Notre Dame Journal of Formal Logic
Volume XII, Number 4, October 1971
NDJFAM

A MODEL FOR LEŚNIEWSKI'S MEREOLOGY IN FUNCTIONS

ROBERT E. CLAY

INTRODUCTION. Mereology, it may be recalled, is Leśniewski's system consisting of:

(1) A system of propositional logic, upon which is based

(2) A system for characterizing the meaning of 'is', upon which is based

(3) A system for characterizing the relation of 'part' to the 'whole'.

The partial system of mereology consisting of just (1) is called protothetic. The partial system consisting of (1) and (2) is called ontology.

Up to now, the models of mereology that have been constructed have given an interpretation for the terms 'part' and 'whole' of (3) but have left the term 'is' of (2) uninterpreted (see [3]). In this paper we give the first model for mereology in which 'is' is interpreted as well. In other words, based on ontology, we have a model of mereology that includes a model of ontology.

(2) consists of a primitive semantical category (logical type) called the category of names, a proposition forming functor, ε (read is), of two name arguments, an axiom system

0. $[Aa] .\therefore A \varepsilon a . \equiv : [\exists B] . B \varepsilon A : [C] : C \varepsilon A . \supset . C \varepsilon a : [CD] : C \varepsilon A . D \varepsilon A . \supset . C \varepsilon D .$[1]

and two new rules, namely the rule of ontological extentionality

E0. $[\sigma\tau\, A_1 \ldots A_n] .\therefore [A] : A \varepsilon \sigma\{A_1 \ldots A_n\} . \equiv . A \varepsilon \tau\{A_1 \ldots A_n\} : \equiv : [\varphi] : \varphi\langle\sigma\rangle . \equiv . \varphi\langle\tau\rangle$

and the ontological rule of definition

D0. $[A_1 \ldots A_n] : A \varepsilon \alpha(A_1 \ldots A_n) . \equiv . A \varepsilon A . \beta(A_1 \ldots A_n)$

1. Strictly speaking, in terms of the rules for protothetic, we should write $\varepsilon\{Aa\}$ instead of $A \varepsilon a$, but the form given in the axiom is easier to read.

Received January 17, 1964

where α is the term being defined and β is some expression in which $A_1 \ldots A_n$ and possibly A, are the free variables. In both *E0* and *D0*, $A_1 \ldots A_n$ may be of any available semantical categories.

Note. Names occurring to the left of the ε are called individual names or just individuals. The third conjunct of the axiom 0 makes this terminology acceptable.

(3) consists of a primitive name forming functor, **el** (read element), of one name argument and an axiom system

M1. $[AB]:A\,\varepsilon\,\mathbf{el}(B)\,.\supset.\,B\,\varepsilon\,B.$

M2. $[A]:A\,\varepsilon A\,.\supset.A\,\varepsilon\,\mathbf{el}(A).$

M3. $[ABC]:A\,\varepsilon\,\mathbf{el}(B)\,.\,B\,\varepsilon\,\mathbf{el}(C)\,.\supset.A\,\varepsilon\,\mathbf{el}(C).$

DM. $[Aa]\,.\because.A\,\varepsilon\,\mathbf{Kl}(a)\,.\equiv:A\,\varepsilon A\,:[D]:D\,\varepsilon\,a\,.\supset.\,D\,\varepsilon\,\mathbf{el}(A):[D]:D\,\varepsilon\,\mathbf{el}(A)\,.$
$\supset.[\exists EF]\,.\,E\,\varepsilon\,a\,.\,F\,\varepsilon\,\mathbf{el}(D)\,.\,F\,\varepsilon\,\mathbf{el}(E)\,.$

M4. $[ABa]:A\,\varepsilon\,\mathbf{Kl}(a)\,.\,B\,\varepsilon\,\mathbf{Kl}(a)\,.\supset.A\,\varepsilon\,B.$

M5. $[Aa]:A\,\varepsilon\,a\,.\supset.[\exists B]\,.\,B\,\varepsilon\,\mathbf{Kl}(a).$

To construct our model, we add to ontology that *0* and *1* are two distinct individuals. Then we interpret non-empty names as (single-valued) functions with domain the non-empty names and range *0* and *1*. The individuals then become interpreted as those functions that take the value *1* exactly once. The analog of ε, which we call η, is then defined and the analogs of 0, *E0*, and *D0* are proved. Next we interpret element by saying that one η-individual is an element of a second η-individual if the name at which the first takes the value *1* is a subname of the name at which the second takes the value *1*. Lastly, an η-individual is called the class (**Kl**) of an η-name if the η-individual takes the value *1* at the union of all the names at which the η-name takes the value *1*. The analogs of *M1-M3, DM, M4,* and *M5* are then proved, and the construction of the model is then complete.

For the definitions of ontological symbols which occur in this paper, see [2]. For more about ontology, see [1].

In [4] the author attempted to construct a model of mereology which included a model of ontology, but was only partially successful because the analog for the ontological rule of definition failed.

Construction of the model.

N0. $0 \neq 1$

Note. In ontology $A \neq B$ is not the same as $\sim(A = B)$;

$$A \neq B\,.\equiv.A\,\varepsilon A\,.\,B\,\varepsilon\,B\,.\sim(A = B)$$

DN1 $[\sigma]::\mathcal{N}\langle\sigma\rangle.\equiv.\because.[a]:!\,(a)\,.\equiv.[\exists A]\,.A\,\varepsilon\,\sigma(a)\,.\because.[Aa]\,.\because.A\,\varepsilon\,\sigma(a)\,.\supset$
$:A = 0\,.\vee.\,A = 1\,:[ABa]:A\,\varepsilon\,\sigma(a)\,.\,B\,\varepsilon\,\sigma(a)\,.\supset.A = B:[\exists a]\,.\,1\,\varepsilon\,\sigma(a)$

DN2 $[\Sigma]\,.\because.\mathcal{I}\langle\Sigma\rangle\,.\equiv:\mathcal{N}\langle\Sigma\rangle:[ab]:1\,\varepsilon\,\Sigma\,(a)\,.\,1\,\varepsilon\,\Sigma\,(b)\,.\supset.a \circ b$

DN3 $[\Sigma\,\sigma]\,.\because.\Sigma\eta\sigma\,.\equiv:\mathcal{I}\langle\Sigma\rangle\,.\,\mathcal{N}\langle\sigma\rangle:[a]:1\,\varepsilon\,\Sigma(a)\,.\supset.\,1\,\varepsilon\,\sigma(a)$

N1 $[\Sigma]:\Sigma\eta\Sigma\,.\equiv.\,\mathcal{I}\langle\Sigma\rangle$ [*DN3, DN2*]

N2 $[\Sigma\sigma]:\Sigma\eta\sigma\,.\supset.\,\Sigma\eta\Sigma$ [*DN3, N1*]

N3 $[\Sigma\sigma a]:\mathcal{I}\langle\Sigma\rangle.\,\mathcal{N}\langle\sigma\rangle\,.\,1\,\varepsilon\,\Sigma(a)\,.\,1\,\varepsilon\,\sigma(a)\,.\supset.\,\Sigma\eta\sigma$

Hyp(4) $\supset$:

5) $[bc]: 1 \varepsilon \Sigma(b) . 1 \varepsilon \Sigma(c) . \supset . b \circ c:$ [DN2, 1]

6) $[b]: 1 \varepsilon \Sigma(b) . \supset . 1 \varepsilon \sigma(b):$ [5, 3, 4]

