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The Published Work of Kurt Gödel: An Annotated Bibliography

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Bibliographies of the published work of Kurt Gödel, all nearly identical and apparently based on Gödel's own bibliography prepared for the Institute for Advanced Study, have previously appeared in [Bulloff, Holyoke, and Hahn, 1969], the *Mathematical Intelligencer*, vol. 1 (1978), p. 185, and [Kreisel, 1980]. None of these are annotated, however, and all are incomplete: in addition to omitting Gödel's published reviews and correspondence, various remarks, abstracts, reprints, and several translations, three short articles ([1933b], [1933c], and [1933i] below) have been uniformly overlooked.

The present bibliography attempts to be comprehensive. The annotations aim to provide chronological and historical background, brief surveys of the contents of lesser-known articles, comparisons of related papers, and references to later developments. No attempt has been made to undertake a detailed critical evaluation. The reader desiring an overview of Gödel's life and work should consult the extensive memoir [Kreisel, 1980], the shorter memoirs [Christian, 1978], [Christian, 1980], [Wang, 1978], or [Wang, 1981], or the survey [Kleene, 1976] and its addendum [Kleene, 1978]. A further memoir by Kleene, incorporating a portrait and bibliography, is forthcoming in the

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Since the initial preparation of this bibliography, I have been invited to catalogue Gödel's *Nachlass* at the Institute for Advanced Study, a task now in progress. I am most grateful for the opportunity thus afforded to revise and extend my original compilation.

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Biographical Memoirs, National Academy of Sciences. Any definitive study of Gödel's work must await scholarly access to his unpublished *Nachlass* at the Institute for Advanced Study and to the archives of the University of Vienna.

This bibliography is divided into six sections, including three appendices. Part I lists all known publications by Gödel (including abstracts and translations) except his reviews of others' papers; the latter are segregated together in Part II. Citations in Part I are by year of publication. Within a given year, articles are distinguished by alphabetical suffixes; within a given issue of a journal, articles are listed in order of appearance. No other ordering is implied by the suffixes; where available, more detailed chronological data are included in the annotations. The reviews in Part II are similarly cited, but with publication dates prefixed by the letter R. Works by other authors are cited in the text by author's name and date; full citations appear in the reference list at the end of the paper. Part III is a guide to remarks and correspondence of Gödel later published by others.

Appendix A is an index to reviews of Gödel's papers. Appendix B is a similar index to published photographs of Gödel. Diligent efforts have been made to ensure completeness, but it is recognized that these appendices are likely to be less comprehensive than Parts I–III. The author will welcome any additions or corrections readers may be able to provide. Appendix C is a cross-reference index to the translations of Gödel's papers.

I Articles, abstracts, and translations

[1930] "Die Vollständigkeit der Axiome des logischen Funktionenkalküls," *Monatshefte für Mathematik und Physik*, vol. 37 (1930), pp. 349-360.

[1930a] "Über die Vollständigkeit des Logikkalküls" (abstract), *Die Naturwissenschaften*, vol. 18 (1930), p. 1068.

Gödel's doctoral dissertation, written at the University of Vienna under the direction of Hans Hahn, was completed in the autumn of 1929. Bearing the same title as [1930a], it established three fundamental results for the first-order predicate calculus (then called the restricted functional calculus): the completeness theorem (that every valid formula of the system is provable within the system), a version of the downward Löwenheim-Skolem theorem (that every formula is either refutable in the system or satisfiable in a denumerable domain of individuals), and the (countable) compactness theorem (that a denumerably infinite set of formulas is satisfiable if and only if every finite subset of them is satisfiable). A rewritten version of the dissertation was submitted to *Monatshefte für Mathematik*, with Hahn as editor, where it was received October 22, 1929 and published as [1930]. Gödel received his degree February 6, 1930 and presented his results at the fifteenth meeting of Menger's Vienna mathematics colloquium, May 14, 1930. (Cf. *Ergebnisse eines mathematischen Kolloquiums*, vol. 2, p. 17.) Later, on September 6, 1930, he spoke in Königsberg at the annual meeting of the Deutsche Mathematiker-Vereinigung, as noted in [1930a].

The question of completeness had been cited as an open problem in

[Hilbert and Ackermann, 1928], p. 68, although, as Gödel noted only much later (cf. his letter to van Heijenoort of August 14, 1964, quoted in [van Heijenoort, 1967], p. 510, and in [Kleene, 1976], pp. 762-763), an implicit solution lay unrecognized in [Skolem, 1923], a work not cited by Hilbert and Ackermann and unknown to Gödel himself at the time (cf. [Kleene, 1978], p. 613). Gödel's demonstration of the completeness theorem, unlike the well-known construction of [Henkin, 1949], is based on ideas of Skolem and Löwenheim and proceeds by induction on the quantifier complexity of normal formulas. For further details and commentary, see [van Heijenoort, 1967], pp. 582-591.

