On CR Mappings of Real Quadric Manifolds

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0. Introduction

According to the well-known theorem of Alexander [Al], each local CR diffeomorphism of the real unit sphere in $\mathbf{C}^n$, for $n>1$, extends to a complex rational mapping on all $\mathbf{C}^n$. This important result was generalized by several authors in different directions. Thus, Tumanov [Tu], Tumanov and Henkin [TH], and Forstnerič [Fo] transferred it to CR mappings of real quadric Cauchy–Riemann manifolds of an arbitrary codimension in $\mathbf{C}^n$. In the present note we generalize these results. Our main tool is the reflection principle due to Lewy [Le], Pinchuk [P1], and Webster [W1; W2; W3].

I express my thanks to S. Pinchuk for his interest in this work.

1. Result

We denote coordinates in $\mathbf{C}^n$ by (z, w), where $z \in \mathbf{C}^k$, $w \in \mathbf{C}^d$, $k+d=n$, and $k, d>0$. Let also $\langle z, \zeta\rangle = \sum_{j=1}^k z_j \zeta_j$. Consider in $\mathbf{C}^n$ a real generic manifold of the form

$$w_j + \bar{w}_j = \langle L_j(z), \bar{z}\rangle, \quad j=1, \dots, d, \tag{1.1}$$

where each $L_j : \mathbf{C}^k \to \mathbf{C}^k$ is a $\mathbf{C}$-linear hermitian operator. To simplify the notation we shall write (1.1) in the form

$$w + \bar{w} = \langle L(z), \bar{z}\rangle, \tag{1.2}$$

setting

$$\langle L(z), \bar{z}\rangle = (\langle L_1(z), \bar{z}\rangle, \dots, \langle L_d(z), \bar{z}\rangle). \tag{1.3}$$

A manifold M of the form (1.2) is said to be a *quadric* Cauchy–Riemann manifold (in short, quadric) of real codimension d in $\mathbf{C}^n$.

Let $T_p^c(M)$ denote the complex tangent space of M at $p \in M$. Recall that $T_p^c(M) = T_p(M) \cap i(T_p(M))$, where $T_p(M)$ is a real tangent space and $i = \sqrt{-1}$. The complex dimension of $T_p^c(M)$ does not depend on p in M, and is equal to k; it is called the *CR dimension* of M. The vector-valued (with values in $\mathbf{R}^d$) hermitian quadratic form (1.3), defined on $\mathbf{C}^k \cong \{(z, w) : w = 0\} =$

Received September 16, 1992.
Michigan Math. J. 41 (1994).

$T_0^c(M)$, is the *Levi form* of M. Each L_j is said to be the *Levi operator*. We also connect with M a convex cone in $\mathbf{R}^d$ of the form

$$\mathrm{Co}\{\langle L(z), \bar{z}\rangle : z \in \mathbf{C}^k\},$$

where Co denotes the linearly convex hull; it is called the *Levi cone* of M.

Together with (1.2) let us consider in $\mathbf{C}^{n'}$ coordinates (z', w'), where $z' \in \mathbf{C}^{k'}$, $w' \in \mathbf{C}^{d'}$, $k' + d' = n'$, and $k', d' > 0$. We define a quadric $M' \subset \mathbf{C}^{n'}$ of the form

$$w_j' + \bar{w}_j' = \langle L_j'(z'), \bar{z}'\rangle', \quad j = 1, \dots, d'. \tag{1.4}$$

Here $\langle z', \zeta'\rangle' = \sum_{j=1}^{k'} z_j' \zeta_j'$. By analogy with (1.2), one can represent (1.4) in the form

$$w' + \bar{w}' = \langle L'(z'), \bar{z}'\rangle', \tag{1.5}$$

where

$$\langle L'(z'), \bar{z}'\rangle' = (\langle L_1'(z'), \bar{z}'\rangle', \dots, \langle L_{d'}'(z'), \bar{z}'\rangle').$$

If $F: M \to M'$ is a mapping of class C^1, we denote by dF_p the differential (tangent mapping) of F at p in M.

Our main result is the following theorem.

THEOREM. *Let $M \subset \mathbf{C}^n$ be a quadric* (1.2) *and let the Levi cone of M have a non-empty interior in $\mathbf{R}^d$. Let also $F: D \to M'$ be a CR mapping of class C^1 from an open connected subset D of M (with $0 \in D$) to a quadric M'* (1.5). *Assume that $F(0) = 0$ and*

$$\sum_{j=1}^{d'} L_j'(dF_0(T_0^c(M))) = T_0^c(M'). \tag{1.6}$$

Then F extends to a complex rational mapping on $\mathbf{C}^n$.