$\Sigma \eta \sigma$ [DN3, 1, 2, 6]

N4 $[\Sigma \sigma]: \Sigma \eta \sigma . \supset . [\exists a] . 1 \varepsilon \sigma(a) . 1 \varepsilon \Sigma(a)$

Hyp (1) $. \supset .$

2) $\mathcal{N}\langle \Sigma \rangle .$ [DN3, 1, DN2]

$[\exists a] .$

3) $1 \varepsilon \Sigma(a) .$ [DN1, 2]

4) $1 \varepsilon \sigma(a) .$ [DN3, 1, 3]

$[\exists a] . 1 \varepsilon \Sigma(a) . 1 \varepsilon \sigma(a)$ [3, 4]

N5 $[\Sigma \sigma]: \Sigma \eta \sigma . \equiv . \mathcal{J}\langle \Sigma \rangle . \mathcal{N}\langle \sigma \rangle . [\exists a] . 1 \varepsilon \Sigma(a) . 1 \varepsilon \sigma(a)$ [DN3, N4, N3]

N6 $[\Sigma \Phi \sigma]: \Sigma \eta \sigma . \Phi \eta \Sigma . \supset . \Phi \eta \sigma$

Hyp(2) $. \supset$:

3) $\mathcal{J}\langle \Phi \rangle :$

4) $[a]: 1 \varepsilon \Phi(a) . \supset 1 \varepsilon \Sigma(a):$ [DN3, 2]

5) $\mathcal{N}\langle \sigma \rangle :$

6) $[a]: 1 \varepsilon \Sigma(a) . \supset . 1 \varepsilon \sigma(a):$ [DN3, 1]

7) $[a]: 1 \varepsilon \Phi(a) . \supset . 1 \varepsilon \sigma(a):$ [4, 6]

$\Phi \eta \sigma$ [DN3, 3, 5, 7]

N7 $[\Sigma \Phi \Psi \sigma]: \Sigma \eta \sigma . \Phi \eta \Sigma . \Psi \eta \Sigma . \supset . \Phi \eta \Psi$

Hyp(3) $. \supset .\cdot.$

4) $\mathcal{J}\langle \Sigma \rangle .$ [DN3, 1]

5) $\mathcal{J}\langle \Phi \rangle .$ [DN3, 2]

6) $\mathcal{J}\langle \Psi \rangle .$ [DN3, 3]

7) $\mathcal{N}\langle \Psi \rangle :$ [DN2, 6]

$[\exists a]:$

8) $1 \varepsilon \Phi(a) .$

9) $1 \varepsilon \Sigma(a) .$ [N4, 2]

$[\exists b] .$

10) $1 \varepsilon \Psi(b) .$

11) $1 \varepsilon \Sigma(b) .$ [N4, 3]

12) $a \circ b .$ [DN2, 4, 9, 11]

13) $1 \varepsilon \Psi(a) .$ [10, 12]

$\Phi \eta \Psi$ [N3, 5, 7, 8, 13]

DN4 $[Aab] :: A \varepsilon \Box\{a\}(b) . \equiv .\cdot. A \varepsilon A . !(a) . !(b) .\cdot. A = 1 . a \circ b : \vee : A = 0 . \sim (a \circ b)$

N8 $[Aab] :: A \varepsilon \Box\{a\}(b) . \equiv .\cdot. !(a) . !(b) .\cdot. A = 1 . a \circ b : \vee : A = 0 . \sim (a \circ b)$ [DN4]

N9 $[ab]: 1 \varepsilon \Box\{a\}(b) . \equiv . !(a) . a \circ b$ (i.e. $a \Box b$) [N8, N0]

N10 $[a]: 1 \varepsilon \Box\{a\}(a) . \equiv . !(a)$ [N9]

N11 $[ab] .\cdot. !(a) \supset : 1 \varepsilon \Box\{a\}(b) . \equiv . a \circ b$ [N9]

N12 $[Aab] .\cdot. A \varepsilon \Box\{a\}(b) . \supset : A = 0 . \vee . A = 1$ [N8]

N13 $[ABab]: A \varepsilon \Box\{a\}(b) . B \varepsilon \Box\{a\}(b) . \supset A = B$

Hyp(2) $\supset$:

3) $A = 1 . a \circ b . \vee . A = 0 . \sim(a \circ b):$ [N8, 1]

4) $B = 1 . a \circ b . \vee . B = 0 . \sim(a \circ b):$ [N8, 2]

$A = B$ [3, 4]

N14	$[ab]:!(a).!(b).\supset.[\exists A].A\,\varepsilon\,\square\{a\}(b)$	
	Hyp(2) $\supset$:	
	3) $1\,\varepsilon\,\square\{a\}(b).\vee.0\,\varepsilon\,\square\{a\}(b):$	[*N8*, 1, 2, *N0*]
	$[\exists A].A\,\varepsilon\,\square\{a\}(b)$	[3]
N15	$[a]\therefore !(a).\supset:!(b).\equiv.[\exists A].A\,\varepsilon\,\square\{a\}(b)$	[*N14*, *N8*]
N16	$[a]:!(a).\supset.\mathcal{N}\langle\square\{a\}\rangle$	[*DN1*, *N15*, *N12*, *N13*, *N10*]
N17	$[a]:!(a).\supset.\mathcal{J}\langle\square\{a\}\rangle$	[*DN2*, *N16*, *N11*]
N18	$[a]:\mathcal{J}\langle\square\{a\}\rangle.\supset.!(a)$	
	Hyp(1) $.\supset.$	
	2) $\mathcal{N}\langle\square\{a\}\rangle.$	[*DN2*, 1]
	3) $[\exists b].1\,\varepsilon\,\square\{a\}(b).$	[*DN1*, 2]
	$!(a)$	[*N8*, 3]
N19	$[a]:!(a).\equiv.\mathcal{J}\langle\square\{a\}\rangle$	[*N17*, *N18*]
N20	$[ab\Sigma]\therefore\mathcal{N}\langle\Sigma\rangle.1\,\varepsilon\,\Sigma(a).1\,\varepsilon\,\Sigma(b):[\Phi\Psi]:\Phi\eta\Sigma.\Psi\eta\Sigma.\supset.\Phi\eta\Psi:\supset.a\circ b$	
	Hyp(4) $\supset.$	
	5) $!(a).$	[*DN1*, 1, 2]
	6) $!(b).$	[*DN1*, 1, 3]
	7) $1\,\varepsilon\,\square\{a\}(a).$	[*N10*, 5]
	8) $1\,\varepsilon\,\square\{b\}(b).$	[*N10*, 6]
	9) $\mathcal{J}\langle\square\{a\}\rangle.$	[*N19*, 5]
	10) $\mathcal{J}\langle\square\{b\}\rangle.$	[*N19*, 6]
	11) $\square\{a\}\eta\Sigma.$	[*N3*, 9, 1, 7, 2]
	12) $\square\{b)\eta\Sigma.$	[*N3*, 10, 1, 8, 3]
	13) $\square\{a\}\eta\square\{b).$	[4, 11, 12]
	14) $1\,\varepsilon\,\square\{b\}(a).$	[*DN3*, 13, 7]
	$a\circ b$	[*DN2*, 10, 14, 8]
N21	$[\theta\Sigma\sigma]\therefore\theta\eta\Sigma:[\Phi]:\Phi\eta\Sigma.\supset.\Phi\eta\sigma:[\Phi\Psi]:\Phi\eta\Sigma.\Psi\eta\Sigma.\supset\Phi\eta\Psi:\supset.\Sigma\eta\sigma$	
	Hyp(3) $.\supset:$	
	4) $\mathcal{N}\langle\Sigma\rangle:$	[*DN3*, 1]
	5) $[ab]:1\,\varepsilon\,\Sigma(a).1\,\varepsilon\,\Sigma(b).\supset a\circ b:$	[*N20*, 4, 3]
	6) $\mathcal{J}\langle\Sigma\rangle.$	[*DN2*, 4, 5]
	7) $\Sigma\eta\Sigma.$	[*N1*, 6]
	$\Sigma\eta\sigma$	[2, 7]
N22	$[\Sigma\sigma]\therefore\Sigma\eta\sigma.\equiv:[\exists\theta].\theta\eta\Sigma:[\Phi]:\Phi\eta\Sigma.\supset.\Phi\eta\sigma:[\Phi\Psi]:\Phi\eta\Sigma.\Psi\eta\Sigma.\supset.\Phi\eta\Psi$	[*N2*, *N6*, *N7*, *N21*]