[1930b] "Einige metamathematische Resultate über Entscheidungsdefinitheit und Widerspruchsfreiheit," *Anzeiger der Akademie der Wissenschaften in Wien*, vol. 67 (1930), pp. 214-215.

[1931] "Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I," *Monatshefte für Mathematik und Physik*, vol. 38 (1931), pp. 173-198. (Received for publication November 17, 1930.)

[1931a] "Diskussion zur Grundlegung der Mathematik," *Erkenntnis*, vol. 2 (1931), pp. 147-151.

Among the highlights of the 1930 meeting in Königsberg was Hilbert's address on "Naturerkennen und Logik," and a symposium September 5 on the foundations of mathematics, featuring a talk by F. Waismann on Wittgenstein's work and addresses by Carnap, Heyting, and von Neumann as advocates of the competing philosophies of logicism, intuitionism, and formalism, respectively. (English translations of the last three, by Erna Putnam and Gerald Massey, are available in [Benacerraf and Putnam, 1964], pp. 31-54; see also Gödel's reviews [R1932a], [R1932b], and [R1932c] below.) Organized by the Vienna Circle, whose meetings Gödel regularly attended, the symposium was followed on September 7 by a discussion on the foundations of mathematics among Hahn, Carnap, Scholz, von Neumann, Heyting, Reidemeister, and Gödel. Subsequently both the symposium (except for Waismann's talk) and the ensuing discussion were published in *Erkenntnis* (the former *Annalen der Philosophie*, taken over and renamed by the Vienna Circle in 1930).

In addition to the discussion, [1931a] includes an extensive bibliography of the foundations of mathematics and a *Nachtrag* by Gödel summarizing his incompleteness results, which by then had already appeared as [1931]. In the *Nachtrag* Gödel notes that [1931] had not yet been submitted for publication at the time of the discussion; however, in the text proper (p. 148, as part of his criticism of consistency as a criterion of adequacy for formalized mathematics) Gödel explicitly states that "under the assumption of the consistency of classical mathematics, one can even give examples of propositions . . . which are really contentually true, but are unprovable in the formal system of classical mathematics".[1] Thus it seems clear that Gödel had by then already discovered a form of his first incompleteness theorem. (But not the second; cf. [R1932c] below.) However, according to [Wang, 1981], pp. 654-655, Gödel's original examples of undecidable propositions were finite combinatorial statements, and Gödel did not at first believe that undecidable statements of a simple arithmetical character could be found. Also, as is well known, [1931] actually invoked the stronger assumption of the ω-consistency of arithmetic, a blemish later remedied in [Rosser, 1936]. Both incompleteness theorems were announced in the abstract [1930b], communicated to the Vienna Academy of Sciences by Hahn October 23, 1930, and in a presentation by Gödel to the Vienna colloquium (see [1932b] below).

On September 15, 1931 Gödel spoke on the incompleteness theorems at the annual meeting of the Deutsche Mathematiker-Vereinigung in Bad Elster (cf. *Jahresbericht der Deutschen Mathematiker-Vereinigung*, vol. 41 (1932), pt. 2, p. 85), and on June 25, 1932 he submitted [1931] to the University of Vienna as his *Habilitationsschrift*. (Hahn's laudatory assessment of Gödel's work, in his report of December 1, 1932 granting the *Habilitation*, is reproduced in [Christian, 1980], p. 263.) Gödel spent the academic year 1933-34 at the Institute for Advanced Study in Princeton; it was his first visit to America and the first year of operation of the Institute. There, from February to May of 1934, he lectured on the incompleteness theorems. Notes on these lectures by Kleene and Rosser were circulated in mimeographed form and eventually appeared in print as [1965a].

The proof of the second incompleteness theorem is only sketched in [1931]; a detailed proof, together with generalizations of the first theorem, was to appear in a sequel which, however, was never written, due both to Gödel's ill health and the ready acceptance of his results. (Cf. [Kleene, 1976], p. 767 and [van Heijenoort, 1967], p. 594 and footnote 68a, p. 616.) Details were finally published in [Hilbert and Bernays, 1939], pp. 283-340.

[Smorynski, 1977] provides an up-to-date survey of the incompleteness theorems and their aftermath. [1961], [1962], [1965], and [1967b] below are translations of [1931]. A partial English translation of [1931a], containing only Hahn's remarks, has recently appeared in [Hahn, 1980], pp. 31-38.

