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MATHEMATICAL MODEL OF ELASTIC CLOSED FLEXIBLE SHELLS WITH NONLOCAL SHAPE DEVIATIONS

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Abstract. A model of deformation and mechanical stability of a thin-walled shell with geometric deviations, which is close to a circular cylindrical shell, under the action of axial compression and normal pressure is developed. The model uses the scheme of a flexible shell of zero Gaussian curvature with a perturbed edge, which makes it possible to apply the methods of the geometrically nonlinear theory of torso shells.

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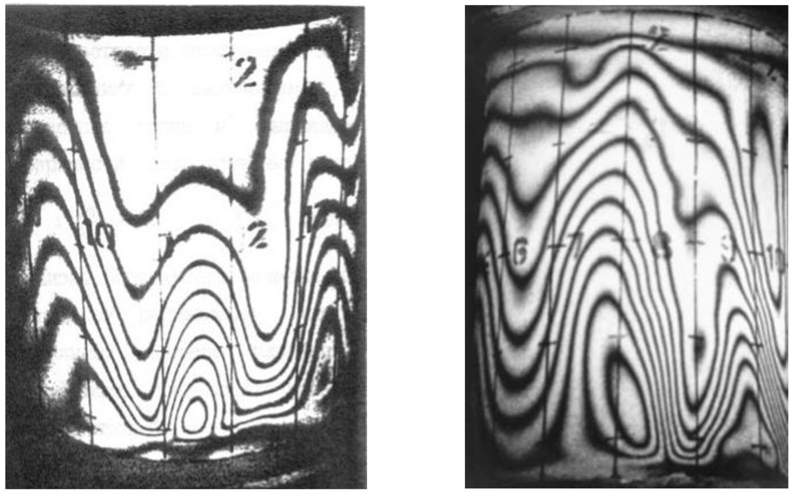
1. Introduction

It is well known that thin-walled shells are very sensitive to the presence of local initial geometrical deviations of the middle surface and ends [24]. The presence of even small deviations within the limits of the wall thickness of cylindrical shells leads to a significant decrease in the carrying capacity of the shell by a factor of 2–3, especially when axial compression is applied.

At the same time, multifactor experimental studies show that nonlocal deviations such as ovality and taper are also significant [11, 21]. Moreover, holographic interferometry of shells with both ovality and conicity (Fig.1) shows surprising proximity of their radial displacements field to the displacements of cylindrical shells with deviation in the form of two waves along the edge. This similarity requires an explanation.

Shells with nonlocal deviations, which are made of sheet material by joining it without stretching the middle surface, are shells with an unfolding principal surface, or, equivalently, with zero Gaussian curvature. The essential complexity of describing the behavior of such shells arises already at the stage of describing the geometry of their principal surface. If the ovality of the opposite ends is not the same, then the principal surface differs from the cylinder and cone. In this case, there is no single point of intersection of the generators and the axis of the shell,



a) b)

Figure 1. Arrangement of radial displacements for shell with both ovality and conicity a) and with deviations of the lower end b).

which was traditionally taken as the origin for the conical shells proper. The introduction of coordinates on such a surface is possible by several methods, the main of which are the following two. According to the first method, a parameterization is introduced whose coordinate lines do not coincide with the lines of the principal curvatures of the principal surface – global Gaussian coordinates [14]. Such an approach in the study of oval cones was demonstrated in the works of Almroth, Brogan and Marlowe [5]. They used an affine coordinate system – the distance from the vertex of the cone along the generatrices and the size of the arc of the cross section. This allowed us to set the boundary conditions in the traditional form. The equations of the nonlinear theory obtained in this case are complicated, cannot be simplified and are solved, mainly, numerically. A similar approach for an arbitrary system of cylindrical coordinates that do not coincide with parameterization along the lines of principal curvatures was used in the work of Melbin and Noordergraaf [19] when considering oval weakly conical shells that mimic the work of blood vessels. In this case the conicity was assumed to be less than 0.1 radian (up to 6°).

