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WALTER BURLEIGH'S HYPOTHETICAL SYLLOGISTIC

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Having stated and clarified the general rules or matatheorems of consequences (TB, pp. 1-20; TL, pp. 60-66), Burleigh proceeds to discuss hypothetical syllogisms and the special rules governing them.. By a hypothetical syllogism he means any deductive argument which utilizes one or more explicitly or implicitly hypothetical propositions; and by the latter he has in mind any compound proposition; that is, not merely those of "if ... then" form, but also those which are constructed by means of syncategoremata such as 'and', 'or', 'because', 'while', 'only', 'except', 'inasmuch as', and many others, whether they have a special name or not (Cf. TL, p. 131). It is the purpose of this paper (a) to give an exposition of Burleigh's treatment of those hypothetical syllogisms which involve explicitly hypothetical propositions—that is, conditional, disjunctive, and copulative (or conjunctive) propositions; (b) to examine the relation of special rules governing these syllogisms to the general rules of propositional logic; and (c) to point out the most obvious indebtedness of Burleigh to Boethius whose writings so greatly influenced the medieval logical tradition.

Implications and Generalized Conditionals

Burleigh recognized two sorts of conditional propositions. One sort had the form 'If *A* then *B*' ('*A*' and '*B*' are *his* propositional variables, although they are not replacable by propositions but by *names* of such propositions), and these are rendered in modern notation by *Cpq*. It must be understood that the interpretation of these conditionals—nor of any other molecular propositions—is not necessarily, and possibly never, truth-functional. Some authors on ancient and medieval logic try to remedy the situation by inventing new notation; Karl Dürr (PLB), for example, uses the lower-case letter '*c*' instead of the usual upper-case '*C*' and lets it represent ambiguously the material and the strict implication; Ernest Moody (TCML) introduces the sign '⥽' to distinguish 'simple' from 'as of now' consequences. Such devices seem to me very important in discussing the nature of the various types of *consequentiae*, but they would not be of any special value to the purposes of the present paper where hypothetical syllogistic modes are expressed in terms of theses in which the *asserted* re-

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lation will be in any case logical and would remain such whether the unasserted, or subordinate, or 'material' occurrences of *Cpq* be interpreted as $p \dashv q$ or as *NKpNq*, whether the Diodoran or Philonian implication be meant. It may be remarked, however, that it is very likely that Burleigh, even when he appears to give a truth-functional interpretation of some molecular propositions, would not read truth tables--if made explicit on the basis of his statement of truth conditions for such propositions--in both directions; to borrow the phrases from Reichenbach (NSAO, p. 3), his interpretation of truth tables would more likely be connective rather than adjunctive.

Characterization of the *Cpq*-type propositions is found in the TB, p. 1 and in the TL, pp. 60 f. The so-called 'purely hypothetical syllogisms' consist of three such conditionals; some mixed conditional syllogisms also employ one *Cpq* proposition in addition to a categorical one; but other mixed conditional syllogisms rest on a second sort of proposition, namely, on the generalized conditional.

Generalized conditionals have the form 'If anything is A, then it is B' or 'If anything is an A, then it is a B'. Here, 'A' and 'B' are no longer propositional but predictae or class variables, and the conditionals of this sort may be rendered as $\Pi x C\phi x\psi x$ and $(x)(x\epsilon\alpha \supset x\epsilon\beta)$ respectively. Burleigh does not treat of these in either of the preserved tracts entitled 'On the general rules of consequences', but introduces them in his discussion of the mixed conditional syllogism (TL, p. 104).

Purely Hypothetical Syllogisms

Dependence of Burleigh on the writings of Boethius in this subject is not negligible; not is there any attempt of concealing it, for not fewer than seven explicit references are made to this ancient authority in the comparatively short tract of forty-three pages devoted to the explicitly hypothetical syllogisms. Like Boethius, Burleigh recognized three figures of such syllogisms, but the latter is more faithful than the former to the foundation for classifying such syllogisms into three figures, namely, the analogy of their form to that of categorical syllogisms. The first figure coincides--or nearly so--in the two authors. I say 'nearly so' because Burleigh prefers to place the major premiss first and the minor second, whereas Boethius reversed this order. Figure I may be represented by the thesis *CKCqrCpqCpr*, resembling the Aristotelian syllogistic schema in the arrangement of variables:

$$\begin{array}{ll} M - P & q - r \\ S - M & p - q \\ S - P & p - r \end{array}$$

In regard to the second and third figures of purely hypothetical syllogism, Burleigh's view coincides with that of Theophrastus (apart from the interchange of premisses and the interpretation of letters '*A*', '*B*', and '*C*', which in Theophrastus serve as predicate variables -- Cf. AFL, p. 75, HFL, pp. 103ff.; but cf. PLB, pp. 7f. for a different view) rather than Boethius:

corresponding to Fig. II of Boethius we have the thesis *CKCpqCNprCNqr*, and corresponding to Fig. III the thesis *CKCpqCrNqCrNp*; whereas the same two figures in Burleigh may be represented by *CKCpqCrNqCrNp* and *CKCpqCNprCNrp* respectively; that is, Figure II of Burleigh is almost identical with Figure III of Boethius, and Figure III almost identical with Figure II of Boethius (only the premisses in the latter's are interchanged). Let us compare the texts:

Boethius: "... primae figurae primus modus...si est A, est B, et si est B., necesse est esse C; tunc enim si est A, etiam C esse necesse est." (GLA I, p. 710, n. 159; cf. also DSH, p. 856 B, where the capital letters 'A', 'B', 'C' are replaced by lower case letters '*a*', '*b*', '*c*' in italics.)

'Sit igitur primus modus secundae figurae ... si est A, est B, si autem non est A, est C; dico quoniam si non est B, est C." (GLA I, p. 712, n. 160; DSH, p. 859 D).

"nunc igitur de tertia figura dicendum est,... primus modus ... si est B, est A, si est C, non est A ... quoniam si est B, non esse C necesse est ..." (GLA,I, p. 714, n. 161; DSH, p. 864 B).

Boethius also recognized indirect conclusions which consist of transposed direct conclusions.

Burleigh: "... est syllogismus in prima figura, quando illud, quod est antecedens in maiore, est consequens in minore." (TL, p. 88).

"... in secunda figura medius terminus est ... consequens in utraque praemissarum" (TL, p. 93).

"medium in hac [tertia] figura est antecedens affirmatum in una praemissarum et negatum in reliqua praemissarum" (TL, p. 98).

It may be noted at this point that Burleigh distinguished between 'affirmative' and 'negative' hypothetical syllogisms, between 'direct' and 'indirect' conclusions, and between the 'perfect' and 'imperfect' hypothetical syllogistic figures. He also recognized the possibility of reduction, by means of certain general rules and the rules derived from them, of imperfect figures to the perfect, this reduction being 'direct' or 'indirect'. We see again how closely Burleigh wished to pattern his hypothetical syllogistic after the categorical syllogistic of Aristotle. The base of the system is quite clearly distinguished from the derivative part of it. It is true that the aim of his logic is to point out and to elucidate the canons of deductive inferences rather than to collect all the principles the statement of which is tautological (cf. L & L, p. 235); more than any other medieval logician Burleigh departed in this regard from Aristotle: his approach is metalogical. As Bochenski remarks, "in the *De puritate artis logicae* of Burleigh not a single variable of the object language is to be found" (HFL, p. 152). Yet since his rules are concerned with the form of statements and arguments rather than with their thought contents, *it is convenient to express* *them as theses and thus exhibit rather than describe the logical structures under consideration.* By so doing we can see more clearly that Burleigh, in spite of his neglect of the categorical syllogistic (Cf. SB), was nevertheless an Aristotelian at heart.

Special Rules of Consequences (applying to H.S.)

Just as in the categorical syllogistic we have general rules of structure, quantity, quality and existential import and special rules, *derived* from the general ones (cf. FAR, pp. 71f.), applying to different figures, so in the propositional logic of Burleigh, general rules of *consequentiae* are distinguished from, and seen to give a rise to, the special rules governing various hypothetical syllogistic figures. For the sake of convenience, the theses analogous to these special rules will be gathered from the text and stated at this point. Since the numbers preceding the general rules discussed in my earlier paper, 'A Study in Burleigh: *Tractatus de regulis generalibus consequentiarum*' (NJFL, Vol. III, no. 1) ranged from 1.00 to 8.00, the numbers preceding the theses of the special rules will begin with 9.10.