This last thesis, *N22*, is the analog of a standard single axiom of ontology. Next we shall prove that, under the hypothesis that the functions used satisfy *DN1*, the analog for the rule of ontological extentionality holds for η.

N23	$[\sigma\tau a]\therefore\mathcal{N}\langle\sigma\rangle:[\Sigma]:\Sigma\eta\sigma.\supset.\Sigma\eta\tau:1\,\varepsilon\,\sigma(a):\supset.1\,\varepsilon\,\tau(a)$	
	Hyp(3) $\supset.$	
	4) $!(a).$	[*DN1*, 1, 3]
	5) $1\,\varepsilon\,\square\{a\}(a).$	[*N10*, 4]
	6) $\mathcal{J}\langle\square\{a\}\rangle.$	[*N17*, 4]
	7) $\square\{a\}\eta\sigma.$	[*N3*, 6, 1, 5, 3]

8) $\square \prec a \succ \eta \tau$. [2, 7]

$1 \varepsilon \tau(a)$ [$DN3$, 5, 8]

$N24$ $[\sigma \tau a] .\because. \mathcal{N}\langle\sigma\rangle . \mathcal{N}\langle\tau\rangle : [\Sigma] : \Sigma\eta\sigma . \supset . \Sigma\eta\tau : 0 \varepsilon \tau(a) : \supset . 0 \varepsilon \sigma(a)$

Hyp(4) $. \supset \because$

5) $[ABb] : A \varepsilon \tau(b) . B \varepsilon \tau(b) . \supset . A = B :$ [$DN1$, 2]

6) $\sim(1 \varepsilon \tau(a))$. [$N0$, 4, 5]

7) $\sim(1 \varepsilon \sigma(a))$. [$N23$, 1, 3, 6]

8) $!(a) :$ [$DN1$, 2, 4]

$[\exists A] :$

9) $A \varepsilon \sigma(a) :$ [$DN1$, 1, 8]

10) $A = 0 . \vee . A = 1 :$ [$DN1$, 1, 9]

$0 \varepsilon \sigma(a)$ [9, 10, 7]

$N25$ $[\sigma\tau] .\because. \mathcal{N}\langle\sigma\rangle . \mathcal{N}\langle\tau\rangle : [\Sigma] : \Sigma\eta\sigma . \equiv . \Sigma\eta\tau : \supset : [a] : 1 \varepsilon \sigma(a) . \equiv . 1 \varepsilon \tau(a) : [a] :$
$0 \varepsilon \sigma(a) . \equiv . 0 \varepsilon \tau(a)$ [$N23$, $N24$]

$N26$ $[\sigma\tau A a] .\because. \mathcal{N}\langle\sigma\rangle . \mathcal{N}\langle\tau\rangle : [\Sigma] : \Sigma\eta\sigma . \equiv \Sigma\eta\tau : A \varepsilon \sigma(a) : \supset . A \varepsilon \tau(a)$

Hyp(4) $. \supset :$

5) $A = 0 . \vee . A = 1 :$ [$DN1$, 1, 4]

$A \varepsilon \tau(a)$ [5, $N25$, 1, 2, 3, 4]

$N27$ $[\sigma\tau] .\because. \mathcal{N}\langle\sigma\rangle . \mathcal{N}\langle\tau\rangle : [\Sigma] : \Sigma\eta\sigma . \equiv . \Sigma\eta\tau : \supset : [Aa] : A \varepsilon \sigma(a) . \equiv . A \varepsilon \tau(a)$
[$N26$]

$DN4a$ $[AA_1 \ldots A_n a \sigma] : A \varepsilon \boldsymbol{K} \prec\sigma\succ \{\!\!|A_1 \ldots A_n a|\!\!\} . \equiv . A \varepsilon \sigma\{A_1 \ldots A_n\}(a)$

$DN4b$ $[\alpha\tau] : \boldsymbol{\mathcal{L}} \prec\alpha\succ \{\!|\tau|\!\} . \equiv . [\exists\sigma] . \circ \{\!|\tau\, \boldsymbol{K}\prec\sigma\succ|\!\} . \alpha\prec\sigma\succ$

$N27a$ $[\alpha\sigma] : \boldsymbol{\mathcal{L}} \prec\alpha\succ \{\!|\boldsymbol{K}\prec\sigma\succ|\!\} . \equiv . \alpha\prec\sigma\succ$ [$DN4b$]

$N27b$ $[\sigma\tau A_1 \ldots A_n] .\because. \mathcal{N}\langle\sigma\{A_1 \ldots A_n\}\rangle . \mathcal{N}\langle\tau\{A_1 \ldots A_n\}\rangle : [\Sigma] :$
$\Sigma\eta\sigma\{A_1 \ldots A_n\} . \equiv . \Sigma\eta\tau\{A_1 \ldots A_n\} : \supset : [\alpha] : \alpha\prec\sigma\succ . \equiv . \alpha\prec\tau\succ$

Hyp(3) $. \supset \because$

4) $[Aa] : A \varepsilon \sigma\{A_1 \ldots A_n\}(a) . \equiv . A \varepsilon \tau\{A_1 \ldots A_n\}(a) :$ [$N27$, 1, 2, 3]

5) $[Aa] : A \varepsilon \boldsymbol{K}\prec\sigma\succ \{\!\!|A_1 \ldots A_n a|\!\!\} . \equiv . A \varepsilon \boldsymbol{K}\prec\tau\succ \{\!\!|A_1 \ldots A_n a|\!\!\} :$ [$DN4a$, 4]

6) $[\varphi] : \varphi\{\!|\boldsymbol{K}\prec\sigma\succ|\!\} . \equiv . \varphi\{\!|\boldsymbol{K}\prec\tau\succ|\!\} :$ [$D0$, 5]

7) $[\alpha] : \boldsymbol{\mathcal{L}}\prec\alpha\succ \{\!|\boldsymbol{K}\prec\sigma\succ|\!\} . \equiv . \boldsymbol{\mathcal{L}}\prec\alpha\succ \{\!|\boldsymbol{K}\prec\tau\succ|\!\} :$ [$DN4b$, 6]

$[\alpha] : \alpha\prec\sigma\succ . \equiv . \alpha\prec\tau\succ$ [$N27a$, 7]

$DN4c$ $[A_1 \ldots A_n \sigma\Sigma] : \boldsymbol{\mathcal{M}} \{\!\!|A_1 \ldots A_n \Sigma|\!\!\} \prec\sigma\succ . \equiv . \Sigma\eta\sigma\{A_1 \ldots A_n\}$