For the comparison with known results recall that a quadric M is said to be *Levi nondegenerate* if $\langle L(z), \bar{\zeta}\rangle = 0$ for all ζ in $\mathbf{C}^k$ implies $z = 0$.

COROLLARY. *Assume that:*

(a) *M is a quadric* (1.2) *and the Levi cone of M has non-empty interior;*
(b) *M' is a Levi nondegenerate quadric* (1.5);
(c) *$F: D \subset M \to M'$ is a CR mapping of class C^1 such that $F(0) = 0$ and dF_0 is a linear isomorphism of $T_0^c(M)$ onto $T_0^c(M')$.*

Then F extends to a complex rational mapping on $\mathbf{C}^n$.

This last result was obtained by Forstnerič [Fo]; the special case when $M = M'$ was considered by Tumanov [Tu], and similar results are in [TH]). When both M and M' are strongly pseudoconvex hyperquadrics in $\mathbf{C}^n$, we obtain the classical theorem of Alexander [Al].

Proof. We have

$$T_0^c(M) = \mathbf{C}^k = \{(z, w) : w = 0\} \quad \text{and} \quad T_0^c(M') = \mathbf{C}^{k'} = \{(z', w') : w' = 0\}.$$

Since dF_0 is an isomorphism, $dF_0(\mathbf{C}^k)=\mathbf{C}^{k'}$. Assume that (1.6) does not hold. Then there is a vector ξ in $\mathbf{C}^{k'}\setminus\{0\}$ orthogonal (for the standard hermitian scalar product on $\mathbf{C}^{k'}$) to $\sum_{j=1}^{d'} L'_j(\mathbf{C}^{k'})$. This implies that $\langle L'_j(\xi),\bar{\eta}\rangle'=\overline{\langle L'_j(\eta),\bar{\xi}\rangle'}=0$ for each η in $\mathbf{C}^{k'}$, $j=1,\dots,d'$. Hence, we obtain a contradiction to the Levi nondegeneracy of M'. Therefore, (1.6) holds and one can apply the Theorem. □

If dF_0 is an isomorphism of $T_0^c(M)$ onto $T_0^c(M')$, the manifolds M and M' have the same CR dimension. We emphasize that in the present theorem the CR dimensions of M and M' can differ. Indeed, (1.6) requires only the restriction $d'k\geq k'$. In particular, the difference $k'-k$ can be arbitrarily large. An equivalent form of condition (1.6) appeared in [De] as "condition H" in connection with another problem. (I am thankful to B. Coupet who brought my attention to this paper.)

Remark. Of course, the condition $F(0)=0$ is not essential for the Theorem. Recall that each quadric is affinely homogeneous. This means that the group of complex linear transformations of $\mathbf{C}^n$, preserving a quadric, acts transitively on this quadric [PS].

2. Tangent Cauchy–Riemann Fields

Recall that a smooth vector field $Y=\sum_{j=1}^{k}\xi_j\partial/\partial z_j+\sum_{r=1}^{d}\omega_r\partial/\partial w_r$ on $\mathbf{C}^n$ is tangent to M if and only if, for each p in M, its value

$$\begin{aligned}Y_p&=\sum_{j=1}^{k}\xi_j(p)\frac{\partial}{\partial z_j}+\sum_{r=1}^{d}\omega_r(p)\frac{\partial}{\partial w_r}\\&\cong(\xi_1(p),\dots,\xi_k(p),\omega_1(p),\dots,\omega_d(p))\end{aligned}$$

at p belongs to $T_p^c(M)$. We call such vector fields *Cauchy–Riemann fields* (or CR fields) on M. (For instance, see details in [Ch].) For M of the form (1.2) we consider CR fields T^q, $q=1,\dots,k$, of the form

$$T^q=\frac{\partial}{\partial z_q}+\sum_{j=1}^{d}\left(\sum_{r=1}^{k}a_{rq}^{j}\bar{z}_r\right)\frac{\partial}{\partial w_j},\tag{2.1}$$

where (a_{rq}^j) is the matrix of L_j. An easy verification shows that vectors T_p^q, $q=1,\dots,k$, generate $T_p^c(M)$ for each p in M.