[1932] "Zum intuitionistischen Aussagenkalkül," *Anzeiger der Akademie der Wissenschaften in Wien*, vol. 69 (1932), pp. 65-66.

This short note, communicated by Hahn to the Vienna Academy on February 25, 1932, established two results concerning Heyting's intuitionistic propositional calculus: (i) that no realization involving only finitely many truth values satisfies those and only those formulas provable in the system; (ii) that there is a monotone decreasing sequence of systems embracing that of Heyting and contained within the classical propositional calculus.

The question of the number of truth values needed to realize Heyting's system had already been cited by Gödel as an open problem in his remarks November 7, 1931 at Menger's colloquium. (See [1933] below; in the reprint [1933g], Gödel attributes the question to Hahn.)

[1932a] "Ein Spezialfall des Entscheidungsproblems der theoretischen Logik," *Ergebnisse eines mathematischen Kolloquiums*, vol. 2 (1929/30), pp. 27-28.

This undated contribution was not presented to a regular meeting of the colloquium, but appeared among the *Gesammelte Mitteilungen* for 1929/30. In the context of the first-order predicate calculus without equality, Gödel describes an effective procedure for deciding whether or not a formula with prenex form

$$(\exists x_1 \ldots x_n)(y_1 y_2)(\exists z_1 \ldots z_n)A(x_i, y_i, z_i)$$

is satisfiable; the procedure is related to the method used in [1930] to establish the completeness theorem. (Essentially the same procedure was found independently somewhat later by both Kalmar and Schütte–see [Kalmar, 1933], [Schütte, 1934], and Gödel's review [R1933b] of the former.)

For the predicate calculus *with* equality, the corresponding decision problem remains open; indeed, "it is essentially the only prefix class [in the predicate calculus, with or without equality,] whose [decidability] status is unknown." [Goldfarb, 1981], from which the above is quoted, gives a concise summary of the status of all other cases. Cf. also [1933j] below, a related paper whose concluding remark suggests that Gödel mistakenly believed that his method settled the case with equality as well. (Goldfarb's paper conclusively establishes that it does not.)

[1932b] "Über Vollständigkeit und Widerspruchsfreiheit," *Ergebnisse eines mathematischen Kolloquiums*, vol. 3 (1930/31), pp. 12-13.

Closely related to [1931], [1932b] notes extensions of the incompleteness theorems to a wider class of formal systems. The system considered in [1931] is based on *Principia Mathematica* and allows variables of all finite types. Here Gödel observes that any finitely-axiomatizable, ω-consistent formal system S with just substitution and implication (*modus ponens*) as rules of inference will possess undecidable propositions whenever S extends the theory Z of first-order Peano arithmetic plus the schema of definition by recursion; and indeed, that the same is true of infinite axiomatizations so long as the class of Gödel numbers of axioms, together with the relation of immediate consequence under the rules of inference, is definable and decidable in Z.

[1932b] was originally presented to the 24th meeting of the Vienna colloquium, January 22, 1931. [1967c] is an English translation.

[1932c] "Eine Eigenschaft der Realisierungen des Aussagenkalküls," *Ergebnisse eines mathematischen Kolloquiums*, vol. 3 (1930/31), pp. 20-21.

In answer to a question of Menger, Gödel shows that given an arbitrary realization of the axioms of the propositional calculus in a structure with operations interpreting the connectives $\sim$ and $\supset$, the elements of the structure can always be partitioned into two disjoint classes behaving exactly like the classes of true and false propositions. (Presented as part of the 31st colloquium, June 24, 1931.)

[1933] Untitled remark following W. T. Parry, "Ein Axiomensystem für eine neue Art von Implikation (analytische Implikation)," *Ergebnisse eines mathematischen Kolloquiums*, vol. 4 (1931/32), p. 6.

During the 33rd session of the colloquium, November 7, 1931, the American visitor Parry introduced an axiom system for "analytic implication," a concept of logical consequence entailing the unprovability of $A \rightarrow B$ whenever B contains a propositional variable not occurring in A. Following Parry's demonstration (via multi-valued truth tables) of this characteristic property, Gödel suggested that a completeness proof be sought for Parry's axioms, while noting that the question whether Heyting's propositional calculus could be realized using only finitely many truth values was then open. Cf. [1932] above.

On p. 4 of this same issue, an article by Alt ("Zur Theorie der Krümmung") mentions an unpublished suggestion by Gödel.

[1933a] "Über Unabhängigkeitsbeweise im Aussagenkalkül," *Ergebnisse eines mathematischen Kolloquiums*, vol. 4 (1931/32), pp. 9-10.