$N27c$ $[A_1 \ldots A_n \sigma\tau\Sigma] .\because. [\alpha] : \alpha\prec\sigma\succ . \equiv . \alpha\prec\tau\succ : \supset : \Sigma\eta\sigma\{A_1 \ldots A_n\} . \equiv . \Sigma\eta\tau\{A_1 \ldots A_n\}$

Hyp(1) $. \supset :$

2) $\boldsymbol{\mathcal{M}} \{\!\!|A_1 \ldots A_n \Sigma|\!\!\} \prec\sigma\succ . \equiv . \boldsymbol{\mathcal{M}} \{\!\!|A_1 \ldots A_n \Sigma|\!\!\} \prec\tau\succ :$ [$DN4c$, 1]

$\Sigma\eta\sigma\{A_1 \ldots A_n\} . \equiv . \Sigma\eta\tau\{A_1 \ldots A_n\}$ [$DN4c$, 2]

$N28$ $[\sigma\tau A_1 \ldots A_n] :: \mathcal{N}\langle\sigma\{A_1 \ldots A_n\}\rangle . \mathcal{N}\langle\tau\{A_1 \ldots A_n\}\rangle . \supset \because [\Sigma] :$
$\Sigma\eta\sigma\{A_1 \ldots A_n\} . \equiv . \Sigma\eta\tau\{A_1 \ldots A_n\} : \equiv : [\alpha] : \alpha\prec\sigma\succ . \equiv . \alpha\prec\tau\succ$ [$N27b$, $N27c$]

This last thesis, $N28$, furnishes us with the law of ontological extentionality for η (for functions satisfying $DN1$). See $E0$. Next we prove a metatheorem which will correspond to the ontological rule of definition, $D0$.

$N29$ $[\Sigma\Phi] : \Sigma\eta\Phi . \Phi\eta\Sigma . \supset . \circ\langle\Sigma\Phi\rangle$

Hyp(2) $. \supset :$

3) $[\Psi] : \Psi\eta\Sigma . \supset . \Psi\eta\Phi :$ [$N6$, 1]

4) $[\Psi] : \Psi\eta\Phi . \supset . \Psi\eta\Sigma :$ [$N6$, 2]

5) $[\Psi] : \Psi\eta\Sigma . \equiv . \Psi\eta\Phi :$ [3, 4]

6) $\mathcal{N}\langle\Sigma\rangle$. [$DN3$, 2]
7) $\mathcal{N}\langle\Phi\rangle$. [$DN3$, 1]
$\circ\langle\Sigma\Phi\rangle$ [$N27$, 6, 7, 5]

$N30$ $[\Sigma a]:\mathcal{I}\langle\Sigma\rangle\,.\,1\,\varepsilon\,\Sigma(a)\,.\supset.\,\circ\langle\Sigma\,\Box\{a\}\rangle$
Hyp(2) . ⊃.
3) $\mathcal{N}\langle\Sigma\rangle$. [$DN2$, 1]
4) $!(a)$. [$DN1$, 3, 2]
5) $1\,\varepsilon\,\Box\{a\}(a)$. [$N10$, 4]
6) $\mathcal{I}\langle\Box\{a\}\rangle$. [$N17$, 4]
7) $\mathcal{N}\langle\Box\{a\}\rangle$. [$DN2$, 6]
8) $\Sigma\eta\,\Box\{a\}$. [$N3$, 1, 7, 2, 5]
9) $\Box\{a\}\,\eta\,\Sigma$. [$N3$, 6, 3, 5, 2]
$\circ\langle\Sigma\,\Box\{a\}\rangle$ [$N29$, 8, 9]

METATHEOREM: *An expression of the form*

$$[A_1\ldots A_n]\,.\!\cdot\,.\,\mathcal{N}\langle\sigma\{A_1\ldots A_n\}\rangle\,.\supset:[\Sigma]:\Sigma\eta\sigma\{A_1\ldots A_n\}\,.\equiv.\,\Sigma\eta\Sigma\,.\,\sigma^*\{A_1\ldots A_n\}\langle\Sigma\rangle,$$

where $A_1\ldots A_n$ *belong to any of the previously introduced semantical categories, can be introduced as a thesis provided that the functor* $\sigma\{A_1\ldots A_n\}$ *has been defined by:*

* $[aAA_1\ldots A_n]::A\,\varepsilon\,\sigma\{A_1\ldots A_n\}(a)\,.\equiv.\!\cdot\,.\,A\,\varepsilon\,A\,.\,!(a)\,.\!\cdot\,.\,A=1\,.$
$\sigma^*\{A_1\ldots A_n\}\langle\Box\{a\}\rangle:\vee:A=0\,.\sim(\sigma^*\{A_1\ldots A_n\}\langle\Box\{a\}\rangle))$

Proof:

*1 $[aAA_1\ldots A_n]::A\,\varepsilon\,\sigma\{A_1\ldots A_n\}(a)\,.\equiv.\!\cdot\,.\,!(a)\,.\!\cdot\,.\,A=1\,.$
$\sigma^*\{A_1\ldots A_n\}\langle\Box\{a\}\rangle:\vee:A=0\,.\sim(\sigma^*\{A_1\ldots A_n\}\langle\Box\{a\}\rangle))$ [*]
*2 $[aA_1\ldots A_n]:1\,\varepsilon\,\sigma\{A_1\ldots A_n\}(a)\,.\equiv.\,!(a)\,.\,\sigma^*\{A_1\ldots A_n\}\langle\Box\{a\}\rangle$ [$N0$, *1]
*3 $[\Sigma A_1\ldots A_n]:\Sigma\eta\sigma\{A_1\ldots A_n\}\,.\supset.\,\sigma^*\{A_1\ldots A_n\}\langle\Sigma\rangle$
Hyp(1) . ⊃
2) $\mathcal{I}\langle\Sigma\rangle$. [$DN3$, 1]
3) $\mathcal{N}\langle\Sigma\rangle$. [$DN2$, 2]
$[\exists a]$.
4) $1\,\varepsilon\,\Sigma(a)$. [$DN1$, 3]
5) $\circ\langle\Sigma\,\Box\{a\}\rangle$. [$N30$, 2, 4]
6) $1\,\varepsilon\,\sigma\{A_1\ldots A_n\}(a)$. [$DN3$, 1, 4]
7) $\sigma^*\{A_1\ldots A_n\}\langle\Box\{a\}\rangle$. [*2, 6]
$\sigma^*\{A_1\ldots A_n\}\langle\Sigma\rangle$. [5, 7]
*4 $[\Sigma A_1\ldots A_n]:\mathcal{N}\langle\sigma\{A_1\ldots A_n\}\rangle\,.\,\Sigma\eta\Sigma\,.\,\sigma^*\{A_1\ldots A_n\}\langle\Sigma\rangle\,.\supset.\,\Sigma\eta\sigma\{A_1\ldots A_n\}$
Hyp(3) . ⊃.
4) $\mathcal{I}\langle\Sigma\rangle$. [$N1$, 2]
5) $\mathcal{N}\langle\Sigma\rangle$. [$DN2$, 4]
$[\exists a]$.
6) $1\,\varepsilon\,\Sigma(a)$. [$DN1$, 5]
7) $!(a)$. [$DN1$, 6]
8) $\circ\langle\Sigma\,\Box\{a\}\rangle$. [$N30$, 4, 6]
9) $\sigma^*\{A_1\ldots A_n\}\langle\Box\{a\}\rangle$. [3, 8]
10) $1\,\varepsilon\,\sigma\{A_1\ldots A_n\}(a)$. [*2, 7, 9]
$\Sigma\eta\sigma\{A_1\ldots A_n\}$ [$N3$, 4, 1, 6, 10]

*5 $[A_1 \ldots A_n] .\therefore \mathcal{N}\langle\sigma\{A_1 \ldots A_n\}\rangle . \supset : [\Sigma] : \Sigma\eta\sigma\{A_1 \ldots A_n\} . \equiv . \Sigma\eta\Sigma .$
$\sigma^{*}\{A_1 \ldots A_n\}\langle\Sigma\rangle$ [$N2$, *3, *4]

Thus we have proved the validity for the ontological rule of definition for η. $N22$, $N28$, and the **METATHEOREM** show that, for functions satisfying $DN1$, η is a complete analog for the primitive ε. Next we give the expected characterization of $\mathcal{I}$.