Complex conjugate vector fields $\bar{T}^q$ also are interesting. A function h of class C^1 on M is said to be a *CR function* if $\bar{T}^q h=0$, $q=1,\dots,k$ on M; these are the *tangential* Cauchy–Riemann equations. Hence, for our mapping $F=(F_1,\dots,F_{n'})\colon D\to M'$ we have

$$T^q\bar{F}_j(z,w)=0,\quad q=1,\dots,k,\ \ j=1,\dots,n',\tag{2.2}$$

where $(z,w)\in D$.

Fix $\epsilon>0$. Let C be an open non-empty convex cone in $\mathbf{R}^d$. We denote by $W(M,C,\epsilon)$ a domain in $\mathbf{C}^n$ of the form

$$\{|(z,w)|<\epsilon : w+\bar{w}-\langle L(z),\bar{z}\rangle \in C\} \tag{2.3}$$

(a "wedge" with "edge" M). According to [BP; Na], F holomorphically continues to some wedge (2.3) under the condition that the Levi cone of M has non-empty interior in $\mathbf{R}^d$. Hence, we may assume without loss of generality that F is holomorphic on some wedge $W(M,C,\epsilon)$ of the form (2.3), and that F is of class C^1 on $W(M,C,\epsilon)\cup M$, or more precisely on $W(M,C,\epsilon)\cup D$.

3. Condition (1.6)

Let $F=(f,g)$, where $f=(f_1,\ldots,f_{k'})=(F_1,\ldots,F_{k'})$ and $g=(g_1,\ldots,g_{d'})=(F_{k'+1},\ldots,F_{n'})$. Since F is CR, the restriction $dF_0\,|\,T_0^c(M)$ is a $\mathbf{C}$-linear mapping. This follows directly from (2.1) and (2.2). Hence $dF_0(\mathbf{C}^k)\subset\mathbf{C}^{k'}$. Therefore,

$$\frac{\partial g_j}{\partial z_q}(0)=0, \quad j=1,\ldots,d', \; q=1,\ldots,k. \tag{3.1}$$

Since $F(D)\subset M'$, for (z,w) in D we have

$$\langle L_j'(f),\bar{f}\rangle' = g_j+\bar{g}_j, \quad j=1,\ldots,d'. \tag{3.2}$$

Acting on both sides of (3.2) with tangent operators T^q, in view of (2.2) we obtain, for (z,w) in D,

$$\langle L_j'(T^q f),\bar{f}\rangle' = T^q g_j, \quad q=1,\ldots,k, \; j=1,\ldots,d',$$

where $T^q f=(T^q f_1,\ldots,T^q f_{k'})$. Then (2.1) and (3.1) imply that

$$L_j'(T^q f)(0)=L_j'(dF_0(e_q)), \tag{3.3}$$

where e_q, $q=1,\ldots,k$, is a standard basis of $\mathbf{C}^k$. Condition (1.6) implies that the rank of the system of vectors $L_j'(dF_0(e_q))$ ($j=1,\ldots,d'$, $q=1,\ldots,k$) is equal to k'. Hence there are couples $(j(s),q(s))$, $s=1,\ldots,k'$, with $1\le j(s)\le d'$ and $1\le q(s)\le k$ such that the vectors (3.3) are linearly independent for $(j,q)=(j(s),q(s))$, $s=1,\ldots,k'$.

For (z,w) in D we have

$$\begin{gathered}\langle L_j'(T^q f),\bar{f}\rangle' = T^q g_j;\\ (j,q)=(j(s),q(s)), \quad s=1,\ldots,k';\\ \bar{g}_j-\langle L_j'(f),\bar{f}\rangle' = -g_j, \quad j=1,\ldots,d'.\end{gathered} \tag{3.4}$$

Consider (3.4) as a system of linear equations for $\bar{F}=(\bar{f},\bar{g})$. Then its determinant, evaluated for $(z,w)=0$, differs from 0. Hence Cramer's rule implies that

$$\bar{F}_j(z,w)=R_j(\bar{z},z,w), \quad j=1,\ldots,n', \tag{3.5}$$

for (z,w) in D. Here each R_j is a (real) analytic function on $W(M,C,\epsilon)$ and is continuous on $W(M,C,\epsilon)\cup D$. Moreover, each R_j is a rational function in $(\bar{z}_1, \dots, \bar{z}_k)$, whose coefficients are holomorphic on $W(M, C, \epsilon)$ and continuous on $W(M, C, \epsilon) \cup D$.

4. Local Holomorphic Continuation

The following proposition is a simple version of the reflection principle.

CLAIM. *The mapping F holomorphically continues to a full neighborhood Ω of 0 in $\mathbf{C}^n$.*

For $d=1$, this is a consequence of the result in [De]. In the general case this claim can also be derived from Derridj's theorem by application of some special version of the "edge-of-the-wedge theorem" due to Airapetian and Henkin [AH]. We give an elementary direct proof.