The application of the second method is connected with the theorem, given, for example, in the work of Vorovich [26], on the existence of such a parameterization of equations for shells of zero Gaussian curvature, for which one of the Lame coefficients of the principal surface is equal to one, and the corresponding principal curvature is zero. The use of such a local coordinate system, so-called Euler coordinates greatly simplifies the considered dependencies, especially if the system is orthogonal, i.e., if its coordinate lines coincide with the lines of the principal curvatures of the principal surface of the shell [2, 25, 26]. The specification of boundary conditions requires in this case close attention to maintain their correspondence with their natural appearance. For cones of arbitrary cross-section, this approach was demonstrated in the works of Agenosov, Orlov and Sachenkov [3,4]. In this case, the first quadratic form of the surface was given in an orthogonal form a priori, which requires a separate verification for an arbitrary cross section.

For torso shells that are not conical and cylindrical, solutions in a linear formulation are given in the works of Krivoshapko and Ivanov [16, 18]. At the same time, the study of the stability of such shells requires a nonlinear calculation.

The study of the deformation of thin–walled structures in most of the works is based on the approximate integration of nonlinear differential equations of the theory of flexible elastic shells. The most frequently used technical theory of Donnell–Mushtari–Vlasov shells [6] in various modifications [15,17,20,23]. As was shown in [7], an analysis of the accuracy of the terms entering into the equations of the geometrically nonlinear theory with respect to the natural small parameters of the structure allows us to obtain a number of simplifications for practically important computational schemes. This makes it possible to apply perturbation methods for such constructions according to natural small parameters.

The equations of the theory of flexible elastic shells can be obtained from the general variation principles of the mechanics of a deformable solid [1] – Lagrange, Castiliano, Reissner, Hu–Vashizu – by constructing the corresponding functional and finding its stationary or extreme points. This problem can be solved either by creating analytical stationary conditions (Euler equations), or by applying variation methods directly to the functional under consideration [6]. If the Lagrange principle implies the equilibrium equations and natural boundary conditions in stresses, then the Reissner principle implies the equations of equilibrium, the relations of elasticity and the natural boundary conditions in stresses and displacements. It is known that the Reissner principle is a Hamiltonian form of the Lagrange principle and, thus, allows the introduction of generalized variables [22]. Reducing the dimensionality of the functional can be achieved by specifying the type of solution in two coordinates, as well as by using hypotheses of applied theories. The Reissner principle allows the use of independent approximations of forces and deflections, which satisfy the boundary conditions. All this makes the Reissner method most suitable for constructing equations of the theory of flexible elastic shells nonlinear deformation.

2. The Mathematical Description of the Shells Surface with Deviations

We consider one of the variants of parameterization for calculating shells of zero curvature with a conicity and different ovality of the ends. For a rectangular coordinate system associated with the shell axis and the axes of base ellipses described by the equations

$$
\begin{aligned}
&x = a_2 \cos t_2, \quad y = b_2 \sin t_2, \quad z = L \\
&x = a_1 \cos t_1, \quad y = b_1 \sin t_1, \quad z = 0
\end{aligned} \tag{1}
$$

when using the results of [18]. The equation of the median surface of the torso shell in the parametric form takes the following form

$$
\begin{aligned}
&x = x(\lambda, t_2) = a_1 \cos t_1 + \lambda(a_2 \cos t_2 - a_1 \cos t_1) \\
&y = y(\lambda, t_2) = b_1 \sin t_1 + \lambda(b_2 \sin t_2 - b_1 \sin t_1) \\
&z = z(\lambda) = \lambda L
\end{aligned} \tag{2}
$$

where t_1, t_2 are the parameters of the corresponding points of the ellipses, t_1 is determined from the uniqueness condition of the torso surface

$$(a_1/b_1)\mathrm{tan} t_1 = (a_2/b_2)\mathrm{tan} t_2.$$

The coordinate lines

$$t_2 = \mathrm{const}$$

generally are not parallel to each other and non-coplanar with the axis of the shell, and the curves

$$\lambda = \mathrm{const}$$

are transverse sections of the principal surface of the shell by planes perpendicular to its axis (Fig. 2 a)). Such a coordinate system is not orthogonal in the general case. The parameters of its quadratic forms are determined by the formulas given in [16, 18].