N31 $[\Sigma] : \mathcal{I}\langle\Sigma\rangle . \supset . [\exists a] . !(a) . \circ\langle\Sigma\,\square\!\prec\! a\!\succ\rangle$
Hyp(1) . $\supset$
2) $\mathcal{N}\langle\Sigma\rangle .$ [$DN2$, 1]
$[\exists a] .$
3) $1\,\varepsilon\,\Sigma(a) .$ [$DN1$, 2]
4) $!(a) .$ [$DN1$, 3]
$\circ\langle\Sigma\,\square\!\prec\! a\!\succ\rangle .$ [$N30$, 1, 4]

N32 $[\Sigma] : \mathcal{I}\langle\Sigma\rangle . \equiv . [\exists a] . !(a) . \circ\langle\Sigma\,\square\!\prec\! a\!\succ\rangle$ [$N31$, $N17$]

We now construct a model for mereology based on the ontological model that we have just constructed. We wish to define element so that one η-individual is an element of a second η-individual precisely when the name at which the first takes the value *1* is contained in the name at which the second takes the value *1*. Thus we wish to arrive at the thesis

$$[\Sigma\Phi] : \Sigma\eta\,\mathbf{el}\prec\!\Phi\!\succ . \equiv \Sigma\eta\Sigma . \Phi\eta\Phi . [\exists bc] . 1\,\varepsilon\,\Sigma(b) . 1\,\varepsilon\,\Phi(c) . b \subset c .$$

According to the **METATHEOREM** proved above we must introduce:

DN5 $[aA\Phi] :: A\,\varepsilon\,\mathbf{el}\prec\!\Phi\!\succ(a) . \equiv .\therefore A\,\varepsilon\,A . !(a) .\therefore A = 1 . \Phi\eta\Phi . [\exists bc] . 1\,\varepsilon\,\square\!\prec\! a\!\succ(b) .$
$1\,\varepsilon\,\Phi(c) . b \subset c : \vee : A = 0 . \sim(\Phi\eta\Phi . [\exists bc] . 1\,\varepsilon\,\square\!\prec\! a\!\succ(b) . 1\,\varepsilon\,\Phi(c) . b \subset c)$

N33 $[\Phi] .\therefore \mathcal{N}\langle\mathbf{el}\prec\!\Phi\!\succ\rangle . \supset : [\Sigma] : \Sigma\eta\,\mathbf{el}\prec\!\Phi\!\succ . \equiv \Sigma\eta\Sigma . \Phi\eta\Phi . [\exists bc] .$
$1\,\varepsilon\,\Sigma(b) . 1\,\varepsilon\,\Phi(c) . b \subset c$ [**METATHEOREM**, $DN5$]

N34 $[aA\Phi] :: A\,\varepsilon\,\mathbf{el}\prec\!\Phi\!\succ(a) . \equiv .\therefore !(a) .\therefore A = 1 . \Phi\eta\Phi . [\exists c] . 1\,\varepsilon\Phi(c) . a \subset c :$
$\vee : A = 0 . \sim(\Phi\eta\Phi . [\exists c] . 1\,\varepsilon\,\Phi(c) . a \subset c)$ [$DN5$, $N11$]

N35 $[\Phi] :\cdot: \Phi\eta\Phi . \supset :: [Aa] :: A\,\varepsilon\,\mathbf{el}\prec\!\Phi\!\succ(a) . \equiv .\therefore !(a) .\therefore A = 1 . [\exists c] . 1\,\varepsilon\Phi(c) .$
$a \subset c : \vee : A = 0 : [c] : 1\,\varepsilon\,\Phi(c) . \supset . \sim(a \subset c)$ [$N34$]

N36 $[aA\Phi] : A\,\varepsilon\,\mathbf{el}\prec\!\Phi\!\succ(a) . \supset . !(a)$ [$N34$]

N37 $[\Phi a] : \Phi\eta\Phi . !(a) . \supset . [\exists A] . A\,\varepsilon\,\mathbf{el}\prec\!\Phi\!\succ(a)$
Hyp(2) . $\supset .\therefore$
3) $\mathcal{I}\langle\Phi\rangle .\therefore$ [$N1$, 1]
$[\exists b] .\therefore$
4) $1\,\varepsilon\,\Phi(b) :$ [$DN2$, 3, $DN1$]
5) $a \subset b . \supset . 1\,\varepsilon\,\mathbf{el}\prec\!\Phi\!\succ(a) :$ [$N35$, 1, 2, $N0$, 4]
6) $[c] : \sim(a \subset b) . 1\,\varepsilon\,\Phi(c) . \supset . \sim(a \subset c) :$ [$DN2$, 3, 4]
7) $\sim(a \subset b) . \supset : [c] : 1\,\varepsilon\,\Phi(c) . \supset . \sim(a \subset c) :$ [6]
8) $\sim(a \subset b) \supset . 0\,\varepsilon\,\mathbf{el}\prec\!\Phi\!\succ(a) .\therefore$ [$N35$, 1, 2, $N0$, 7]
$[\exists A] . A\,\varepsilon\,\mathbf{el}\prec\!\Phi\!\succ(a)$ [5, 8]

N38 $[aA\Phi] .\therefore A\,\varepsilon\,\mathbf{el}\prec\!\Phi\!\succ(a) . \supset : A = 0 . \vee . A = 1$ [$N34$]

N39 $[aAB\Phi] : A\,\varepsilon\,\mathbf{el}\prec\!\Phi\!\succ(a) . B\,\varepsilon\,\mathbf{el}\prec\!\Phi\!\succ(a) . \supset . A = B$ [$N34$]

N40 $[\Phi] : \Phi\eta\Phi . \supset . [\exists a] . 1\,\varepsilon\,\mathbf{el}\prec\!\Phi\!\succ(a)$
Hyp(1) . $\supset$.