Proof. Consider a real analytic, totally real, n-dimensional submanifold M^* of M defined by the equations

$$w+\bar{w} = \langle L(z), \bar{z} \rangle, \quad z+\bar{z} = 0.$$

Then M^* can be represented in the form $\bar{z} = -z$, $\bar{w} = -w + \langle L(z), z \rangle$. Let $G_j(z, w) = R_j(-z, z, w)$. Then $G = (G_1, \dots, G_{n'})$ is a holomorphic mapping on $W(M, C, \epsilon)$ and G is continuous on $W(M, C, \epsilon) \cup D$. Hence (3.5) implies

$$\bar{F}(z, w) = G(z, w) \tag{4.1}$$

for (z, w) in $M^* \cap \Omega$. Here Ω is some neighborhood of 0 in $\mathbf{C}^n$. Since M^* is a real analytic totally real n-dimensional manifold, there is some neighborhood $\tilde{\Omega} \ni 0$ in $\mathbf{C}^n$ and a biholomorphism $\Psi : \Omega \to \tilde{\Omega}$ with $\Psi(0) = 0$ such that $\Psi(M^* \cap \Omega) = \mathbf{R}^n \cap \tilde{\Omega}$ (see for instance [W1; W3]). Evidently, the image $\Psi(W(M, C, \epsilon))$ contains some wedge of the form $(\mathbf{R}^n + i\tilde{C}) \cap \tilde{\Omega}$ where $\tilde{C}$ is an open non-empty cone in $\mathbf{R}^n$. The mappings $F(\Psi^{-1}(x))$ and $\overline{G(\Psi^{-1}(\bar{x}))}$ are holomorphic for x in $(\mathbf{R}^n + i\tilde{C}) \cap \tilde{\Omega}$ and x in $(\mathbf{R}^n - i\tilde{C}) \cap \tilde{\Omega}$, respectively. Both mappings are also continuous on $\mathbf{R}^n \cap \tilde{\Omega}$ and their values coincide on $\mathbf{R}^n \cap \tilde{\Omega}$ in view of (4.1). By the usual "edge-of-the-wedge theorem" [R1], $F \circ \Psi$ (and, certainly, F) holomorphically continues to some neighborhood of 0 in $\mathbf{C}^n$. □

REMARK. Of course, this claim holds when M and M' are real analytic manifolds of the form (1.2) and (1.5), respectively, with the third-degree terms. The proof is the same with evident modifications; one can use the implicit function theorem instead of Cramer's rule. In such a form this claim generalizes well-known results [Le; P1; W1; W3] on the analytic continuation of holomorphic mappings.

5. Segre Surfaces and Partial Rationality

Fix a point (ζ, ω) in $\mathbf{C}^n$, where $\zeta \in \mathbf{C}^k$ and $\omega \in \mathbf{C}^d$, and define an affine linear subspace of $\mathbf{C}^n$ of the form

$$Q(\zeta, \omega) = \{(z, w) \in \mathbf{C}^n : w + \bar{\omega} = \langle L(z), \bar{\zeta} \rangle\}.$$

The subspace is said to be the *Segre surface* of M corresponding to (ζ, ω). (See [Se] and [W2].)

The crucial point of our proof is the following lemma.

LEMMA. *For each (ζ, ω) belonging to some neighborhood of 0 in $\mathbf{C}^n$, the restriction $F \mid Q(\zeta, \omega) = F(z, \langle L(z), \bar{\zeta} \rangle - \bar{\omega})$ extends to a rational mapping in z.*

Proof. (1.2) and (3.5) imply that

$$F_j(z, \langle L(z), \bar{z} \rangle - \bar{w}) = \tilde{R}_j(z, \bar{z}, \bar{w}) \tag{5.1}$$

for (z, w) in $M \cap \Omega$. Here $\tilde{R}_j(z, \bar{z}, \bar{w}) = \overline{R_j(\bar{z}, z, w)}$. Hereafter we omit the tilde in this notation.