The special rules (or the theses analogous to them) listed below are derivable, according to Burleigh, from two general ones:

[2.00] CCpqCCqrCpr
[2.10] CCpqCCrpCrq.

Yet judging on the basis of some passages in which he himself attempts to make a sample derivation it is clear that at least one other general rule is to be added, namely the one exhibited by

[3.00] CCpqCNqNp

Derived Rules For Figure I (of the pure H. S.)

(a) Affirmative Modes
Rule [2.00] or rule [2.10] is to be applied directly

(b) Negative Modes
Invalid.

Derived Rules For Figure II

(a) Affirmative Modes

9.20 CCpqCCrNqCrNp

Quidquid antecedit ad oppositum consequentis, illud idem ante- ad oppositum antecedentis (Whatever implies the opposite of the consequent implies the opposite of the antecedent). (TL, p. 94). Derived from 2.10.

Nota: The last word in the sentence expressing the rule in the text reads 'consequentis', but the text is obviously corrupt at this point; if it were not, we would have a thesis

CCpqCCrNqCrNq which is certainly valid, but irrelevant in the context.

(b) Negative Modes

9.21 CCpqCNCrqNCrp

Quod non antecedit ad consequens non antecedit ad antecedens (Whatever does not imply the consequent does not imply the antecedent). (TL, p. 96).
Derived from [2.00]

Derived Rules For Figure III

(a) Affirmative Modes

9.30 CCpqCCNprCNqr

Quidquid sequitur ad oppositum antecedentis, illud idem sequitur ad oppositum consequentis (Whatever follows from the opposite of the antecedent follows from the opposite of the consequent). (TL, p. 99).
Derived from [2.00]

(b) Negative Modes

9.31 CCpqCNCprNCqr

Quod non sequitur ad antecedens, non sequitur ad consequens (Whatever is not implied by the antecedent is not implied by the consequent). (TL, p. 102)
Derived from [2.10].

As it turns out, 9.20 happens to be the same as [7.00] which was listed in the above mentioned paper on the general rules of consequences; it is interesting to note that Burleigh derived theses [7.00] from [3.00], rather than from [2.10]. Similarly, 9.30 is derivable according to the tract on general rules, from [3.00]. But if we disregard this fact and observe that [2.00], [2.10], and [3.00] alone are sufficient, we can better appreciate Burleigh's conception of axiomatization of the hypothetical syllogistic.

As a concession to the topical tradition,* Burleigh speaks in connection with mixed conditional syllogisms of maxims rather than simply of the rules. Two maxims govern such syllogisms:

9.40 CKCpqpq

Posito antecedente ponitur et consequens (If the antecedent is posited, the consequent is to be posited). (TL, p. 104).

9.41 *Destructo consequente destruitur antecedens* (If the consequent is sublated, the antecedent is to be sublated). (TL, p. 104).

*Cf. three interesting and important papers on the subject by O. Bird: "Topic and Consequence in Ockham's Logic", *NDJFL*, Vol. II. (65-78); "The Formalizing of the Topics in Mediaeval Logic", *NDJFL*, Vol. I (138-149); and "The Rediscovery of Topics", *MIND* 70(1961), 534-539.

Structure and Modes of the First Figure

"Conditional syllogism is in the first figure whenever that which is the antecedent in the major is consequent in the minor premiss." (TL, p. 88). For instance:

If a man is running, an animal is running.
If Socrates is running, a man is running.
Therefore, if Socrates is running, an animal is running.

We are arguing here, Burleigh says either, according to the general rule: 'Whatever implies the antecedent implies the consequent' ($\vdash CCpqCCrpCrq$), or according to the general rule: 'Whatever follows from the consequent follows from the antecedent' ($\vdash CCpqCCqrCpr$), depending on whether we look from the point of view of the major or of the minor premiss.

Conditional sorites or *a primo ad ultimum* argument is also reducible to the first figure. If we have, for instance, four conditional propositions constituting a sorites, we can analyze the chain into two purely hypothetical syllogisms. For such an argument to be valid it is indispensable that the consequent of the preceding conditional and the antecedent of the subsequent one are exactly the same.

There are eight affirmative modes with direct conclusion in the first figure: four in which the antecedent of the minor premiss is affirmative and four in which the antecedent of the minor is negative. Burleigh presents them schematically: 'Si A est, B est; si B est, C est; ergo si A est, C est', and similarly for the remaining modes. The following theses, then, may be given:

11.1 $CKCpqCqrCpr$

11.2 $CKCpqCqNrCpNr$

11.3 $CKCpNqCNqrCpr$

11.4 $CKCpNqCNqNrCpNr$

11.5 $CKCNpqCqrCNpr$

11.6 $CKCNpqCqNrCNpNr$

11.7 $CKCNpNqCNqrCNpr$

11.8 $CKCNpNqCNqNrCNpNr$

In his explicit statement of the modes in the first figure Burleigh follows Boethius in that he places the minor premiss first; but he adds that "if the second proposition were placed first and the first after it, the syllogism would be even more evident, because then it would more clearly appear that the antecedent of the major is the consequent in the minor" (TL, p. 91). The first mode, then, would have the form: $CKCqrCpqCpr$; the second: $CKCqNrCpqCpNr$; etc. In stating the modes of the remaining figures, he actually follows this 'more evident' way.

If we compare the statement of the first mode of Burleigh with that of

Boethius ('If A is, B is, and if B is, C must be; but then: if A is, C must be'—cf. GLA I, p. 710, n. 159) it is difficult to account for the fact that K. Dürr (PLB, "The Fourth Class of Inference Schemes, p. 44) as well as I. M. Bocheński in AFL, p. 107, but not in the HFL, p. 139, in a verbal form, import the antecedent of the conditional conclusion into the premiss-set; the former writer renders the first mode as *igitur et et cpqcqrpr*; the latter as $p \supset q \cdot q \supset r \cdot p \cdot\supset\cdot r$. Now considering that '*igitur*' has roughly the function of our asserted relation of implication, '*et*' the function of our '*K*' and '*c*' the function of our '*C*' it is clear that the conclusion is not conditional. It may be that Dürr followed the translation of Prantl: 'Wenn A ist, ist B, und wenn B ist, ist C. A ist./C ist' (GLA, p. 711). We should add that Bocheński, in stating two modes of Boethius' second and two of the third figure does express the conclusion as a conditional, thus giving a genuine 'totally hypothetical syllogism' rather than an (valid) analogue of it (AFL, 107).

The eight modes given above are the only explicitly stated valid modes in the first figure. Both premisses, Burleigh says, have to be affirmative. If one or both premisses were negated, no conclusion would follow: if both, no conclusion could be drawn because nothing follows from two negative premisses; if only the major premiss were negative, no conclusions would follow because we could easily point to a substitution instance which is patently invalid, consisting of true premisses and a false conclusion: 'It is not the case that if a man is running, donkey is running; if every animal is running, man is running; therefore, it is not the case that if every animal is running, donkey is running' (TL, p. 92). Syllogisms represented by *CKNCqrCpqNCpr*, then, are rejected as invalid. For they presuppose an invalid rule that if something does not follow from the consequent it does not follow from the antecedent ($\dashv$*CCpqCNCqrNCpr*). Nor is a hypothetical syllogism valid if the conditional major premiss is affirmed and the minor negated. Burleigh gives the following argument with true premisses and false conclusion: 'If a man is running, an animal is running; but it is not the case that if a donkey is running a man is running; therefore, it is not the case that if a donkey is running, an animal is running' (TL, p. 92). Thus syllogisms of the form *CKCqrNCpqNCpr* are rejected as invalid, for they are seen to presuppose an invalid rule: 'What does not imply the antecedent does not imply the consequent' ($\dashv$*CCqrCNCpqNCpr*). We do have two valid rules which resemble these last two mentioned, namely, 'What does not follow from the antecedent does not follow from the consequent' ($\vdash$*CCpqCNCprNCqr*) and 'What does not imply the antecedent does not imply the consequent' ($\vdash$*CCpqCNCrpNCrq*) (TL, p. 92) which are directly derivable from [2.00] and [2.10] respectively by the application of rule [3.00] to the conditional consequents *CCqrCpr* and *CCrpCrq*. But they have no significant role to play in the derivation of valid conclusions in the first figure. One could of course always employ the *reductio ad absurdum* method, proving for instance, that from one of the premisses and the contradictory of the conclusion the contradictory of the remaining premiss follows if the original argument was valid. But this would merely amount to applying to conditional syllogism a rule which Aristotle already had applied to categorical syllogisms, viz. $\vdash CCKpqrCNrANpNq$ (cf. SB, [3.50]–[3.63]); no new rules are required for that method of testing.