2) $\mathcal{N}\langle\Phi\rangle$. [DN3, 1]
$[\exists a]$.
3) $1\,\varepsilon\,\Phi(a)$. [DN1, 2]
4) $!\,(a)$. [DN1, 3]
$[\exists a]\,.\,1\,\varepsilon\,\mathbf{el}\{\Phi\}(a)$ [N35, 4, N0, 3]

N41 $[\Phi]:\Phi\eta\Phi.\supset.\mathcal{N}\langle\mathbf{el}\{\Phi\}\rangle$ [DN1, N37, N36, N38, N39, N40]

N42 $[\Sigma\Phi]:\Sigma\eta\,\mathbf{el}\{\Phi\}\,.\equiv.\,\Sigma\eta\Sigma\,.\,\Phi\eta\Phi\,.\,[\exists bc]:1\,\varepsilon\,\Sigma(b)\,.\,1\,\varepsilon\,\Phi(c)\,.\,b\subset c$. [N33, DN3, N41]

Next we wish an η-individual, Σ, to be the class of an η-name, σ, if and only if Σ takes the value 1 at the name that is the union of all the names at which σ takes the value 1. That is

$$[\Sigma\sigma]:\Sigma\eta\,\mathbf{Kl}\{\sigma\}\,.\equiv.\,\Sigma\eta\Sigma\,.\,\mathcal{N}\langle\sigma\rangle\,.\,1\,\varepsilon\,\Sigma(\bigcup\langle\sigma\rangle)$$

where $\bigcup$ is defined by:

D0. $[A\sigma]:A\varepsilon\bigcup\langle\sigma\rangle\,.\equiv.\,[\exists a]\,.\,1\,\varepsilon\,\sigma(a)\,.\,A\,\varepsilon\,a$

According to the **METATHEOREM** we must introduce:

DN6 $[aA\sigma]::A\,\varepsilon\,\mathbf{Kl}\{\sigma\}(a)\,.\equiv.\therefore A\,\varepsilon\,A\,.\,!\,(a)\,.\therefore A=1\,.\,\mathcal{N}\langle\sigma\rangle\,.\,1\,\varepsilon\,\square\{a\}(\bigcup\langle\sigma\rangle):\vee:$
$A=0\,.\sim(\mathcal{N}\langle\sigma\rangle\,.\,1\,\varepsilon\,\square\{a\}(\bigcup\langle\sigma\rangle))$

N43 $[\sigma]\therefore\mathcal{N}\langle\mathbf{Kl}\{\sigma\}\rangle\,.\supset:[\Sigma]:\Sigma\eta\,\mathbf{Kl}\{\sigma\}\,.\equiv.\,\Sigma\eta\Sigma\,.\,\mathcal{N}\langle\sigma\rangle\,.\,1\,\varepsilon\,\Sigma(\bigcup\langle\sigma\rangle)$ [**METATHEOREM**, *DN6*]

N44 $[aA\sigma]::A\,\varepsilon\,\mathbf{Kl}\{\sigma\}(a)\,.\equiv.\therefore\,!\,(a)\,.\therefore A=1\,.\,\mathcal{N}\langle\sigma\rangle\,.\,a\circ\bigcup\langle\sigma\rangle:\vee:A=0\,.$
$\sim(\mathcal{N}\langle\sigma\rangle\,.\,a\circ\bigcup\langle\sigma\rangle)$ [DN6, N11]

N45 $[\sigma]:\cdot:\mathcal{N}\langle\sigma\rangle\,.\supset::[Aa]::A\,\varepsilon\,\mathbf{Kl}\{\sigma\}(a)\,.\equiv.\therefore\,!\,(a)\,.\therefore A=1\,.\,a\circ\bigcup\langle\sigma\rangle:\vee:$
$A=0\,.\sim(a\circ\bigcup\langle\sigma\rangle)$ [N44]

N46 $[a\sigma]:\mathcal{N}\langle\sigma\rangle\,.\,!\,(a)\,.\supset.\,[\exists A]\,.\,A\,\varepsilon\,\mathbf{Kl}\{\sigma\}(a)$
Hyp(2) . $\supset$:
3) $a\circ\bigcup\langle\sigma\rangle\,.\supset.\,1\,\varepsilon\,\mathbf{Kl}\{\sigma\}(a)$: [N45, 1, 2, N0]
4) $\sim(a\circ\bigcup\langle\sigma\rangle)\,.\supset.\,0\,\varepsilon\,\mathbf{Kl}\{\sigma\}(a)$: [N45, 1, 2, N0]
$[\exists A]\,.\,A\,\varepsilon\,\mathbf{Kl}\{\sigma\}(a)$ [3, 4]

N47 $[aA\sigma]:A\,\varepsilon\,\mathbf{Kl}\{\sigma\}(a)\,.\supset.\,!\,(a)$ [N44]

N48 $[aA\sigma]\therefore A\,\varepsilon\,\mathbf{Kl}\{\sigma\}(a)\,.\supset:A=0\,.\vee.\,A=1$ [N44]

N49 $[aAB\sigma]:A\,\varepsilon\,\mathbf{Kl}\{\sigma\}(a)\,.\,B\,\varepsilon\,\mathbf{Kl}\{\sigma\}(a)\,.\supset.\,A=B$ [N44]

N50 $[\sigma]:\mathcal{N}\langle\sigma\rangle\,.\supset.\,1\,\varepsilon\,\mathbf{Kl}\{\sigma\}(\bigcup\langle\sigma\rangle)$
Hyp(1) . $\supset$:
- $[\exists a]$:
2) $1\,\varepsilon\,\sigma(a)$. [DN1, 1]
3) $!\,(a)$. [DN1, 2]
$[\exists A]$.
4) $A\,\varepsilon\,a$. [3]
5) $A\,\varepsilon\bigcup\langle\sigma\rangle$. [D0, 2, 4]

	6) $!(\bigcup\langle\sigma\rangle)$.	[5]
	$1\,\varepsilon\,\mathbf{Kl}{\prec}\sigma{\succ}(\bigcup\langle\sigma\rangle)$	[$N45$, 1, 6, $N0$]
$N51$	$[\sigma]:\mathcal{N}\langle\sigma\rangle\,.\supset.\,\mathcal{N}\langle\mathbf{Kl}{\prec}\sigma{\succ}\rangle$	[$DN1$, $N46$, $N47$, $N48$, $N49$, $N50$]
$N52$	$[\Sigma\sigma]:\Sigma\eta\mathbf{Kl}{\prec}\sigma{\succ}\,.\equiv.\,\Sigma\eta\Sigma\,.\,\mathcal{N}\langle\sigma\rangle\,.\,1\,\varepsilon\,\Sigma(\bigcup\langle\sigma\rangle)$	[$N43$; $DN3$, $N51$]

Now we shall proceed to verify an axiom system for mereology.

