detail proofs that cannot be readily expressed through analytic formulas. For instance, they use the natural device of introducing, as conformal modules, the real periods of a differential dZ (with given imaginary periods) and the integrals

$$\int_{p_1}^{p_2} dZ$$

between the zeros p_2 of Z'. In the situation that arises through a variation they assert, as a triviality, that these quantities determine the Riemann surface. The reviewer agrees that a precise proof is not difficult, but it will necessarily involve considerations that the text does not even touch upon. Apart from such minor inconsistencies the proofs seem satisfactory and the results are very far-reaching.

Numerous applications are given, and it is shown, in particular, that the period matrix depends differentiably on the moduli. There is no attempt, however, to introduce a complex structure on the space of Riemann surfaces.

The last chapter is rather loosely connected with the rest of the book. It gives a concise and readable presentation of the Hodge theory of harmonic differentials on Kählerian manifolds, together with its extension to manifolds with boundary.

The authors must be congratulated on having got out of their hands and before the eyes of the mathematical public a volume that must have been very difficult to edit. It contains a plethora of ideas, each interesting in its own right, and on the whole they have been tied together in a successful manner. At a first reading the wealth of formulas is almost forbidding, especially since the authors have not been very fortunate in their choice of notations. However, the patient reader will be richly rewarded and will become aware of many challenging problems that remain to be solved.

The publishers have gone out of their way to give the formulas an attractive appearance, and the proofreading is excellent.

LARS V. AHLFORS

Topological dynamics. By W. H. Gottschalk and G. A. Hedlund. American Mathematical Society Colloquium Publications, vol. 36. Providence, American Mathematical Society, 1955. 8+148 pp. $5.10.

The authors begin by explaining that by *topological dynamics* they mean "the study of transformation groups with respect to those topological properties whose prototype occurred in classical dynamics." Thus, they say, *topological* has reference to the mathematical content of the subject and *dynamics* to its historical origin.

Part I (*The theory*) defines transformation groups and studies their recursion properties; Part II (*The models*) applies the results to some classically important special cases.

A transformation group is a topological space X, a topological group T, and a continuous mapping $(x, t) \rightarrow xt$ from $X \times T$ into X such that if e is the identity element of T, then $xe = x$ for all x in X, and if x and t are arbitrary elements of T, then $(xs)t = x(st)$ for all x in X. Section 1 introduces the basic concepts associated with transformation groups (e.g., isomorphism, transformation subgroup, direct product, invariance, orbit), and establishes the connections among them. The main purpose of section 2 is to conclude, under various sets of appropriate hypotheses, that the orbit-closures constitute a partition of X. A subset A of the group T is called *syndetic* (or, rather, left syndetic) if there exists a compact set K such that $AK = T$. A typical theorem states that if X is compact and minimal under T, and if S is a syndetic invariant subgroup of T, then the orbit-closures under S constitute a star-closed decomposition of X. Explanation of terms: minimality means that the orbit under T of every point of X is dense in X; a decomposition is a partition of X into compact sets; and a partition is star-closed if, for every closed set E in X, the union of all those sets of the partition that intersect E is a closed set.

Section 3 defines various generalizations of periodicity. The period of the group T at a point x in X is the set P of those elements t for which $xt = x$; the group T is said to be periodic at x if P is a syndetic subset of T. Sample theorem: if X is a Hausdorff space, and if T is locally compact and separable, then a necessary and sufficient condition that T be periodic at a point x is that the orbit xT be compact. The broadest generalization of periodicity is called recursion; its special cases include almost periodicity, regular almost periodicity, and recurrence. The group T is said to be almost periodic at x if, corresponding to every neighborhood U of x, there exists a (left) syndetic subset A of T such that $xA \subset U$. If, in this definition, "syndetic subset" is replaced by "syndetic invariant subgroup," the resulting concept is called regular almost periodicity; if, on the other hand, "syndetic subset" is replaced by "extensive subset," the resulting concept is called recurrence. Explanation of terms: a subset of T is extensive if it intersects every replete semigroup of T; a subset of T is replete if it contains some (bilateral) translate of each compact subset of T. The authors treat all these concepts (and several others) simultaneously by the device of defining recursion in terms of the neutral phrase "admissible set." Thus, for example, the special concept of recurrence is obtained from the general concept of recursion by interpreting "admissible" to mean "extensive." As far as possible the theorems concerning recursion are proved in the general setting. The principal result (the inheritance theorem) asserts that certain subgroups of T are recursive if and only if T is recursive.

Section 4 specializes recursion to almost periodicity. The inheritance theorem asserts here that if T is locally compact and if S is a closed syndetic invariant subgroup of T, then a necessary and sufficient condition that S be almost periodic at a point x is that T be almost periodic at x. The main purpose of the section is to study the relations of almost periodicity to various equicontinuity conditions, and, toward the end, to prove the existence of a generalized mean for certain almost periodic functions with values in a uniform space. Sections 5 and 7 do for regular almost periodicity and for recurrence, respectively, what section 4 does for almost periodicity; section 6 supplies the necessary auxiliary facts about replete semigroups. To ensure the existence of sufficiently many replete semigroups, the authors restrict the treatment of recurrence to generative groups, i.e., to abelian groups generated by a compact neighborhood of the identity.

