies

$$V(A, c) = F$$

for each c with $(a \cdot b)^* \leq c < 1$ – so that $V(\bar{A}, a \cdot b) = T$ as required.

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VOLUME 14

Corrigendum to 'Diagonalization and the recursion theorem', by James C. Owings, Jr., *Notre Dame Journal of Formal Logic*, vol. 14 (1973), pp. 95–99.

It has been recently pointed out to me by Maurizio Negri that Application 4 of the abovementioned paper contains a serious error. It is the purpose of this note to rectify this mistake. I sincerely thank Professor Negri for bringing this matter to my attention. In the original treatment it was falsely claimed that there existed a formula $\delta(v)$ of elementary number theory such that, for any $n \in N$, $\vdash \delta(\mathbf{n}) \leftrightarrow \Phi_n(\mathbf{n})$. However, if there were such a formula, then, letting $\neg\delta$ be Φ_k, we would have $\vdash \delta(\mathbf{k}) \leftrightarrow \neg\delta(\mathbf{k})$, implying that number theory was inconsistent. A corrected version follows.

Application 4 (Feferman's fixed-point theorem for elementary number theory). Let $S = N$, let $\Phi_0, \Phi_1, \Phi_2, \ldots$ be the customary enumeration of all formulas of elementary number theory with at most one free variable v, and, if Ψ is such a formula, let $\ulcorner \Psi \urcorner = e$, where $\Psi = \Phi_e$. Also, let $\phi_0, \phi_1, \phi_2, \ldots$ be a standard enumeration of all partial recursive functions of one variable. If $p, q \in N$, let $p \Box q = \phi_p(q), p * q = p \cdot q = \ulcorner \Phi_p(\mathbf{q}) \urcorner, p \circ q = \ulcorner \exists z(\Phi_p(z) \wedge \theta_q(v, z)) \urcorner$, where θ_q is a formula which strongly represents the partial recursive function ϕ_q (i.e., for all $m, n \in N$, $\phi_q(m) = n \Leftrightarrow \vdash \theta_q(\mathbf{m}, \mathbf{n})$ and, for all $m \in N$, $\vdash \forall y \forall z((\theta_q(\mathbf{m}, y) \wedge \theta_q(\mathbf{m}, z)) \to y = z)$). Let δ be any number such that, for all $p, \phi_\delta(p) = \ulcorner \Phi_p(\mathbf{p}) \urcorner$ and let $p \equiv q$ mean $\vdash \Phi_p \leftrightarrow \Phi_q$.

By definition of δ, $\delta \Box p = p * p$. We have that $(p \circ q) * r) = \ulcorner \Phi_{p \circ q}(\mathbf{r}) \urcorner = \ulcorner \exists z(\Phi_p(z) \wedge \theta_q(\mathbf{r}, z)) \urcorner$; so $\Phi_{(p \circ q) * r} = \exists z(\Phi_p(z) \wedge \theta_q(\mathbf{r}, z))$. On the other hand, $p \cdot (q \Box r) = \ulcorner \Phi_p(\underline{q \Box r}) \urcorner = \ulcorner \Phi_p(\underline{\phi_q(r)}) \urcorner$, so $\Phi_{p \cdot (q \Box r)} = \Phi_p(\phi_q(r))$. One now easily shows that $\vdash \Phi_{(p \circ q) * r} \leftrightarrow \Phi_{p \cdot (q \Box r)}$; i.e., $(p \circ q) * r \equiv p \cdot (q \Box r)$. So, by Theorem 1 of this paper, given any formula $\Psi(v)$ there exists a sentence θ such that $\vdash \Psi(\underline{\ulcorner \theta \urcorner}) \leftrightarrow \theta$, namely $\theta = \exists z(\Psi(z) \wedge \theta_\delta(\underline{\ulcorner \exists z(\Psi(z) \wedge \theta_\delta(z, v)) \urcorner}, z))$.

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