Conclusion: the book is a mine of information, but you sure have to dig for it.

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Theorie der linearen Operatoren im Hilbert-Raum. By N. I. Achieser and I. M. Glasmann. Berlin, Akademie-Verlag, 1954. 13+369 pp. 28.00 DM.

The book under review is a translation from the Russian. The original version was published in 1950; a detailed review of it (by Mackey) appears in vol. 13 of Mathematical Reviews. There are by now well over a dozen books one of whose chief purposes is to introduce the reader to the concepts and methods of operator theory in Hilbert space; in the last five or six years they have been appearing at the rate of slightly more than one a year. In view of these facts, a detailed, discursive review of still another contribution to the expository literature of the subject does not seem necessary. What follows is a list of the standard topics that are treated, a brief description of some of the special topics that the authors chose to include, and an appraisal of the didactic value of the book.

Standard topics: definition of Hilbert space, subspaces, bases, linear functionals and their representation, bounded operators, projections, unitary operators, unbounded operators, spectrum, resolvent, graph, the spectral theorem for not necessarily bounded self-adjoint operators and for unitary operators, defect indices, Cayley transforms, extensions of symmetric operators. Comments: infinite-dimensionality is built into the definition of Hilbert space; separability is not part of the definition, but is usually assumed; weak convergence is treated from the sequential point of view only.

Special topics: completely continuous normal operators; Neumark's generalized extension theory for symmetric operators; Krein's generalized resolvents; differential operators. The spectral theory of completely continuous normal operators is treated in detail before the more general spectral representations are attacked; it is made to serve, quite effectively, as a psychological stepping stone. This occurs in a chapter in the main body of the book. The Neumark-Krein theory and differential operators appear in two appendices. The first appendix states and proves Neumark's theorem on positive operator measures (they are compressions of spectral measures) and Neumark's extension theorem for symmetric operators (they are compressions of self-adjoint operators). The same appendix presents Krein's representation theorem for the generalized resolvents of symmetric operators with defect index (1, 1) (in terms of analytic mappings of the upper half-plane into itself) and a study of quasi-self-adjoint extensions. (If A is symmetric with defect index (m, m), $m < \infty$, a quasi-self-adjoint extension is an operator B such that $A \subset B \subset A^*$ and such that the co-dimension of the domain of A in the domain of B is equal to m.) The second appendix studies self-adjoint extensions of differential operators; it concludes with some concrete examples (such as the differential equations satisfied by the Bessel functions).

A student would find the detailed reading of the book highly profitable. If the authors need an analytic fact, they prove it. They prove, in particular, the completeness of L_2, they prove Plancherel's theorem, and they prove Bochner's theorem on the representation of positive definite functions. (The latter is used to prove the spectral theorem for self-adjoint operators; similarly, the spectral theorem for unitary operators is made to follow from the solution of the trigonometric moment problem.) The book contains many refreshing comments. Examples: the distance from a vector y to the span of the linearly independent vectors $x_1, \cdots, x_n$ is the quotient of the Gramian of $x_1, \cdots, x_n, y$ by the Gramian of $x_1, \cdots, x_n$; a Jacobi matrix represents a completely continuous operator if and only if its coefficients tend to zero; if a one-to-one mapping of a Hilbert space onto itself preserves inner products, then it is linear (and therefore unitary). The book also contains many illuminating examples worked out in complete detail; they include multiplication and differentiation on function spaces and shifts on sequence spaces. The authors' treatment of cyclic (self-adjoint or unitary) operators provides a very good introduction to multiplicity theory (which they do not treat). The style throughout is unhurried, precise, and clear.

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Quadratische Formen und orthogonale Gruppen. By Martin Eichler. (Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, vol. 63.) Berlin, Göttingen, Heidelberg, Springer, 1952. 12+220 pp. 24.60 DM; bound, 27.60 DM.

This is a competent and highly original monograph on the algebraic and arithmetic theory of quadratic forms from the modern point of view of Witt's Crelle 176 paper (1937). Much of the work is done in an arbitrary classical product formula (c.p.f.) field—namely a finite algebraic extension of the rational field or a field of rational functions of one variable over a Galois field—subject only to the perpetual assumption that the characteristic is not 2. It contains the classical theory of equivalence of forms under linear transformations with co-