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INFINITE RESISTIVE NETWORKS

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An *infinite resistive network* N consists of a connected, locally finite, oriented, infinite graph with branches $\boldsymbol{B}_1, \boldsymbol{B}_2, \cdots$. To each branch $\boldsymbol{B}_i$ is associated a resistance $r_i \geqq 0$. We are also given a voltage source, i.e., a finite 1-cochain $\boldsymbol{E}'$, and a current source, i.e., a finite 0-chain $\boldsymbol{i}$ satisfying $\partial_0 \boldsymbol{i} = 0$.

For each (real) 1-chain $\boldsymbol{C} = \sum a_i \boldsymbol{B}_i$, define $\|\boldsymbol{C}\|$ by $\|\boldsymbol{C}\|^2 = \sum a_i^2 r_i$.

THEOREM 1. *There exists a unique* 1-*chain* $\boldsymbol{I}$ *such that*:

(i) (*Kirchhoff's current law*). $\partial \boldsymbol{I} + \boldsymbol{i} = \boldsymbol{0}$.

(ii) (*Kirchhoff's voltage law*). *For each finite cycle* $\boldsymbol{Z}$,

$$\langle \boldsymbol{E}', \boldsymbol{Z} \rangle = \langle R(\boldsymbol{I}), \boldsymbol{Z} \rangle,$$

where if $\boldsymbol{I} = \sum a_j \boldsymbol{B}_j$, *then* $R(\boldsymbol{I})$ *denotes the* 1-*cochain* $R(\boldsymbol{I}) = \sum a_j \boldsymbol{B}'_j$. *Of course* $(\boldsymbol{B}'_j, \boldsymbol{B}_i) = \delta_{ij}$.

(iii) (*Finite power*). $\boldsymbol{I}$ *is square summable, i.e.*, $\|\boldsymbol{I}\| < \infty$.

(iv) *There is a sequence* $\{\boldsymbol{C}_j\}$ *of finite* 1-*chains such that* $\partial \boldsymbol{C}_j + \boldsymbol{i} = \boldsymbol{0}$ *and* $\|\boldsymbol{C}_j - \boldsymbol{I}\| \to 0$.

THEOREM 2. *Let* N_j *be any sequence of subnetworks such that* $N_1 \subset N_2 \subset \cdots$ *and* $\bigcup N_j = N$. *Suppose* N_1 *is large enough to support the voltage source* $\boldsymbol{E}'$ *and the current source* $\boldsymbol{i}$. *Let* $\boldsymbol{I}_j$ *be the unique current on* N_j *given by Theorem* 1. *Then* $\|\boldsymbol{I}_j - \boldsymbol{I}\| \to 0$, *where* $\boldsymbol{I}$ *is the unique current on* N.

The proofs of these results, corollaries, and a full discussion will appear shortly in the IEEE Trans. Circuit Theory.

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