Conditional syllogisms which consist only of affirmative propositions, that is, of conditionals in which the *nota conditionis* is not itself negated, are labeled by Burleigh 'affirmative syllogisms'; as opposed to those that do contain a negative premiss, which are 'negative syllogisms'. All the modes in the first figure must be of the former class, or else they are invalid.

In his discussion of the first figure Burleigh does not even mention 'indirect' modes, that, is, those in which the conclusion is transposed. That he did not recognize such modes would be very unlikely in view of the fact that he knew Boethius who certainly recognized them or at least analogues of them (AFL, p. 107: thesis $*18.32 - p \supset q \cdot q \supset r \cdot - r \cdot \supset . - p$; PLB, the fifth and the sixth class of inference schemes). But Burleigh did recognize 'indirect' modes, since he speaks of, and even states, indirect conclusions in the third figure. Does he admit such modes in the first figure? Yes! We find a passage in his chapter on the third figure: "Et est sciendum, *quod in qualibet trium figurarum* potest sequi duplex conclusio, una directa... Potest etiam concludi conclusio indirecta, si una conditionalis in qua ex opposito conclusionis directae infertur oppositum antecedentis eiusdem conclusionis" (TL, pp. 101f.; italics mine). We could therefore add to our list eight additional theses corresponding to the indirect modes, but two examples will suffice:

11.9 $CKCpqCqrCNrNp$

11.13 $CKCNpqCqrCNrp$

Structure and Modes of the Second Figure

A mode is in the second figure if the middle term is a proposition affirmative in one premiss and negative in the other, occurring as consequent in both: 'Si A est, B est; si C est, B non est; ergo si C est, A non est' (TL, p. 95). No conclusion can be drawn if the middle "term" is of the same quality in both premisses; else we would argue according to the invalid rule: 'Whatever implies the consequent implies the antecedent' ($\dashv CCqrCCprCpq$) (TL, p. 93). Burleigh voices his disagreement with Boethius' contention that the middle "term" of a hypothetical syllogism in the second figure is either the subject or the predicate term of the consequent: "it seems to me more plausible that the whole proposition, affirmative in one and negative in the other, ought to be held as the middle term, however much Boethius seems to be saying to the contrary" (TL, p. 94). There is no doubt that the logic of Burleigh is a propositional logic, whereas there might be some dispute concerning Boethius' hypothetical syllogistic; judging from the latter's view of the middle "term", he may have had in mind theses such as $\Pi x CKC\phi x\psi xC\theta xN\psi xC\theta xN\phi x$, or Camestres in its standard form $CKAabEcbEca$. Burleigh also notes that his second figure was considered by Boethius as the third (TL, p. 98).

The discussion of modes in Figure II that follows is divided into two parts: the first part deals with the eight affirmative, the second with the eight negative modes. If the modes of the first figure may be said to be in the *ponendo ponens* mood, those of the second figure are in the *tollendo tollens* mood; that is, all the variants may be seen to be corollaries of the thesis [2.00] (or its analogue), derived by the principle [3.00]. But Burleigh explains the same fact in a different although equivalent way. A valid affirmative mode in this figure, he says, must conform to the rule depicted by 9.20 (*CCpqCCrNqCrNp*), and this rule is derivable from [2.10]: "For if we have a valid consequence (*Cpq*), then the contradictory of the consequent follows from the contradictory of the antecedent (*CNqNp*) and, as a result, the opposite of the consequent is the antecedent to the opposite of the antecedent; thus, whatever implies the contradictory of the consequent (*CrNq*) implies the contradictory of the antecedent (*CrNp*)" (TL, p. 94). Modes of the second syllogistic figure do conform to this rule since the antecedent of the minor implies the contradictory of the consequent of the major, consequently, the antecedent of the minor implies the contradictory of the antecedent of the major. The direct conclusion of this figure, then, is a conditional in which the antecedent of the minor premiss implies the contradictory of the antecedent of the major premiss.

The reduction of any affirmative mode of the second figure to a mode of the first and consequently to [2.00] or [2.10] could be shown in still another way: let us preserve the original schematic form and place the premisses and the conclusion of the first mode in columns:

Major:	*Cpq* = *CNqNp* by [3.00]
Minor:	*CrNq*
Conclusion:	*CrNp*

The transposed major premiss together with the unchanged minor and the conclusion constitute precisely 11.4 of the first figure—if we transpose its conclusion and thus automatically interchange the order of the premisses so that the first one becomes the major and the second one the minor; for Burleigh, we noted, followed 'the more evident way' of placing the major premiss first only in stating the modes of the second and third figures.

Four of the eight affirmative modes of the second figure are based on the propositional from *KCpqCrNq*, that is, on the premiss-set in which the consequent of the major is affirmative:

12.1 CKCpqCrNqCrNp

12.2 CKCpqCNrNqCNrNp

12.3 CKCNpqCrNqCrp

12.4 CKCNpqCNrNqCNrp

The remaining four affirmative modes are based on the propositional form *KCpNqCrq*, that is, on the premiss-set in which the consequent of the major is negative:

12.5 CKCpNqCrqCrNp

12.6 CKCpNqCNrqCNrNp

12.7 CKCNpNqCrqCrp

12.8 CKCNpNqCNrqCNrp

In accord with the instructions given elsewhere (TL, p. 101f.), eight more theses corresponding to the indirect modes could be listed; let us state only two by way of example:

12.9 CKCpqCrNqCpNr

12.13 CKCpNqCrqCpNr

Some negative modes in the second figure are also valid. They are characterized by the fact that one of the premisses is negative; obviously, the conclusion itself has to be negation of a conditional. If the *minor* premiss is negated, the conclusion follows directly. For instance, 'If A is, B is; it is not the case that if C is, B is; therefore it is not the case that if C is, A is' (TL, p. 96). The inference-schema conforms to the rule: 'Whatever does not imply the consequent does not imply the antecedent' ($\vdash$ *CCpqCNCrqNCrp*) which is derived from [2.10]. The conclusion always calls for a denial of the conditional which denotes that the antecedent of the minor implies the antecedent of the major.

If, on the other hand, the major conditional premiss be negated, the conclusion follows only indirectly: it consists of a denial of the conditional denoting that the antecedent of the minor *is implied by* the antecedent of the major. E.g., 'It is not the case that if animal is running, man is running; if Socrates is running, man is running; therefore it is not the case that if animal is running, Socrates is running.' If we should try to conclude directly, we would commit a fallacy; it does not follow, for instance, from the premisses of the above example: 'therefore it is not the case that if Socrates is running, man is running'; for the premisses are true and the conclusion false. We may conclude only indirectly, and by doing so we are in fact merely commuting the premisses, thus making the original major premiss a minor one, and conversely.

The inference-schema with a negative minor premiss (1) and the inference-schema with a negative major premiss (2) are:

(1)	(2)
$\vdash Cpq$	$\vdash NCpq$
$\vdash NCrq$	$\vdash Crq$
$\vdash NCrp$	$\vdash NCpr$

The conclusion in (2) is drawn indirectly. Now if we invert the order of the premisses in (2) we get (3):

(3)

$$\frac{\begin{array}{l}\vdash Crq\\ \vdash NCpq\end{array}}{\vdash NCpr,}$$

and this is an inference-schema identical with (1), which can easily be proved by substituting p for r and r for p. Thus, if the conclusion in (2) be considered as direct, it is such only in relation to the original premisses commuted.