Since all functions in (5.1) are analytic, functions $F_j(\bar{\xi}, \langle L(\bar{\xi}), \bar{\zeta} \rangle - \bar{\omega})$ and $R_j(\bar{\xi}, \bar{\zeta}, \bar{\omega})$ are anti-holomorphic in some neighborhood of 0 in $\mathbf{C}^{n+k}$, where we take ξ in $\mathbf{C}^k$. Then (5.1) implies that these functions coincide on the manifold

$$\hat{M} = \{(\xi, \zeta, \omega) \in \mathbf{C}^k \times \mathbf{C}^k \times \mathbf{C}^d : \bar{\xi} = \zeta, \ (\zeta, \omega) \in M\}.$$

Evidently, $\hat{M}$ is generic in some neighborhood of 0 in $\mathbf{C}^{n+k}$. Hence the uniqueness theorem of [P2] or [R2] implies that

$$F_j(\bar{\xi}, \langle L(\bar{\xi}), \bar{\zeta} \rangle - \bar{\omega}) = R_j(\bar{\xi}, \bar{\zeta}, \bar{\omega})$$

in some neighborhood of 0 in $\mathbf{C}^{n+k}$. Therefore

$$F_j(z, \langle L(z), \bar{\zeta} \rangle - \bar{\omega}) = R_j(z, \bar{\zeta}, \bar{\omega})$$

for every fixed (ζ, ω). But each $R_j(z, \bar{\zeta}, \bar{\omega})$ is a rational function in z. □

6. Completion of the Proof

Using the Lemma, we reduce our proof to the classical theorem on the separate rationality [BM]. This idea together with the reflection principle has been used by several authors (see e.g. [P1] and [Fo].) Below we reproduce exactly the arguments of Forstnerič [Fo].

From the Lemma, choose a neighborhood $\Omega \ni 0$ of the form $\Omega = U \times V$, where $U \subset \mathbf{C}^k$ and $V \subset \mathbf{C}^d$. Fix ζ^* in U. Then Segre surfaces $Q(\zeta^*, \omega)$ for ω in V form a family of parallel complex k-dimensional affine subspaces. These planes fill some full neighborhood of 0 in $\mathbf{C}^n$. Let t in $Q(\zeta^*, 0)$ be an arbitrary complex line through 0. Then for each p in $\mathbf{C}^n$ in some neighborhood of 0, the line $(p+t)$ lies in some $Q(\zeta^*, \omega)$. According to the Lemma, the restriction $F \mid (p+t)$ extends to a complex rational mapping on $(p+t)$ for each line $(p+t)$.

Consider a basis $t_\nu(\bar{\zeta})$, $\nu = 1, \ldots, k$, of the complex linear subspace $Q(\zeta, 0)$, where

$$\begin{aligned} t_\nu(\bar{\zeta}) &= (e_\nu \mid \langle L(e_\nu), \bar{\zeta}\rangle) \\ &= (0, \ldots, 1, \ldots, 0, \langle L_1(e_\nu), \bar{\zeta}\rangle, \ldots, \langle L_d(e_\nu), \bar{\zeta}\rangle). \end{aligned}$$

Here 1 is in the νth position and e_ν, $\nu = 1, \ldots, k$, is a standard basis of $\mathbf{C}^k$. We shall show that the vectors $t_\nu(\bar{\zeta})$, for $\nu = 1, \ldots, k$ and ζ in U, span $\mathbf{C}^n$.

Assume that this is not true. Then there exists some α in $\mathbf{C}^n \setminus \{0\}$ such that, for all ζ and ν,

$$\begin{aligned} \langle \alpha, t_\nu(\bar{\zeta})\rangle &= \alpha_\nu + \sum_{j=1}^{d} \langle L_j(e_\nu), \bar{\zeta}\rangle \alpha_{k+j} \\ &= \alpha_\nu + \left\langle \sum_{j=1}^{d} \alpha_{k+j} L_j(e_\nu), \bar{\zeta} \right\rangle = 0. \end{aligned}$$

Since ζ in U is arbitrary, we obtain that $\alpha_\nu = 0$ and $\sum_{j=1}^{d} \alpha_{k+j} L_j(e_\nu) = 0$ for $\nu = 1, \ldots, k$. Hence the operators L_j, $j = 1, \ldots, d$, are linearly dependent. This contradicts the condition that the Levi cone of M has non-empty interior.

Thus, there exist linearly independent complex lines $t^1, \ldots, t^n$ containing 0 and such that each restriction $F \mid (p + t^j)$ extends to a complex rational mapping on all of $(p + t^j)$ for every p in some neighborhood of 0. Some nondegenerate $\mathbf{C}$-linear transformation of coordinates maps the t^j, $j = 1, \ldots, n$, onto coordinate lines. Then, by the theorem on the separate rationality, F extends to a complex rational mapping on all $\mathbf{C}^n$. This completes the proof. $\square$

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