Analysis of equations (1)–(2) shows that the deviation of the axial section of the middle surface from the straight line δW for shells of medium length L and thickness h for different ovality of the ends reaches $2h$ (Fig. 2 b), c)), the angle of the section is indicated near the curves). Thus, the shell with different ovality ends has a significant initial flexure, and its deformation should be modeled on the basis of geometrically nonlinear relationships between deformations and displacements.

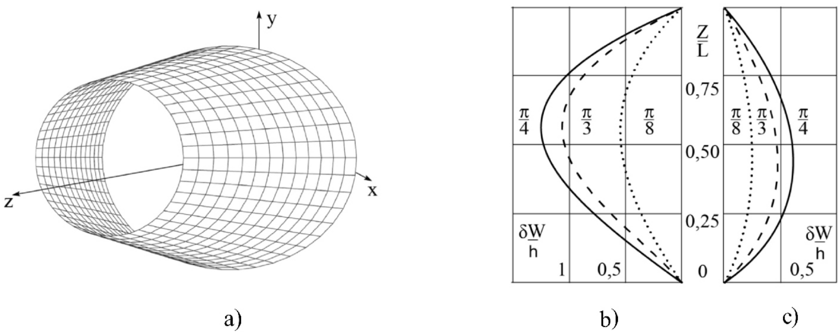


Figure 2. Illustrative arrangement of generatrices a) and deviations of the axial section of the torso shell from a straight line with greater ovality of the upper b) or lower c) end.

The conditions given in [18] were used to determine the principal directions on the principal surface of the shell. The first quadratic form in the considered reference frame has the form

$$A^2 \mathrm{d}t_2^2 + 2F \mathrm{d}\lambda \mathrm{d}t_2 + B^2 \mathrm{d}\lambda^2.$$

The obtained orthogonal reference frame includes the straight-line coordinate lines

$$y = t_2 = \mathrm{const}$$

considered earlier, coinciding with the generators and the direction of zero curvature, and curvilinear guides along the lines of maximum positive curvature

$$x = \lambda + \int\limits_{t_0}^{t} \frac{F}{A^2} \mathrm{d}t_2 = \mathrm{const}$$

and the first quadratic form has the form

$$A^2 \mathrm{d}y^2 + A_2^2 \mathrm{d}\lambda^2$$

where

$$A_2^2 = B^2 - F^2/A^2 \cdot$$

The magnitude of the deflection of the guides from the plane of the cross-section of the shell ΔL is greater $10h$ for medium-length shells (Fig. 3). Hence, even with

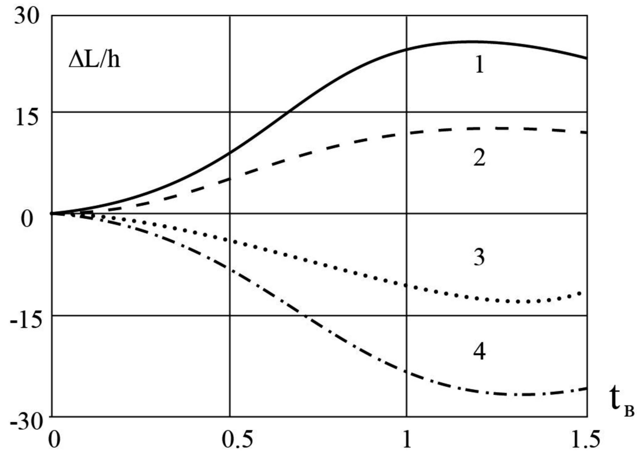


Figure 3. Deviations of the coordinate lines from the lines of the principal curvature for small (1, 4) and large (2, 3) conicity with a larger (1, 2) or smaller (3, 4) ovality of the smaller end.

uniform axial compression, the oval-conical shells are inhomogeneously loaded and have a non-planar edge with respect to the line of principal curvature.

Thus, shells with different ovality of ends and with conicity have a noncanonical shape, non-planar edges and an initial flexure. Their calculation must be done by special methods, for example, by method of perturbation according to the parameter [11, 21], on the basis of geometrically nonlinear equations of shells bending. Such equations are constructed below.

3. Shell Models in the Form of Geometrically Nonlinear Equations with Complex Boundary Conditions

Let us consider the deformation of a thin elastic shell of zero Gaussian curvature of constant thickness h, made of an elastic isotropic material (Fig. 4).