Burleigh does not himself state all the negative modes of the second figure; he leaves this task "industriae cogitantis" (TL, p. 97). But the directions for constructing them he does give. Among the eight thesis-analogues of these modes we have, for example

12.17 $CKCpqNCrqNCrp$

12.21 $CKCpNqNCrNqNCrp$

And if we commuted the order of propositions constituting the antecedents, we would exhibit modes with indirect conclusions; for instance:

12.25 $CKNCpqCrqNCpr$

12.29 $CKNCpNqCrNqNCpr$

The conformity of negative modes to the rule 9.21 is simple enough and explained clearly. But one is tempted to take another approach. Let us examine 12.17:

(1)	Cpq	premisses
(2)	$NCrq$	
		$NCrp$ (conclusion)
(3)	$KrNq$	from (2) by [4.20] (Cf. SB)
(4)	Nq	from (3) by *a fortiori* principle
(5)	Np	from (1) and (4) by 9.41
(6)	r	from (3) by *a fortiori* principle
(7)	$NrNp$	from (6) and (5) by conjunction

Up to this point there is little room for dispute as far as Burleigh is concerned; but we cannot deduce from step (7) the conclusion $NCrp$, unless we assume that we are dealing with implicative statements which in their subordinate positions are material or 'ut nunc'; if stronger implication be meant, and Burleigh certainly recognized 'simple' implication, the conclusion does not follow. In Lewis' system, for example, $KpNq$ does not entail $\sim(p \prec q)$, although it itself *is* entailed by it.

The same remarks are to be made about dealing with the same problem in this way:

(1) $Cpq \longrightarrow CNqNp$

(2) $NCrq \longrightarrow CrNq$

(3) $NCrp \longrightarrow CrNp$

For it is very unlikely that Burleigh would approve the contention that, for instance, if p does not imply q, then q implies p, or that if p does not imply q, then p implies the denial of q, etc.

Structure and Modes of the Third Figure

The third figure is characterized by its middle "term," which is a proposition, once affirmative and once negative, serving as the antecedent in the two premisses. The fundamental propositional form on which the inference-schemata in this figure are based is *KCpqCNpr*. "For just as in categorical syllogisms the middle term is the subject of each premiss in the third figure, so in hypothetical syllogisms the medium consists of the antecedent of both premisses in this figure" (TL, p. 98).

Burleigh thought that no inference-schema could be correlated with *KCpqCpr*, that is, with a propositional form in which both antecedents are of the same quality, for we would presuppose an invalid rule, 'whatever is implied by the antecedent is implied by the consequent'. The following arguments, for example, are invalid: 'If you are everywhere, you are in Rome; if you are everywhere, you are here; therefore, if you are here, you are in Rome'; and: 'If you are not an animal, you are not a man; if you are not an animal, you are not an ass; therefore if you are not an ass, you are not a man' (TL, p. 99). In both, the premisses are true and the conclusion false.

Burleigh either does not know--or else he thinks it too trivial to notice—the law of the multiplication of consequents, viz. *CKCpqCprCpKqr*.

Just as in other figures, so in the third, we must distinguish between affirmative and negative modes, and within each of these between the direct and the indirect ones.

The affirmative syllogisms in this figure hold in virtue of the rule 9.30: 'Whatever is implied by the opposite of the antecedent is implied by the opposite of the consequent' (TL, p. 99), which is derived from [2.00]. For if a consequence, *Cpq*, is valid then, by [3.00] *CNqNp*; since [2.00] says that whatever is implied by the consequent is implied by the antecedent, we derive from the initial supposition *Cpq* another proposition, namely *CCNprCNqr*, which is precisely what 9.30 asserts.

We have eight affirmative direct modes: four in which the antecedent of the major is affirmative, four in which it is negative. An affirmative mode in this figure is direct if the conclusion's antecedent consists of the contradictory of the consequent of the minor premiss and its consequent is the consequent of the major premiss. Examples of the direct modes in the thesis-form are:

13.1 CKCpqCNprCNrq

13.5 CKCNpqCprCNrq

All affirmative modes of this figure, like those of the second, are reducible to the modes of the first figure solely by the use of [3.00]. Take mode 13.1:

(1) *Cpq*

(2) *CNpr* = *CNrp*

(3) *CNrq*

By transposing (2) we get the desired premiss which changes this mode to one of the first figure, namely 11.5.

In addition to affirmative direct modes, Burleigh lists the indirect ones. Third figure is the only one in which he explicitly does this (TL, pp. 100f.) Expressed as theses, the following two examples should suffice:

13.9 CKCpqCNprCNqr

13.13 CKCNpqCprCNqr

All negative syllogistic modes in this figure are warranted by the rule 9.31 which is itself derivable from 2.00 (TL, p. 102). If the minor premiss is negative, the conclusion follows directly: the conclusion states that the consequent of the minor does not imply the consequent of the major; if the major is negative, the conclusion follows only indirectly and it states that the consequent of the major does not imply the consequent of the minor (TL, p. 102). Burleigh notes that Aristotle used this sort of argument to prove that the knowable, or object of knowledge, is prior to the knowledge of it (cf. *Categoriae*, lc, cap. 7; 7b 28ff.). He does not himself state the negative modes, but limits himself to giving an example and only points out that "many modes could be posited just as in the affirmative [mood] as is evident to any intelligent person" (TL, p. 103). Two examples of each, in the thesis form, are given here:

(a) Direct modes

13.17 CKNCpqCprNCrq

13.21 CKNCNpqCNprNCrq

(b) Indirect modes

13.25 CKCpqNCprNCqr

13.29 CKCNpqNCNprNCqr

Mixed Conditional Syllogism

This syllogism may be defined as one whose premiss-set consists of a conditional and a categorical proposition. The latter may be either the antecedent or the contradictory of the consequent of the conditional. If the contradictory of the antecedent, or the consequent be posited, no conclusion can be drawn. We meet here the first two of the Stoics' indemonstrables (AFL, p. 98), and the modes well-known to Boethius (PLB, "The First Class of Inference Schemes"). Burleigh calls the former (positing) type "hypothetical syllogism of the first form', the latter 'hypothetical syllogism of the second form'. Each of the two, he says, has two sub-classes:

(1a) The categorical proposition may be exactly the same as the antecedent of the conditional (major) premiss; for instance: 'If a man is running, an animal is running; but a man is running; therefore an animal is running' (TL, p. 103).

(1b) The categorical proposition may be the contradictory of the consequent; for example: 'If a man is running, an animal is running; but no animal is running; therefore no man is running' (TL, p. 104). The former is warranted by the maxim, 'If the antecedent is posited, the consequent is to be posited'; the latter by the maxim 'If the consequent is sublated, the antecedent is to be sublated' (TL, p. 104).

(2a) The subject of the antecedent of the major premiss may be a transcendental term and the minor a categorical proposition in which the predicate of the antecedent of the major premiss is attributed to the subject of the minor premiss in exactly the same way as to the transcendental subject of the major; for instance, 'If something is a man, it is an animal; Socrates is a man; therefore Socrates is an animal'; or, 'Whatever is a man is an animal; Socrates is a man; therefore Socrates is an animal' (TL, p. 104).

(2b) The minor may be a categorical proposition contradicting the consequent of the major: 'Whatever is a man is an animal; wood is not animal; therefore wood is not man' (TL, p. 104).

The example given for (2a) is not on the same plane as the one given for (2b), not because the former is in positing, and the latter in the sublating mood, but because in the example of (2a) the subject of the categorical proposition is a singular term, a proper name, whereas the subject in (2b) is not. One example has the form $CK\Pi xCMxAxMsAs$, whereas the other example has the form $\Pi xCKCMxAxCWxNAxCWxNMx$ (Celarent modified, rather than a special case of *modus tollendo tollens* for predicates). It is possible, of course, that Burleigh wants us to read the second premiss as a singular proposition, 'This piece of wood is not an animal', in which case we would get a formula parallel to (2a): $CK\Pi xCMxAxNAwNMw$.