$N53$	$[\Sigma\Phi\Psi]:\Sigma\eta\,\mathbf{el}{\prec}\Phi{\succ}\,.\,\Phi\eta\,\mathbf{el}{\prec}\Psi{\succ}\,.\supset.\,\Sigma\eta\,\mathbf{el}{\prec}\Psi{\succ}$	
	Hyp(2) $\supset$:	
	3) $\Sigma\eta\Sigma$.	[$N2$, 1]
	4) $\Psi\eta\Psi$.	[$N2$, 2]
	5) $\mathcal{J}\langle\Phi\rangle$:	[$DN3$, 2]
	$[\exists ab]$:	
	6) $1\,\varepsilon\,\Sigma(a)$.	
	7) $1\,\varepsilon\,\Phi(b)$.	[$N42$, 1]
	8) $a\subset b$.	
	$[\exists cd]$.	
	9) $1\,\varepsilon\,\Phi(c)$.	
	10) $1\,\varepsilon\,\Psi(d)$.	[$N42$, 2]
	11) $c\subset d$.	
	12) $b\circ c$.	[$DN2$, 5, 7, 9]
	13) $a\subset d$.	[8, 12, 11]
	$\Sigma\eta\,\mathbf{el}{\prec}\Psi{\succ}$	[$N42$, 3, 4, 6, 10, 13]
$N54$	$[\Sigma\Phi\sigma]:\Sigma\eta\,\mathbf{Kl}{\prec}\sigma{\succ}\,.\,\Phi\eta\sigma\,.\supset.\,\Phi\eta\,\mathbf{el}{\prec}\Sigma{\succ}$	
	Hyp(2) . $\supset$.	
	3) $\Sigma\eta\Sigma$.	
	4) $1\,\varepsilon\,\Sigma(\bigcup\langle\sigma\rangle)$.	[$N52$, 1]
	5) $\Phi\eta\Phi$.	[$N2$, 2]
	6) $\mathcal{N}\langle\Phi\rangle$.	[$DN3$, 5]
	$[\exists a]$.	
	7) $1\,\varepsilon\,\Phi(a)$.	[$DN1$, 6]
	8) $1\,\varepsilon\,\sigma(a)$.	[$DN3$, 2, 7]
	9) $a\subset\bigcup\langle\sigma\rangle$.	[$D0$, 8]
	$\Phi\eta\mathbf{el}{\prec}\Sigma{\succ}$.	[$N42$, 5, 3, 7, 4, 9]
$N55$	$[\Sigma\Phi\sigma]:\Sigma\eta\mathbf{Kl}{\prec}\sigma{\succ}\,.\,\Phi\eta\mathbf{el}{\prec}\Sigma{\succ}\supset.\,[\exists\Psi X]\,.\,\Psi\eta\sigma\,.\,X\eta\,\mathbf{el}{\prec}\Phi{\succ}\,.\,X\eta\,\mathbf{el}{\prec}\Psi{\succ}$	
	Hyp(2) $\supset.\because$	
	3) $\Sigma\eta\Sigma$.	
	4) $\mathcal{N}\langle\sigma\rangle$.	[$N52$, 1]
	5) $1\,\varepsilon\,\Sigma(\bigcup\langle\sigma\rangle)$.	
	6) $\mathcal{J}\langle\Sigma\rangle$.	[$N1$, 3]
	7) $\Phi\eta\Phi$.	[$N2$, 2]
	8) $\mathcal{N}\langle\Phi\rangle\,.\because$	[$DN3$, 7]
	$[\exists ab]\,.\because$	
	9) $1\,\varepsilon\,\Phi(a)$.	
	10) $1\,\varepsilon\,\Sigma(b)$.	[$N42$, 2]
	11) $a\subset b$.	

12) $b \circ \bigcup\langle\sigma\rangle$. [*DN2*, 6, 5, 10]
13) $a \subset \bigcup\langle\sigma\rangle$: [11, 12]
14) $!(a)$. [*DN1*, 8, 9]
$[\exists A]$:
15) $A \,\varepsilon\, a$. [14]
16) $A \,\varepsilon \bigcup\langle\sigma\rangle$. [13, 15]
17) $!(A)$. [15]
18) $1 \,\varepsilon\, \square\{A\}(A)$. [*N10*, 17]
19) $\square\{A\}\, \eta\, \square\{A\}$. [*N17*, 17, *N1*]
$[\exists c]$.
20) $1 \,\varepsilon\, \sigma(c)$.
21) $A \,\varepsilon\, c$. [*D0*, 16]
22) $!(c)$. [21]
23) $1 \,\varepsilon\, \square\{c\}(c)$. [*N10*, 22]
24) $\mathcal{J}\langle\square\{c\}\rangle$. [*N17*, 22]
25) $\square\{c\}\, \eta\, \square\{c\}$. [*N1*, 24]
26) $\square\{c\}\, \eta\, \sigma$. [*N3*, 24, 4, 23, 20]
27) $\square\{A\}\, \eta\, \mathbf{el}\{\square\{c\}\}$. [*N42*, 19, 25, 18, 23, 21]
28) $\square\{A\}\, \eta\, \mathbf{el}\{\Phi\}$.˙. [*N42*, 19, 7, 18, 9, 15]
$[\exists \Psi\chi] . \Psi\eta\sigma . \chi\eta\, \mathbf{el}\{\Phi\} . \chi\eta\, \mathbf{el}\{\Psi\}$ [26, 28, 27]

N56 $[\Sigma\sigma]$.˙. $\Sigma\eta\Sigma : [\Phi] : \Phi\eta\, \mathbf{el}\{\Sigma\} . \supset . [\exists\Psi] . \Psi\eta\sigma : \supset . \mathcal{N}\langle\sigma\rangle$
Hyp(2) . ⊃:
3) $\mathcal{N}\langle\Sigma\rangle$. [*DN3*, 1]
4) $[\exists a] . 1 \,\varepsilon\, \Sigma(a)$. [*DN1*, 3]
5) $\Sigma\eta\, \mathbf{el}\{\Sigma\}$. [*N42*, 1, 1, 4, 4]
6) $[\exists\Psi] . \Psi\eta\sigma$. [2, 5]
$\mathcal{N}\langle\sigma\rangle$ [*DN3*, 6]

N57 $[\Sigma\sigma Aa]$.˙. $\Sigma\eta\Sigma : [\Phi] : \Phi\eta\sigma . \supset . \Phi\eta\, \mathbf{el}\{\Sigma\} : [\Phi] : \Phi\eta\, \mathbf{el}\{\Sigma\} . \supset [\exists\Psi] . \Psi\eta\sigma :$
$1 \,\varepsilon\, \Sigma(a) . A \,\varepsilon \bigcup\langle\sigma\rangle : \supset . A \,\varepsilon\, a$
Hyp(5) . ⊃:
6) $\mathcal{N}\langle\sigma\rangle$: [*N56*, 1, 3]
$[\exists b]$:
7) $A \,\varepsilon\, b$.
8) $1 \,\varepsilon\, \sigma(b)$. [*D0*, 5]
9) $!(b)$. [7]
10) $1 \,\varepsilon\, \square\{b\}(b)$. [*N10*, 9]
11) $\mathcal{J}\langle\square\{b\}\rangle$. [*N17*, 9]
12) $\square\{b\}\, \eta\, \sigma$. [*N3*, 11, 6, 10, 8]
13) $\square\{b\}\, \eta\, \mathbf{el}\{\Sigma\}$. [2, 12]
$[\exists cd]$.
14) $1 \,\varepsilon\, \square\{b\}(c)$.
15) $1 \,\varepsilon\, \Sigma(d)$. [*N42*, 13]
16) $c \subset d$.
17) $b \circ c$. [*N9*, 14]
18) $a \circ d$. [*N1*, 1, *DN2*, 4, 15]

19) $b \subset a.$ [16, 17, 18]
$A\, \varepsilon\, a$ [7, 19]