Section 8 proves that periodicity or recurrence properties of a generative group are equivalent to incompressibility properties. Call a subset M of X compressible (this is not the authors' terminology) if there exists a replete semigroup P in T such that $MP \subset M$ and such that the set $M - MP$ is of second category; in the contrary case call M incompressible. Sample theorem: a necessary and sufficient condition that every subset of X be incompressible is that the set of those points at which T is not periodic be a set of the first category.

Sections 9 and 10 study transitivity and asymptoticity, respectively, and the relations of these concepts to various forms of recursion. The group T is transitive at x if the orbit of x is dense in X; related concepts are regional transitivity (each non-empty open set intersects some transform of every non-empty open set), universal transitivity (every orbit in X is X), etc., etc. To define asymptoticity, suppose that X is a compact metric space and that f is a homeomorphism from X onto X. A point x of X is said to be (positively) asymptotic to a closed f-invariant subset B of X if the distance from the nth f-transform of x to B tends to 0 as n tends to $+\infty$.

Part I concludes with section 11 on function spaces; this is essentially a standard treatment of the subject, with special emphasis on uniform spaces, and, correspondingly, on questions of equicontinuity and completeness.

Part II has three sections. Section 12 treats symbolic dynamics. This means, by definition, the combinatoric study of a particular transformation σ on a particular space X. The space X is the set of all sequences $\{x_n\}$, where $x_n \in S$ for $n=0, \pm 1, \pm 2, \cdots$, and where S is a finite set containing more than one element; the transformation σ (the shift) is defined by $(\sigma x)_n = x_{n+1}$. Section 13 culminates in the theorem concerning the (topological, not measure-theoretic) transitivity properties of the geodesic flow associated with a surface of constant negative curvature. Section 14 studies the recursion and transitivity properties of certain cylinder homeomorphisms. A cylinder homeomorphism ϕ is determined by a topological space Y, a homeomorphism θ from Y onto Y, and a continuous function f from Y into $\mathcal{R}$ (= the real line). The homeomorphism ϕ acts on the product space $Y \times \mathcal{R}$ by allowing θ to act on the first coordinate y and by translating the second coordinate by the amount $f(y)$.

The most striking virtue of the book is its organization. The authors' effort to arrange the exposition in an efficient order, and to group the results together around a few central topics, was completely successful; they deserve to be congratulated on a spectacular piece of workmanship. The results are stated at the level of greatest available generality, and the proofs are short and neat; there is no unnecessary verbiage. The authors have, also, a real flair for the "right" generalization; their definitions of periodicity and almost periodicity, for instance, are very elegant and even shed some light on the classical concepts of the same name. The same is true of their definition of a syndetic set, which specializes, in case the group is the real line, to Bohr's concept of a relatively dense set.

The chief fault of the book is its style. The presentation is in the brutal Landau manner, definition, theorem, proof, and remark following each other in relentless succession. The omission of unnecessary verbiage is carried to the extent that no motivation is given for the concepts and the theorems, and there is a paucity of illuminating examples. The striving for generality (which, for instance, has caused the authors to treat uniform spaces instead of metric spaces whenever possible) does not make for easy reading. The same is true of the striving for brevity; the shortest proof of a theorem is not always the most perspicuous one. There are too many definitions, especially in the first third of the book; the reader must at all times keep at his finger tips a disconcerting array of technical terminology. The learning of this terminology is made harder by the authors' frequent use of multiple statements, such as: "The term $\{$*asymptotic*$\}$ $\{$*doubly asymptotic*$\}$ means negatively $\{$or$\}$ $\{$and$\}$ positively asymptotic."

Conclusion: the book is a mine of information, but you sure have to dig for it.

PAUL R. HALMOS

Theorie der linearen Operatoren im Hilbert-Raum. By N. I. Achieser and I. M. Glasmann. Berlin, Akademie-Verlag, 1954. 13+369 pp. 28.00 DM.

The book under review is a translation from the Russian. The original version was published in 1950; a detailed review of it (by Mackey) appears in vol. 13 of Mathematical Reviews. There are by now well over a dozen books one of whose chief purposes is to introduce the reader to the concepts and methods of operator theory in Hilbert space; in the last five or six years they have been appearing at the rate of slightly more than one a year. In view of these facts, a detailed, discursive review of still another contribution to the expository literature of the subject does not seem necessary. What follows is a list of the standard topics that are treated, a brief description of some of the special topics that the authors chose to include, and an appraisal of the didactic value of the book.

Standard topics: definition of Hilbert space, subspaces, bases, linear functionals and their representation, bounded operators, projections, unitary operators, unbounded operators, spectrum, resolvent, graph, the spectral theorem for not necessarily bounded self-adjoint operators and for unitary operators, defect indices, Cayley transforms, extensions of symmetric operators. Comments: infinite-dimensionality is built into the definition of Hilbert space; separability is not part of the definition, but is usually assumed; weak convergence is treated from the sequential point of view only.

Special topics: completely continuous normal operators; Neumark's generalized extension theory for symmetric operators; Krein's generalized resolvents; differential operators. The spectral theory of completely continuous normal operators is treated in detail before the more general spectral representations are attacked; it is made to serve, quite effectively, as a psychological stepping stone. This occurs in a chapter in the main body of the book. The Neumark-Krein theory and differential operators appear in two appendices. The first appendix states and proves Neumark's theorem on positive operator measures (they are compressions of spectral measures) and Neumark's extension theorem for symmetric operators (they are compressions of self-adjoint operators). The same appendix presents Krein's representation theorem for the generalized resolvents of symmetric operators with defect index (1, 1) (in terms of analytic