While arguments expressed by $CKCpqpq$ and $CKCpqNqNp$ may be said to be of the first and of the second *form*, but not of any standard figure (TL, p. 103), those that employ a generalized conditional may be said to be in the first figure if in positing mood, and in the second figure if in sublating mood: in the former, the predicate term of the antecedent of the major premiss serves as the middle term; in the latter, the predicate term of the consequent has that function. Burleigh does not seem to think that there is a considerable difference between singular and general terms. For there is no *prima facie* analogy between the standard analogue of *Barbara* (involving constants) $CK\Pi xC\phi x\psi x\phi a\psi a$ and $CK\Pi xC\phi x\psi xC\theta x\phi x\Pi xC\theta x\psi x$ (standard Barbara), but there is one between the verbal example in (2a) and any other verbal example of categorical syllogistic: the argument 'If something is a man, it is an animal; Socrates is a man; therefore Socrates is an animal' is not greatly dissimilar from 'Every mammal has a heart; every cat is a mammal; therefore every cat has a heart.' In both we can discern the arrangement of categoremata in the configuration

$$\begin{array}{c} M - P \\ S - M \\ S - P \end{array}$$

Similarly, there is no *prima facie* similarity between the standard analogue, involving constants, of *Baroco*, $CK\Pi xC\phi x\psi xN\psi aN\phi a$ and the *Baroco* itself, $CK\Pi xC\phi x\psi x\Sigma xN\psi x\Sigma xN\phi x$, but there is one if we take verbal examples: 'If something is man, it is animal; this wood is not animal; therefore this wood is not man' and 'Every man is rational; some animal is not rational; therefore some animal is not man'. In either case we detect the form

$$\begin{array}{c} P - M \\ S - M \\ S - P \end{array}$$

When Burleigh states the modes of the mixed conditional syllogism, he seems to have kept in mind only those based on a generalized conditional, for instance: 'Si aliquid est A, illud est B; C est A; ergo C est B' (TL, p. 105); of those based on a Cpq-type of premiss he only gives an instance; yet he refers to modes of the latter type in plural, evidently thinking of inference-schemata analogous to the following theses:

14.1 $CKCpqpq$

14.2 $CKCpNqpNq$

14.3 $CKCNpqNpq$

14.4 $CKCNpNqNpNq$

14.5 $CKCpqNqNp$

14.6 $CKCpNqqNp$

14.7 $CKCNpqNpp$

14.8 $CKCNpNqqp$

14.1 to 14.4 are in the positing, 14.5 to 14.8 in the sublating mood. The eight modes that involve generalized conditionals are:

14.9 $CK\Pi C\phi x\psi x\Sigma x\phi x\Sigma x\psi x$

14.10 $CK\Pi xC\phi xN\psi x\Sigma x\phi x\Sigma xN\psi x$

14.11 $CK\Pi xCN\phi x\psi x\Sigma xN\phi x\Sigma x\psi x$

14.12 $CK\Pi CN\phi xN\psi x\Sigma xN\phi x\Sigma N\psi x$

14.13 $CK\Pi xC\phi x\psi x\Sigma xN\psi x\Sigma xN\phi x$

14.14 $CK\Pi xC\phi xN\psi x\Sigma x\psi x\Sigma xN\phi x$

14.15 $CK\Pi xCN\phi x\psi x\Sigma xN\psi x\Sigma x\phi x$

14.16 $CK\Pi xCN\phi xN\psi x\Sigma x\psi x\Sigma x\phi x$

Burleigh explicitly rejects mixed conditional arguments of this sort if the subject term in the antecedent and in the consequent is not a transcendental term. Yet, what he is rejecting here appears to be not any special (i.e. categorematic) term, but any term which does not have a distributive supposition. For he listed in the TB (p. 4; cf. also TL, p. 202) a rule derived from [2.00] stating that 'in any valid consequence one may descend from the antecedent to any of its [inferiors] with respect to the same consequent'. That is, if we have a conditional whose antecedent has a distributive supposition, we may take any of its inferiors and keep the same consequent, e.g.

If a man is running, an animal is running;
therefore, if John is running, an animal is running.

But the subject of the antecedent must have a distributive supposition, which is to say that the antecedent must be either a particular or an indefinite proposition. ("in conditionali, cuius antecedens est propositio indefinita vel particularis, supponit subjectum respectu consequentis confuse et distributive"—TB, p. 4; TL, p. 202). If the antecedent is a universal proposition, the sequence will not hold: 'If every man is an animal, every man is a body; Socrates is an animal; therefore Socrates is a body' (TL, p. 106), is not a valid argument; it would, of course, be valid, if the minor premiss posited the antecedent of the major *in toto*.

Third Figure; *Reducibility*

In the third figure, no mixed conditional syllogism is admitted as valid. From 'If a man is running, something risible is running; every man is an animal'—no conclusion may be drawn: not a conditional one, 'If an animal is running, something risible is running', for then the premisses would be true and the conclusion false; and not a categorical one, 'Animal is running', because the premisses would be necessary and the conclusion contingent (and this violates Burleigh's general law [1.41] *CKNMNpMNqNMCpq*, or a law related to it, *CKNMNpNMNCpqNMNq*).

Mixed conditional syllogisms of the first form are considered as 'perfect and *per se* evident' (TL, p. 106); those of the second form are 'inevident, yet necessary' (*ibid.*). The necessity of the latter is demonstrated *per impossibile*:

If A is, B is;	1. *Cpq*
but B is not;	2. *Nq*
therefore A is not.	3. *Np*
If it be supposed that the conclusion does not follow, the contradictory of the conclusion is to be posited, namely, 'A is';	4. *p*

therefore these would stand together: 'if A is, B is; and 'B is not'; nevertheless A is.	5. *KKCpqNqp*
Therefore with this one: 'B is not' stands this one: 'A is'.	6. *KNpq*
But it follows: 'A is, therefore B is', as the major premiss says;	7. *Cpq*
and whatever is consistent with the antecedent is consistent with the consequent.	8. *CCpqCKprKqr*
Thus, if with 'A is' stood 'B is not',	9. *KpNq*
then 'B is' would stand with 'B is not'.	10. *KqNq*
And thus the contradictories will stand together, which is impossible (TL, p. 106).	

Hypothetical Propositions Other Than Conditional

Many arguments in ordinary discourse are based on propositions other than conditional, especially on the conjunctive or copulative, and the disjunctive (in the sense of 'either ... or, or both', *Apq*, and 'either ... or, but not both', *Jpq*). Burleigh deals at some length, not so much with syllogisms themselves, but with the definition, truth, possibility, necessity, and contradictoriness of copulative (*Kpq*) and disjunctive (*Apq*) propositions. He also notes that there are in fact many more hypothetical propositions than there are names for them; e.g., 'Master is to the school as captain is to the ship' and 'Socrates is moving where he is running' are hypothetical in character, yet none of them belongs to those hypothetical propositions which are usually treated in medieval texts, and specially named, such as causal, temporal, exclusive, exceptive, and reduplicative.

One division of hypothetical propositions may be made on the basis of the number of categorical propositions constituting them: if only two propositions are conjoined by means of a connective or an adverb, the hypothetical proposition is simple; if more than two, it is complex. Only one connective can be the principal connective (the *formale* or the *dictio principalis*); others occurring in the subordinate positions are taken materially. This distinction enables Burleigh to solve certain sophismata arising from the ambiguities of the natural language. 'Either Socrates or Plato is running and Cicero is disputing' may be considered either as *KApqr* where the conjunction-sign, or as *ApKqr* where the alternation-sign, is the *formale*. The truth conditions for the two are obviously quite different. Similarly, 'Socrates or Plato is running if either of the two is running' is ambiguous. It may be considered as an implication of a disjunction *Apq* by another disjunction *Apq*, or as a disjunction of a conditional *CApqq* with a categorical proposition *p*. In the former case there is no interpretation which would make the statement of such form false; in the latter case there is, in the sense that *CApqq* as well as *p* may be false and that, consequently, their disjunction would be false. It must be added, however, that if we *actually* *make* the required disjunction *ApCApqq* we end up with a tautologous statement. (TL, p. 108).

