SURFACES WITH A PARALLEL ISOPERIMETRIC SECTION

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This announcement is a continuation of Chen [1] (also, Yau [3]). We shall present additional theorems relating surfaces in a space form with a parallel normal section.

Let M be a surface in an m-dimensional Riemannian manifold R^m with the induced normal connection D. For a unit normal section ξ on M (that is, a unit normal vector field of M in R^m), let A_{ξ} be the second fundamental tensor with respect to ξ ; if we have $D\xi = 0$ identically, then ξ is called a parallel section; if the trace of A_{ξ} is constant (respectively, zero), then ξ is called an isoperimetric section (respectively, minimal section) on M; if the determinant of A_{ξ} is nowhere zero, then ξ is called a nondegenerate section; if A_{ξ} vanishes identically, then ξ is called a geodesic section; and if A_{ξ} is not proportional to the identity transformation everywhere, then ξ is called a umbilical-free section.

Theorem 1. Let M be a closed surface in an m-dimensional Riemannian manifold R^m of constant sectional curvature such that the Gaussian curvature of M does not change its sign. If there exists a parallel umbilical-free isoperimetric section on M, then M is flat.

THEOREM 2. Let M be a closed surface of a 4-dimensional Riemannian manifold R^4 of constant sectional curvature $c \le 0$ such that the Gaussian curvature of M does not change its sign. If there exists a parallel non-degenerate minimal section on M, then the mean curvature vector of M is parallel.

THEOREM 3. Let M be a surface in an m-dimensional simply-connected complete Riemannian manifold R^m of constant sectional curvature c such that the Gaussian curvature of M is constant. If there exists a parallel isoperimetric section on M, then either M is contained in a (small or great) hypersphere of R^m or M is flat.

Theorem 4. Let M be a surface in a 4-dimensional simply-connected complete Riemannian manifold R^4 of constant sectional curvature $c \le 0$ such that the Gaussian curvature of M is constant. If there exists a parallel minimal section on M, then either M is contained in a great hypersphere of R^4 or the mean curvature vector H of M is parallel and M is flat.

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600 B.-Y. CHEN

The proofs of these four theorems use the theory of analytic functions. From Theorems 2 and 4 we have immediately the following characterization theorems of standard flat tori in euclidean 4-space.

COROLLARY 1. Let M be a closed surface in a euclidean 4-space such that the Gaussian curvature of M does not change its sign. Then M is the product surface of two plane circles if and only if there exists a parallel nondegenerate minimal section on M (Chen [2]).

COROLLARY 2. Let M be a surface in a euclidean 4-space such that the Gaussian curvature of M is constant. Then M is an open piece of a product surface of two plane circles if and only if there exists a nongeodesic parallel minimal section on M.

The detailed proofs of these results will be published in the author's forthcoming book *Geometry of submanifolds* published by Marcel-Dekker Inc., New York.

REMARKS. 1. If R^m is euclidean, Theorem 1 was also obtained by B. Wegner.

2. It was pointed out by H. B. Lawson that Corollary 2 of [1] should add the following assumption "M is not minimal surface of any hypersphere of E^{m} ". The author would like to express his thanks to Professor Lawson.

REFERENCES

- 1. B.-Y. Chen, Surfaces with parallel mean curvature vector, Bull. Amer. Math. Soc. 78 (1972), 709-710.
- 2. _____, A characterization of standard flat tori, Proc. Amer. Math. Soc. 37 (1973), 564-567
 - 3. S.-T. Yau, Submanifolds with constant mean curvature (to appear).

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