We introduce on the shell surface S a curvilinear orthogonal basis with axes x, y, z. In this case, the coordinate lines x, y coincide with the lines of the principal curvature of the surface, the axis z is normal to it. We connect the coordinate direction of the axis with the line of principal zero curvature. We confine ourselves to the case of small, in comparison with unity, deformations, finite displacements, finite but small angles of rotation of the normal. We also accept Kirchhoff's kinetic hypothesis on plane sections.

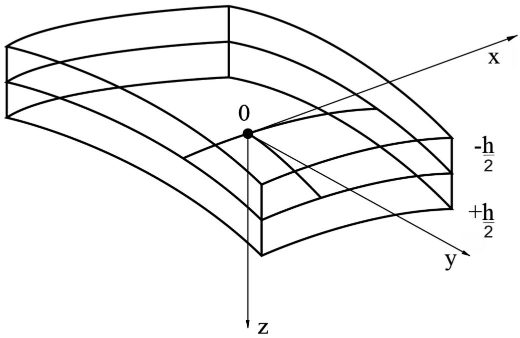


Figure 4. A layout of the coordinate axes of the shell element.

Deformations ε_{ij} and changes in the curvature k_{ij} of the principal surface are expressed through displacements along the axes u, v, w in form

$$\begin{aligned}
&\varepsilon_{11} = u_{,x} + \frac{1}{2}\left(w_{,x}^2 + \frac{1}{4A_2^2}\Big((A_2 v)_{,x} - u_{,y}\Big)^2\right)\\
&\varepsilon_{22} = \frac{uA_{2,x}}{A_2} + \frac{v_{,y}}{A_2} + \frac{w}{R} + \frac{1}{2}\left(Y^2 + \frac{1}{4A_2^2}\Big((A_2 v)_{,x} - u_{,y}\Big)^2\right)\\
&\varepsilon_{12} = \varepsilon_{21} = \frac{1}{2}\left(v_{,x} + \frac{u_{,y}}{A_2} - \frac{vA_{2,x}}{A_2} - w_{,x}Y\right)\\
&k_{11} = -w_{,xx}, \quad k_{22} = \frac{1}{A_2}Y_{,y} - \frac{w_{,x}A_{2,x}}{A_2}\\
&k_{12} = k_{21} = \frac{1}{2}\left(Y_{,x} - \frac{w_{,xy}}{A_2} - Y\frac{A_{2,x}}{A_2} + \frac{1}{2A_2R}\Big((A_2 v)_{,x} - u_{,y}\Big)\right)
\end{aligned} \tag{3}$$

where $Y = \left(-\frac{w_{,y}}{A_2} + \frac{v}{R}\right)$, $R = R(x,y)$ – the radius of curvature of the non–deformed median surface in the direction of the coordinate line y and displacement v, and $A_1, A_2$are Lame parameters of the median surface for shells with developable surface, $A_1 \equiv 1$, $\frac{\partial}{\partial x} = (\,)_{,x}$, $\frac{\partial}{\partial y} = (\,)_{,y}$, v – Poisson's ratio.

For an elastic isotropic solid, the relations between the forces T_{ij}, moments M_{ij} and strains distributed over the shell thickness are

$$\begin{aligned} T_{11} &= (\varepsilon_{11} + \mu\varepsilon_{22})D, & T_{22} &= (\varepsilon_{22} + \mu\varepsilon_{11})D \\ T_{12} &= \varepsilon_{12}D(1-\mu), & M_{11} &= \frac{Dh^3}{12}(k_{11} + \mu k_{22}) \\ M_{22} &= \frac{Dh^3}{12}(k_{22} + \mu k_{11}), & M_{12} &= k_{12}(1-\mu)\frac{Dh^3}{12} \end{aligned} \tag{4}$$

where E – Young's modulus of material elasticity, $D = \dfrac{Eh}{1-\mu^2}$.