In spite of the fact that several categorical propositions enter a hypothetical one, Burleigh sides with Aristotle in holding that a hypothetical proposition does possess a unity of its own, and he does so in order to provide for the possibility of there being a single denial to every proposition. This is especially important in the attempt to determine what precisely constitutes a contradictory of an exponible proposition. Burleigh treats exponibles as conjunctions and, consequently, their denials as disjunctions of the negations of components. If an exponible proposition be translatable into *KKpqr,* then its denial will not be *KKNpNqNr* but *AANpNqNr.* 'Only man is running' is translated into 'Man is running and nothing other than man is running'; the contradictory of the original, then, is: 'Either no man is running or something other than man is running' (TL, pp. 109f.). If the conjunction of propositions into which an exponible proposition is resolved exhausts the sense of it, then the denial of one conjunct constitutes the foundation for the truth of the contradictory of the original in the sense that it entails it (e.g. $Np \rightarrow NKpq$); but since the converse is illegitimate (one may not, for instance, argue from *NKpq* to *Np*), the contradictory of an exponible proposition must be a disjunction of denials, just as in the case of ordinary conjunctive propositions (cf. [4.00]).

Copulative Proposition and Syllogisms

Hypothetical proposition "in which two or more categorical propositions are conjoined by means of 'and' or its equivalent in such a way that the connective 'and' is the principal connective" is called by Burleigh 'copulative' proposition (TL, p. 110). Although the term 'conjunctive proposition' had been used to denote a proposition represented by *Dpq* (matrix 0111)(AFL, p. 91), we shall use the term to denote a logical product of two propositions; in symbolic form, *Kpq* (matrix 1000). Burleigh investigates what are the requirements for truth, possibility, impossibility, and for a contradictory of such proposition, and then makes a very brief remark concerning the way of arguing on its basis.

For TRUTH of a conjunction it is required, not that each categorical proposition occurring in it be true, but that each *principal* component be true. *KApqr,* for instance, is true on the assumption that '*p*' is false as well as on the assumption that '*q*' is false; only '*r*' *must* be true, since it occurs as one of the principal components, the other one being *Apq*.

Since it is both necessary and sufficient for the truth of a copulative that each principal component be true, the following theses hold:

15.10 CKpqp

15.11 CKpqq

And from any two non-modal expressions asserted separately, we may infer a conjunction of the two:

15.12 $(p,q) \rightarrow Kpq$

When Burleigh speaks of possibility, necessity, etc., in connection with copulative proposition, he has in mind alethic modalities: what is in question for him is not whether a proposition is verified or falsified,but whether two or more states of affairs are such as to enable us to say that the propositions expressing them *must* be true, *cannot* be true, *could* be true (unilateral possibility), *are true, but could also not be true* (bilateral possibility) in conjunction. And since it is facts that determine the mode of the propositions expressing them, *Mp* appears to be stronger than that of Lewis (i.e. '*p* is logically conceivable'), and *NMp* weaker than that of Lewis (i.e. '*p* is logically inconceivable'). Possibility and necessity in Burleigh seem to have what Lewis calls 'relative' meaning (L & L, p. 161) in that we are to consider propositions in relation to facts. Those writers who distinguish between *de dicto* and *de re* modalities would classify Burleigh's modes as *de re* modes (EML, p. 1 *et passim*; FL, pp. 185ff.). This contention is based on the following observation: when Burleigh introduces the notion of compossibility, he chooses 'Socrates is white and Socrates is black' as example of a proposition which is impossible on the basis of incompossibility of the components; he does not give, in other words, an example of the denial of *KpNp* but of *Kpq*, the incompossibility of which can be determined only on extralogical grounds, that is, by comparing the two propositions with facts.

In order that a conjunction be POSSIBLE it is required that each principal constituent be possible; if one of them is impossible, the whole conjunction is impossible. Consequently, from *MKpq* we may infer *Mp, Mq,* as well as *KMpMq*:

15.20 CMKpqMp

15.21 CMKpqMq

15.22 CMKpqKMpMq

But the converse of 5.22 does not hold, because the possibility of the conjuncts is only a necessary, but not a sufficient condition for the possibility of a conjunction. Like Lewis in modern times, Burleigh, as mentioned above, introduces an idea similar to the idea of consistency, namely that of compossibility: "Sed adhuc ad possibilitatem copulativae non sufficit possibilitas cuiuslibet pa tis, sed cum hoc requiritur, quod omnes partes sint inter se compossibiles" (TL, p. 111). 'Socrates is white' is a possible proposition; 'Socrates is black' is a possible proposition; but their conjunction is not possible. The copulation of '*p*' and '*q*', then, is not a possibility function of their conjunction.

It may be remarked that the converse of 15.22 would hold if it were a question of necessity rather than of possibility, but Burleigh does not treat of this mode in connection with copulative proposition.

For the IMPOSSIBILITY of a conjunction less is required, namely, the impossibility of either of the components. The following laws hold:

15.31 CNMpNMKpq

15.32 CNMqNMKpq

15.33 CANMpNMqNMKpq.

There could, of course, be a third factor making a conjunction impossible, namely, the incompossibility of the components (even if each of them taken separately be possible): thus the converses of 15.31-15.33 do not hold.

In the tract on the general theory of consequences, Burleigh stated the rule that 'from something contingent [and thus from something possible] does not follow something impossible' [1.40] and that 'from something necessary does not follow something contingent' [1.41] and, *a fortiori*, something impossible. But he nevertheless admits that from possible propositions impossible ones may follow if these possible ones are not compatible: for "when premisses are incompossible, the whole antecedent is impossible and from the impossible may well follow the impossible" (TL, p. 111; cf. also [1.20]). So the rule [1.40] still holds, since from possible *and* compossible propositions an impossible one was never admitted to follow.

The CONTRADICTORY of a conjunctive, Burleigh observes, is "a disjunction of the denials of the original components" (TL, p. 113); he had already observed this in his study of general rules of consequences (cf. [4.00]). Since *NKpq* and *ANpNq* imply each other, we have an equivalence relation: *ENKpqANpNq*.

The contention that the negation of a conjunction is another conjunction is emphatically rejected. Burleigh was well aware of the series of statements each of which is sufficient to falsify *Kpq*, namely, *KNpq*, *KpNq*, *KNpNq*, and of the perfect normal form for the conjunction of p and q, namely *Kpq* (TL, p. 113). While *Kpq* and *KpNq* are certainly incompatible since they could not both be true, they are nevertheless not contradictories, since they could both be false; they are in fact contraries. Burleigh demonstrates this by pointing out that on the assumption that Socrates is running and Plato is not, "each of the following is false: 'Socrates is running and Plato is running' " (TL, p. 113).

Only a small paragraph is devoted to arguments based on copulative proposition. If either or both of the premisses are conjunctions, the syllogism may be said to be 'copulative,' although Burleigh does not consider such reasoning to by syllogistic in the strict sense (TL, p. 115). What he probably means is that it is not *hypothetical* syllogistic reasoning, as may be judged from his example:

Every man is running and every ass is sleeping;
Socrates is a man and Brunellus is an ass;
therefore Socrates is running and Brunellus is sleeping.

We do not have here an inference of propositional logic but of the logic of classes,

$\vdash [(x)(x\epsilon\alpha . \supset . x\epsilon\beta) \cdot (x)(x\epsilon\gamma . \supset . x\epsilon\delta)]$

$\vdash (a\epsilon\alpha \cdot b\epsilon\gamma)$

$\vdash (a\epsilon\ \beta \cdot b\epsilon\delta)$

or of the logic of predicates:

$\vdash K\Pi x C\phi x\psi x \Pi x\theta x\chi x$

$\vdash K\phi a\theta b$

$\vdash K\psi a\chi b$

Examples of other forms of copulative syllogism are:

Socrates is running and Plato is running;
Every man is Socrates or Plato;
therefore every man is running.

$\vdash K\phi a\phi b$

$\vdash \Pi x C\psi x A(x=a)(x=b)$

$\vdash \Pi x C\psi x\phi x$

Every man is running and an ass is running;
Socrates is a man;
therefore Socrates is running and an ass is running.