N58 $[\Sigma\sigma A a] \therefore \Sigma\eta\Sigma : [\Phi] : \Phi\eta\, \mathbf{el}\prec\Sigma\succ . \supset . [\exists\Psi X] . \Psi\eta\sigma . X\eta\, \mathbf{el}\prec\Phi\succ .$
$X\eta\, \mathbf{el}\prec\Psi\succ : 1\, \varepsilon\, \Sigma(a) . A\, \varepsilon\, a . \supset . A\, \varepsilon \mathbf{U}\langle\sigma\rangle$
Hyp(4) . $\supset$:
5) $!(A) .$ [4]
6) $1\, \varepsilon \square\prec A\succ(A) .$ [*N10*, 5]
7) $\mathcal{J}\langle\square\prec A\succ\rangle .$ [*N17*, 5]
8) $\square\prec A\succ \eta \square\prec A\succ .$ [*N1*, 7]
9) $\square\prec A\succ \eta\, \mathbf{el}\prec\Sigma\succ :$ [*N42*, 8, 1, 6, 3, 4]
$[\exists\Psi X] :$
10) $\Psi\eta\sigma .$
11) $X\eta\, \mathbf{el}\prec\square\prec A\succ\succ .$ [2, 9]
12) $X\eta\, \mathbf{el}\prec\Psi\succ .$
13) $\mathcal{J}\langle X\rangle .$ [*DN3*, 11]
$[\exists bc] .$
14) $1\, \varepsilon\, X(b) .$
15) $1\, \varepsilon \square\prec A\succ(c) .$ [*N42*, 11]
16) $b \subset c .$
17) $A \circ c .$ [*N9*, 15]
18) $b \subset A .$ [16, 17]
19) $!(b) .$ [*DN2*, 13, *DN1*, 14]
20) $b = A .$ [18, 19, 4]
21) $1\, \varepsilon\, X(A) .$ [14, 20]
$[\exists de] .$
22) $1\, \varepsilon\, X(d) .$
23) $1\, \varepsilon\, \Psi(e) .$ [*N42*, 12]
24) $d \subset e .$
25) $A\, \varepsilon\, d .$ [*DN2*, 13, 21, 22, 4]
26) $A\, \varepsilon\, e .$ [24, 25]
27) $1\, \varepsilon\, \sigma(e) .$ [*DN3*, 10, 23]
$A\, \varepsilon \mathbf{U}\langle\sigma\rangle$ [*D0*, 25, 26]

N59 $[\Sigma\sigma] \therefore \Sigma\eta\Sigma : [\Phi] : \Phi\eta\sigma . \supset . \Phi\eta\, \mathbf{el}\prec\Sigma\succ : [\Phi] : \Phi\eta\, \mathbf{el}\prec\Sigma\succ . \supset [\exists\Psi X] .$
$\Psi\eta\sigma . X\eta\, \mathbf{el}\prec\Phi\succ . X\eta\, \mathbf{el}\prec\Psi\succ : \supset . \Sigma\eta\, \mathbf{Kl}\prec\sigma\succ$
Hyp(3) . $\supset$:
4) $\mathcal{N}\langle\sigma\rangle .$ [*N56*, 1, 3]
5) $\mathcal{N}\langle\Sigma\rangle .$ [*DN3*, 1]
$[\exists a] .$
6) $1\, \varepsilon\, \Sigma(a) .$ [*DN1*, 5]
7) $a \circ \mathbf{U}\langle\sigma\rangle .$ [*N58*, 1, 3, 6, *N57*, 1, 2, 3, 6]
8) $1\, \varepsilon\, \Sigma(\mathbf{U}\langle\sigma\rangle) .$ [7, 6]
$\Sigma\eta\, \mathbf{Kl}\prec\sigma\succ$ [*N52*, 1, 4, 8]

N60 $[\Sigma\sigma] \therefore \Sigma\eta\, \mathbf{Kl}\prec\sigma\succ . \equiv : \Sigma\eta\Sigma : [\Phi] : \Phi\eta\sigma . \supset . \Phi\eta\, \mathbf{el}\prec\Sigma\succ : [\Phi] : \Phi\eta\, \mathbf{el}\prec\Sigma\succ .$
$\supset . [\exists\Psi X] . \Psi\eta\sigma . X\eta\, \mathbf{el}\prec\Phi\succ . X\eta\, \mathbf{el}\prec\Psi\succ$ [*N2*, *N54*, *N55*, *N59*]

N61 $[\Sigma\Phi\sigma] : \Sigma\eta\, \mathbf{Kl}\prec\sigma\succ . \Phi\eta\, \mathbf{Kl}\prec\sigma\succ \supset . \Sigma\eta\Phi$
Hyp(2) . $\supset$
3) $\mathcal{J}\langle\Sigma\rangle .$ [*DN3*, 1]

4) $1\,\varepsilon\,\Sigma(\mathbf{U}\langle\sigma\rangle)\,.$ $[N52,\ 1]$
5) $\mathcal{J}\langle\Phi\rangle\,.$ $[DN3,\ 2]$
6) $\mathcal{N}\langle\Phi\rangle\,.$ $[DN2,\ 5]$
7) $1\,\varepsilon\,\Phi(\mathbf{U}\langle\sigma\rangle)\,.$ $[N52,\ 2]$
$\Sigma\eta\Phi$ $[N3,\ 3,\ 6,\ 4,\ 7]$

N62 $[\Sigma\sigma]:\Sigma\eta\sigma\,.\supset.\,[\exists\Phi]\,.\,\Phi\eta\,\mathbf{Kl}\{\sigma\}\,.$
Hyp(1) . ⊃:
2) $\mathcal{N}\langle\sigma\rangle:$ $[DN3,\ 1]$
$[\exists a]:$
3) $1\,\varepsilon\,\sigma(a)\,.$ $[DN1,\ 2]$
4) $!\,(a)\,.$ $[DN1,\ 2,\ 3]$
$[\exists A]\,.$
5) $A\,\varepsilon\,a\,.$ $[4]$
6) $A\,\varepsilon\,\mathbf{U}\langle\sigma\rangle:$ $[D0,\ 3,\ 5]$
7) $!(\mathbf{U}\langle\sigma\rangle)\,.$ $[6]$
8) $1\,\varepsilon\,\square\{\mathbf{U}\langle\sigma\rangle\}(\mathbf{U}\langle\sigma\rangle)\,.$ $[N10,\ 7]$
9) $\square\{\mathbf{U}\langle\sigma\rangle\}\ \eta\ \square\{\mathbf{U}\langle\sigma\rangle\}.$ $[N19,\ 7,\ N1]$
10) $\square\{\mathbf{U}\langle\sigma\rangle\}\ \eta\ \mathbf{Kl}\{\sigma\}\,.$ $[N52,\ 9,\ 2,\ 8]$
$[\exists\Phi]\,.\,\Phi\eta\,\mathbf{Kl}\{\sigma\}$ $[10]$

N63 $[\Sigma\Phi]:\Sigma\eta\,\mathbf{el}\{\Phi\}\,.\supset.\,\Phi\eta\Phi$ $[N42]$

N64 $[\Phi]:\Phi\eta\Phi\,.\supset.\,\Phi\eta\,\mathbf{el}\{\Phi\}$ $[N42,\ 2\mid\Phi,\ DN3,\ DN1]$

N63, *N64*, *N53*, *N60*, *N61* and *N62* give us an analog of the axiom system for mereology given in the introduction of this paper.

REFERENCES

[1] Canty, J. T., "Leśniewski's ontology and Gödel's incompleteness theorem," Ph.D. thesis, University of Notre Dame, June 1967, Notre Dame, Indiana.

[2] Clay, R. E., "The relation of weakly discrete to set and equinumerosity in mereology," *Notre Dame Journal of Formal Logic*, vol. VI (1965), pp. 325-340.

[3] Clay, R. E., "The consistency of Leśniewski's mereology relative to the real number system," *The Journal of Symbolic Logic*, vol. 33 (1968), pp. 251-257.

[4] Clay, R. E., "Contributions to mereology," Ph.D. thesis, University of Notre Dame, August 1961, Notre Dame, Indiana.

University of Notre Dame
Notre Dame, Indiana