In the mixed formulation, the stress function F is introduced

$$T_{11} = \frac{(F_{,y}/A_2)_{,y} + A_{2,x}F_{,x}}{A_2}, \quad T_{22} = F_{,xx}, \quad T_{12} = \frac{(F_{,x}/A_2)_{,y} + F_{,x}A_{2,x}/A_2}{A_2}.$$

If we neglect the effect of tangential displacements on the angles of rotation of the normal at a point, which corresponds to the theory of shallow shells [20, 27], then equations (3) will be simplified as follows

$$\begin{aligned} &\varepsilon_{11} = u_{,x} + \frac{1}{2}w_{,x}^2, \quad \varepsilon_{22} = \frac{v_{,y}}{A_2} + \frac{uA_{2,x}}{A_2} + \frac{w}{R} + \frac{1}{2}\left(\frac{w_{,y}}{A_2}\right)^2 \\ &\varepsilon_{12} = \varepsilon_{21} = \frac{1}{2A_2}(A_2 v_{,x} + u_{,y} - A_{2,x}v + w_{,x}w_{,y}) \\ &k_{11} = -w_{,xx}, \quad k_{22} = -\frac{1}{A_2}\left(\left(\frac{w_{,y}}{A_2}\right)_{,y} + w_{,x}A_{2,x}\right) \\ &k_{12} = k_{21} = \frac{1}{2}\left(\left(-\frac{w_{,y}}{A_2}\right)_{,x} - \frac{w_{,xy}}{A_2} + \frac{w_{,y}A_{2,x}}{A_2^2}\right). \end{aligned} \tag{5}$$

From the Gauss–Codazzi equations we get $A_{2,xx} = 0$.

The coordinate system is orthogonal. In this case, the coordinate lines are curvilinear and do not coincide with the boundary of the shell. Thus, the border is different from the location of the coordinate lines $x = \mathrm{const}$.

In the system of longitudinal orthogonal coordinates, which coincide with the lines of the main curvatures, for the shell of zero Gaussian curvature, the equilibrium equations will take the form

$$
\begin{aligned}
T_{11,x} + T_{21,y} + A_2^{-1} A_{2,x}(T_{11} - T_{22}) &= 0 \\
T_{22,y} + T_{12,x} + A_2^{-1} A_{2,x} T_{21} + k_2 Q_2 &= 0 \\
Q_{1,x} - Q_{2,y} + A_2^{-1} A_{2,x} Q_1 - A_2^{-1} A_{2,x} T_{22} &= q \\
M_{11,x} + A_2^{-1} A_{2,x}(M_{11} - M_{22}) - k_2 M_{22} - Q_1 &= 0 \\
M_{22,y} - Q_2 = 0, \quad M_{11,x} - M_{22,y} + A_2^{-1} A_{2,x} M_{11} &= 0\cdot
\end{aligned} \tag{6}
$$

Shear forces Q_i can be explicitly expressed and excluded from the system of equations of equalities. It can be seen that the use of longitudinal coordinates for shells of zero Gaussian curvature allows us to consider a simplified model with constant coefficients for higher derivatives. The shell model with a developable surface can be represented in the following general form for convenience of calculation by the modified method of parameter continuation (MMCP) [9, 10, 12, 13].

4. The Shell Model in a General Case

Let the closed simply connected curvilinear trapezium Ω in the Euclidean space E^2 of two variables $\{\eta, \xi\} \in E^2$ is close to a rectangle Ω_0 and meet the following relations

$$
\begin{aligned}
&\Omega = \{-\pi < \eta < \pi \ ; \ -f_1(\eta) < \xi < 1 + f_2(\eta)\} \\
&\Omega_0 = \{-\pi < \eta < \pi \ ; 0 < \xi < 1\} \\
&\Omega_0 \subset \Omega, \qquad f_i(\eta) \geq 0, \qquad \max_{\eta \in [-\pi, \pi]} f_i(\eta) = 1, \qquad i = 1, 2
\end{aligned} \tag{7}
$$

where $f_i(\eta)$ are continuous functions over the interval $[-\pi, \pi]$.