$\vdash K\Pi x C\phi x\psi x\Sigma x K\theta x\psi x$

$\vdash \phi a$

$\vdash K\psi a\Sigma x K\theta x\psi x$

Since the second conjunct did not undergo any internal operation, it appears that we have here a tacit recognition of one of the laws of confinement, namely, $E\Pi x K\phi x p K\Pi x\phi x p$.

While the three examples of 'copulative' reasoning are the only ones given, and no inferential schemata are presented ("because syllogisms of this sort are not much in use"—TL, p. 115), it might be remarked that in his treatment of disjunctive propositions Burleigh recognizes that if both components are true, the (non-exclusive) disjunction of the same components is true, i.e., that

15.41 $CKpqApq$

and, a foriori,

15.42 $CKpqp$

15.43 $CKpqq$

is valid.

These last three theses are extremely important since the many exponible propositions utilized in every-day discourse are considered by Burleigh as conjunctions and thus the laws applying to the latter apply to the former.

Disjunctive Proposition and Syllogism

Having defined disjunctive proposition in a vague manner as "one which is composed of several categorical propositions by means of the disjunctive connective [*vel, an*]'' (TL, p. 115), Burleigh considers next the question of *proper* disjunction: should we mean 'either ... or ..., or both', or 'either ... or ..., but not both'. He observes that Boethius made the latter choice, but he decides to disagree with the ancient authority and proposes to interpret disjunction in a non-exclusive sense (matrix 1110): ''For the truth of a disjunctive proposition it is sufficient and necessary that one component be true. Whether the other component is false or not, the disjunction is true if one component is true'' (TL, p. 116). And: ''I say that if both parts of a disjunctive are true, the whole disjunction is true'' (TL, p. 245). Bochenski remarks that Burleigh *rediscovered* non-exclusive disjunction after it has been forgotten, or was rejected as illegitimate, since its first discoverer Galen (cf. HFL, p. 303).

While Burleigh agrees with Boethius that '*App*' is not a proper disjunction (TL, p. 116; cf. also p. 119), he does so because a proposition cannot properly be said to be disjoined with itself (''Inter idem et seipsum proprie non fit disjunctio'') and not because both components will necessarily have identical truth value. Nevertheless, interpreting disjunction as he does, *App* can occur in his logic, while it cannot in the logic of Boethius: *Jpp* (matrix 0110) is always false. Burleigh, of course, was not interested in having such an 'improper' disjunction as App for its own sake in his system, but he had to face it when he applied certain rules of disjunctive syllogism: accordingly to the rule that whatever is said of the inferiors may be said of the corresponding superiors one can infer from 'Socrates is running or Plato is running' the proposition 'Socrates is running or a man is running', and from this one the proposition 'A man is running or a man is running'. ''The impropriety of the ordinary discourse does not preclude its [i.e. of *App*] truth, nor does it preclude its consequence'' (TL, p. 119).

If the truth of either component is sufficient to make a disjunction true, the following laws hold:

16.10 CpApq

16.11 CqApq.

The problem of disjunctive propositions referring to future contingent events is also discussed. Peter Aureoli (Burleigh's older contemporary) maintained that in such disjunctions neither component needs to be true, and yet the disjunctive will be true. Aristotle, too, *appears*, according to Burleigh, to hold the same view. Yet, Burleigh defends the general rule that for the truth of a disjunctive it is necessary that one of its components be true, and by doing so he claims to be giving in fact the correct interpretation of Aristotle's *Perihermeneias*: in propositions concerning future contingent events one of the contradictories is true and the other false, except that the truth of such propositions is not determined in the same way as it is in those concerning present events.

There are at least three senses in which a proposition may be said to be 'determinately true' (on 'determinate verum' see TDP, p. 48): (1) necessary proposition is determinately true, since it is impossible for it to be false, and this in two cases: (a) when it excludes the possibility of falsehood for all times, and (b) when it excludes the possibility of falsehood for some instant. (An example of the latter: 'I am sitting', and, generally, any proposition relating to some present event not dependent on the future.) (2) when it "excludes falsehood, but not the possibility of falsehood, for the instant for which the proposition is true" (TL, p. 117; the same distinctions are made in his work *In artem veterem*: cf. TDP, p. 83). It is in this last sense that Burleigh holds a proposition concerning contingent future to be determinately true.

Let us suppose that tomorrow will be the reign of Antichrist and that this is a contingent event. "Then, 'Antichrist will be tomorrow' is determinately true for this instant in that it is not false in this instant; and yet it is possible for it to be false in the sense that since it is contingent that Antichrist will be tomorrow, it is possible that he will not be tomorrow; and thus it is possible for 'Antichrist will be tomorrow' to be false" (TL, p. 117).

Thus, both Burleigh and Aristotle maintain that even propositions concerning future contingents are determinately true; what they deny is merely that they are determinately true in the same way as propositions concerning present events not dependent on the future. Does this commit them to determinism? Burleigh, at least, would say that it does not. Like all medievals of his time (cf. TM, p. 3; FL, p. 211; TDP, Boehner's commentary) he had at his disposal a distinction between *necessity of the consequence* and *necessity of the consequent*. *ApNp* is equivalent to *CNpNp* as well as to its converse *Cpp*. While *ApNp* seems to lead us into an impass as far as the possibility of undetermined events is concerned, the latter two ways, in terms of consequence, of expressing the same idea do nothing of the sort. *CNpNp* says that if an event will not take place, then it will not take place. The consequence is determinately true, since it as a tautology; but we do not know anything about the consequent: it all depends on the future events to which it refers. Compare this with a disjunctive proposition concerning the present moment: 'I am either sitting or not sitting'; this is equivalent to 'If I am not sitting I am not sitting', which is certainly a determinately true consequ*ence*. Now suppose I am not sitting. Then, by the *de post facto* necessity, the consequ*ent* too is necessary. The only difference between disjunctions concerning present events and those concerning future contingent events is that in the former we can at least in principle verify one of the disjuncts and thus know whether the consequent is determinately true, while in the latter we cannot. The same results could be obtained if we availed ourselves of some three-valued logic in which *Apq is* definable as *CNpq*: if p has value $\frac{1}{2}$ and *Np* has value $\frac{1}{2}$, *ApNp* as well as *CNpNq* and *Cpq* will have value 1. In either case the consequence holds, but the consequent is determinately true only conditionally; it will all depend on what the future will be.

The same view is maintained by Burleigh when he treats of temporal propositions (TL, p. 130; cf. also TDP, p. 75, n. 27 where a similar discussion from the *In artem veterem* is quoted by Boehner). The statement 'Everything which is when it is is necessary' might be taken in two ways: in a divisive way (or in a divided sense) when the meaning is: everything which is necessarily is when it is; but it may also be taken in a composite way when the meaning is: everything which is when(ever) it is is necessarily. Burleigh claims that the proposition is true only in the former case.

* * * *

For the NECESSITY of a disjunctive "it is sufficient and necessary that either of the components be necessary or that components contradict each other or are equivalent to contradictories" (TL, p. 117). Defining the shorter symbolic expression '*Lp*' by *NMNp* we have

16.20 CLpLApq

16.21 CLqLApq

16.22 CALpLqLApq

16.23 CApNpLApNp

The first two laws are not convertible, the latter two are. One is tempted to interpret the last clause of the above definition ("... that components ... are equivalent to contradictories") as

CNMKpqLApq

but this thesis is not logically true unless we make a special assumption and conjoin to the antecedent either '*p*' or '*q*' (It is not necessary to conjoin to it *Lp* or *Lq*: "non enim requiritur ad necessitatem disiunctivae necessitas alterius partis"—TL, p. 177); without this assumption we would be entitled to posit as consequent only *LANpNq* (or a stroke function of '*p*' and '*q*' preceded by the sign of necessity). It may be worthwhile to notice also that Burleigh bases his inference-schemata 16.20-16.22 on the general rule [1.41] which says that a necessary proposition does not imply a contingent one.

The denial of a disjunction whose components are each other's contradictories leads to an affirmation of a self-contradictory proposition (TL, p. 118):

16.24 CNApNpKNpp,

and conversely. This follows by general rule [4.10], governing the relationship between denials of disjunctions and conjunctions.

