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NOTICE

The issue number on the cover and on the spine of the March 1982 Annals of Statistics was misprinted and should have read

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and

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