One can map onto Ω the middle surface of a closed shell having edges smoothly perturbed in the plane perpendicular to the shell axis [8]. The trapezium boundary $\partial\Omega$ consists of the following four parts

$$
\begin{aligned}
&\partial\Omega_1 = \partial\Omega|_{\eta=-\pi}, \qquad &&\partial\Omega_2 = \partial\Omega|_{\eta=\pi} \\
&\partial\Omega_3 = \partial\Omega|_{\xi=f_1(\eta)}, \qquad &&\partial\Omega_4 = \partial\Omega|_{\xi=1+f_2(\eta)}
\end{aligned}
$$

and the four parts of the rectangle Ω_0 boundary $\partial\Omega_0$ are

$$
\begin{aligned}
&\partial\Omega_{01} = \partial\Omega|_{\eta=-\pi}, \qquad &&\partial\Omega_{02} = \partial\Omega|_{\eta=\pi} \\
&\partial\Omega_{03} = \partial\Omega_0|_{\xi=0}, \qquad &&\partial\Omega_{04} = \partial\Omega_0|_{\xi=1}.
\end{aligned}
$$

Let U is a set of uniformly bounded functions U_i defined over the region $\overline{\Omega} = \Omega \bigcup \partial\Omega$

$$U = \{U_i \in U,\ i = 1, \ldots, N\}. \tag{8}$$

Let each of these functions U_i have derivatives everywhere over $\overline{\Omega}$

i) the uniformly bounded and continuous partial derivatives of order not less than n_i of the following form

$$\begin{gathered} U^{(n)} = \{U_{ikp} \in U^{(n)},\ U_{ikp} = \frac{\partial^{k+p} U_i}{\partial \eta^k \partial \xi^p}\} \\ 0 \leq k + p \leq n_i, \qquad k, p = 1, \ldots, n_i, \qquad i = 1, \ldots, N \end{gathered} \tag{9}$$

ii) the set of continuous partial derivatives of order not less than $n_i - 1$ of the form

$$\begin{gathered} U^{(n-1)} = \{U_{ikp} \in U^{(n)}\}, \qquad i = 1, \ldots, N \\ 0 \leq k + p \leq n_i - 1, \qquad k, p = 1, \ldots, n_i - 1 \end{gathered} \tag{10}$$

iii) a set of partial derivatives of a higher order

$$\begin{gathered} U^{(max)} = \{U_{ikp} \in U^{(max)}, U_{ikp} \in U^{(n)}\} \\ k + p = n_i, \qquad k, p = 1, \ldots, n_i, \qquad i = 1, \ldots, N. \end{gathered} \tag{11}$$

Besides that, $U_i = U_{i00},\ i = 1, \ldots, N$.

Let there is a system of N nonlinear PDEs

$$\lambda_j \left(\eta, \xi, U^{(n)}\right) = \Phi_j \left(\eta, \xi, U^{(n-1)}\right), \qquad j = 1, \ldots, N \tag{12}$$

where $\lambda_j,\ \Phi_j$ are the algebraic analytical functions uniformly bounded over $\overline{\Omega}$ and represented by the Maclaurin series with respect to η, ξ, U_{ikp} (all variables are considered as independent). One more assumption is that λ_j are the linear with respect to $U^{(\max)}$.

For the problem completeness, there must be also some boundary conditions defined over $\partial\Omega$. Let one of them be the condition of the solution periodicity with respect to η

$$U^{(n)}|_{\partial\Omega_1} = U^{(n)}|_{\partial\Omega_2} \tag{13}$$

which reflects the conditions of closed shell modeling, and below are the boundary conditions for parts $\partial\Omega_3$ and $\partial\Omega_4$, where $\partial\Omega_k$ such that

$$G_j^k\left(\eta, U^{(n-1)}|_{\partial\Omega_k}\right) = 0, \qquad j = 1, \ldots, n_k, \qquad k = 3, 4 \tag{14}$$

where G_j^k are the bounded piece-wise continuous algebraic functions regarding all their arguments. The aim is to find the system (12) solution comprised of functions from the set U and satisfying the boundary conditions (13)–(14) over $\partial\Omega$. The form of the studied system of equations defines the number of independent boundary conditions (14). They should provide a possibility to define all arbitrary functions included in general solution to the system.

In the general case, the problem (13)–(14) is a nonlinear boundary value problem with complex boundary conditions over non–canonical (perturbed) domain. Equations (12)–(14) describe boundary value problems of the theory of plates and torso shells.

5. Conclusions

The presented model can be applied to describe nonlinear deformation of the shell of zero Gaussian curvature with geometric deviations under complex loading and large deflections.

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