For the POSSIBILITY of a disjunctive proposition "it is sufficient and necessary that either of its components be possible" (TL, p. 118). Thus:

16.30 CMpMApq

16.31 CMqMApq

16.32 CAMpMqMApq

16.32 holds also in the converse form:

16.33 CMApqCAMpAMq

and thus as an equivalence:

16.34 EAMpMqMApq.

IMPOSSIBILITY imposes a stronger requirement: every component must be impossible if the disjunction is to be impossible (TL, p. 118):

16.41 CKNMpNMqNMApq

16.42 CNMApqKNMpNMq

16.43 EKNMpNMqNMApq

CONTRADICTORY of a disjunctive is subject to the general rule [4.10] (TB, p. 10; TL, p. 118).

In his treatment of disjunctive SYLLOGISMS, Burleigh distinguishes between those which have for their major premiss a disjunctive proposition constructed with the connective *vel* and those based on disjunctives constructed with *an*. *Vel* ('or') merely disjoins two propositions (In natural language it may also disjoin subjects and predicates), *an* ('whether') may, in addition to serving as a disjunctive connective, introduces a question.

Syllogisms whose validity depends on the internal structure of the propositions *a* and *b* which, in a disjunctive form '*a* vel *b*', serve as the major premiss, obey the same rules as categorical syllogisms. For example the rule: 'What is affirmed of the inferiors may be affirmed of the corresponding superior' may be applied here:

(1) Man is running or man is white

(2) Animal is running or man is white

(3) Animal is running or animal is white.

From (1), each of the remaining two propositions may be deduced; from (2), the third one may be deduced. But not conversely. We have here two categorical syllogisms in enthymematic form, and a rule governing them. We can apply that rule as soon as the missing premiss is added:

(1) $CA\Sigma xKMxRx\Sigma xKMxWx$

(1a) $\Pi xCMxAx$ (missing premiss)

(1) and (1a) is conjunction yield

(2) $A\Sigma xKAxRx\Sigma xKMxWx$,

and

(3) $A\Sigma xKAxRx\Sigma xKAxWx$

If we should treat propositions constituting the enthymeme as universally quantified, the deduction of (2) and (3) from (1) would be illegitimate. Burleigh explains this elsewhere (TB, p. 4) by stating that the subject of the proposition to which the rule 'ab inferiori ad superius' applies must have a destribut*ive* supposition. Now a quantifier (implicit or explicit)—as explained so well by Prior—distributes a term "when *fQb* entails *fHb*"

(SCWB), where '*fHb*' may be read as 'this *f* is a *b*', and this does not happen in the case of propositions quantified universally but only in those quantified particularly or indefinitely.

The ordinary disjunctive syllogism is also explicitly given (TL, p. 119f.), and, in accordance with the inclusive interpretation of the disjunctive connective, only the sublating modes are admitted. Eight modes are possible; depicted as theses, we have, for instance:

16.51 CKApqNpq

16.55 CKApqNqp; and so on.

If there be more than two disjuncts, then, sublating each of them in turn except one, the remaining one is to be posited (TL, p. 119), which is a law anticipated by the Stoics (AFL, p. 98):

16.60 CKAApqrKNpNqr.

The positing mood is explicitly rejected. Even in the case of disjunction consisting of contradictories, where the disjunctive connective is as a matter of fact exclusive, Burleigh never gives examples in which the minor premiss would posit. He examines, for example, *CKApNpNpp* and rejects it as invalid because it does not posit *Np* (the second disjunct) by sublating the first one, and not because it does not sublate the first disjunct by positing the second one (TL, p. 120).

* * *

Although the particle 'whether' sometimes introduces a question, its principal function is to disjoin propositions. It may occur once or it (in English, its equivalent) may be repeated. In either case it is a binary connective. The forms of propositions in question are: 'You know (or some other propositional attitude) whether ...' and 'You know whether ... or ...'. Whenever the disjunction-sign occurs only once, the disjunction which it effects is contradictory. 'You know whether Socrates is running' is equivalent to 'You know whether Socrates is running or not running'. If the disjunction sign is repeated, it usually does not effect a disjunction of a proposition with its contradictory; it disjoins any two propositions.

On the basis of "whether"—disjunctions we may not apply the rule 'Whatever is said of inferiors may be said of the superior'. 'You know whether a man is running, therefore you know whether an animal is running' is not a valid inference, since it is possible that you know that no man is running, but not know whether any other animal is running. Nor is this sequence valid: 'You know whether every man is running, therefore you know whether Socrates is running', since it is possible that you know that not every man is running (because you yourself are sitting) and nevertheless not know whether anyone else is running.

Syllogism based on disjunctions constructed with *an* ('whether') are much too complicated to deal with symbolically. We have here an intentional logic, in the etymological sense of the term, although there is no doubt that formalization could be carried out to a greater degree than it has been so far.

Conclusion

It seems appropriate to bring out the main points of the foregoing inquiry in the light of the purposes stated in the beginning. (a) Burleigh's conditional syllogistic is developed more fully than other parts of his hypothetical syllogistic. All the direct affirmative modes of the three figures are stated in the schematic form. The indirect modes are stated explicitly only in the third figure, but we do find a prescription as to how to construct them in any figure. The negative modes are not all made explicit, but again the directions for stating them are fully given. Mixed conditional syllogisms, too, are elaborated upon, and the realization that there is a difference between ordinary and generalized conditionals, without a simultaneous attempt to reduce the latter to categorical propositions, shows Burleigh's keen sense for making the necessary distinctions.

In the chapters on copulative and disjunctive propositions Burleigh fails to make explicit the properties of commutativity, associativity, and idempotency of the logical product and the logical sum; nor do we find there the laws of simplification. It may be, however, that Burleigh considered these matters too obvious, and since he did not intend to set up a *logistic* system, he felt justified in omitting to state the "trivial" logical truths. Another lack in his hypothetical syllogistic is a treatment of inferential schemata utilizing both conditional and conjunctive premisses (dilemmas and the laws of composition). We do find, however, a statement of relations between conjunctions and disjunctions (when one of the two is negated) sufficient to construct a square of opposition analogous to that for categorical propositions: *Kpq* and *ANpNq* are contradictories; so are *Apq* and *KNpNq*. *Kpq* and *KNpNq* are contraries (see the second paragraph on the CONTRADICTORY of the conjunctive); that *Apq* and *ANpNq* are subcontraries could be arrived at by a reflection similar to the one that led Burleigh to the discovery of contraries. Finally, *Kpq* is super-altern of *Apq* [cf. 15.41], and the same could be said of *KNpNq* in relation to *ANpNq* either on the basis of knowledge of the relations determined thus far, or on the basis of a direct reflection similar to that which led to the discovery of 15.41.

(b) The reduction—either direct or indirect—of special rules to the general ones and of modes of imperfect figures to those of the perfect one is carried out quite far in both the pure and the mixed conditional syllogistic. Rules [2.00], [2.10] and [3.00] are considered as basic, although of the first two only one is actually required. Copulative and disjunctive propositions appear to be considered as primitive.

(c) Burleigh's indebtedness to Boethius is greatest in the case of pure conditional syllogism and in the case of disjunctive proposition, although repeated observations of departure in interpretation had been made.

Summary of Theses From SB Utilized or Referred-to In the Present Paper

1.40 *CKMpNMqNLCpq*

1.41 *CKLpMNqNLCpq*

2.00 *CCpqCCqrCpr*

2.10 *CCpqCCrpCrq*

3.00 *CCpqCNqNp*

3.50 *CCKpqrCKqNrNp*

3.51 *CCKpqrCKpNrNq*

3.52 *CCKpqrCNrANpNq*

3.60 *CCKNrApqANpNqCKpqr*

3.61 *CCKNrpNqCKpqr*

3.62 *CCKNrqNpCKpqr*

4.00 *ENKpqANpNq*

4.10 *ENApqKNpNq*

4.20 *ENCpqKpNq*

7.00 *CCpqCCNprNqr*

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NSAO Reichenbach, H., *Nomological Statements and Admissible Operations*, North-Holland, 1954.

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