To Hahn's question, "Can every independence proof for statements of the propositional calculus be carried out by means of finite multi-valued truth tables?" Gödel provides a negative answer. Specifically, using infinitely many truth values he demonstrates the independence of $p \supset \sim\sim p$ from the set of axioms $p \supset p$, $(p \supset \sim\sim q) \supset (p \supset q)$, and $(\sim\sim p \supset \sim\sim q) \supset (p \supset q)$, while showing that any finite realization of those axioms must also realize $p \supset \sim\sim q$. (Delivered to the 37th colloquium, December 2, 1931.)

[1933b] "Über die metrische Einbettbarkeit der Quadrupel des R_3 in Kugelflächen," *Ergebnisse eines mathematischen Kolloquiums*, vol. 4 (1931/32), pp. 16-17.

[1933c] "Über die Waldsche Axiomatik des Zwischenbegriffes," *Ergebnisse eines mathematischen Kolloquiums*, vol. 4 (1931/32), pp. 17-18.

With the exception of [1933h], Gödel's contributions to geometry have been overlooked by bibliographers. Both [1933b] and [1933c] formed part of the 42nd colloquium, held February 18, 1932. In the former, Gödel answers a question raised by Laura Klanfer at the 37th colloquium, December 2, 1931: he shows that whenever a quadruple of points in a metric space is isometric to four noncoplanar points of R_3, the quadruple is isometric, under the *geodesic* metric, to four points on the surface of a sphere. (The corresponding result for the usual metric on R_3 is trivial.) In the second paper Gödel reformulates Wald's axiomatization of the betweenness concept as a theorem about triples of real numbers, assigning the triple of distances $(\overline{ab}, \overline{bc}, \overline{ac})$ to a triple (a, b, c) in a given metric space. The theorem states that b lies between a and c in the sense of Menger if and only if $(\overline{ab}, \overline{bc}, \overline{ac})$ lies in that part of the plane $x + y = z$ for which each of the four quantities x, y, z, and $(x + y - z)(x - y + z)(-x + y + z)$ is nonnegative.

[1933d] "Zur Axiomatik der elementargeometrischen Verknüpfungsrelationen," *Ergebnisse eines mathematischen Kolloquiums*, vol. 4 (1931/32), p. 34.

Only two brief comments were published from the discussion with the above title held as the 51st colloquium, May 25, 1932. In translation, Gödel's remark reads in full: "[Some]one should investigate the system of all those statements about fields that in normal form contain no existential prefixes. The concepts of point and line, which are definable by application of existential prefixes (e.g., a point is an element for which there exists no nonempty element that is a proper part of it), are undefinable in this more restricted system. Investigations in this direction are to be found in Wernick (Crelle's Journal, 161)."

[1933e] "Zur intuitionistischen Arithmetik und Zahlentheorie," *Ergebnisse eines mathematischen Kolloquiums*, vol. 4 (1931/32), pp. 34-38.

In this short but important paper, Gödel shows that although the intuitionistic propositional calculus is customarily regarded as a subsystem of the classical, by a different translation the reverse is true, not only for the propositional calculus but for arithmetic and number theory as well. (Independently and slightly later the same result was discovered by Gentzen and Bernays—see [Gentzen, 1969], note 46, p. 315.) Specifically, with each formula A of Herbrand's system of arithmetic Gödel associates a translation A' in an extension of Heyting's arithmetic, such that A' is intuitionistically provable whenever A is classically provable. Since it provides an intuitionistic consistency proof for classical arithmetic, Gödel's translation gives classical mathematicians grounds for maintaining that insofar as arithmetic is concerned, intuitionistic qualms amount to "much ado about nothing"; for intuitionists, however, the issue is not so much consistency as it is matters of proper interpretation and methodology. (Delivered to the 52nd colloquium, June 28, 1932. [1965b] is an English translation.)

[1933f] "Eine Interpretation des intuitionistischen Aussagenkalküls," *Ergebnisse eines mathematischen Kolloquiums*, vol. 4 (1931/32), pp. 39-40.

By formalizing the concept "p is provable" via a unary predicate Bp satisfying the axioms $Bp \rightarrow p$, $Bp \rightarrow BBp$, and $Bp \rightarrow (B(p \rightarrow q) \rightarrow Bq)$, Gödel shows that Heyting's propositional calculus can be given a natural classical interpretation. Specifically the intuitionistic notions $\neg p$, $p \supset q$, $p \vee q$, and $p \wedge q$ are to be interpreted by $\sim Bp$, $Bp \rightarrow Bq$, $Bp \vee Bq$, and $p \cdot q$. (Published as part of the *Gesammelte Mitteilungen* for 1931/32. [1969] is an English translation.)

Recently, similar axiomatizations have led to a serendipitous reinterpretation of modal logic. See [Boolos, 1979] for details.

[1933g] Reprint of [1932], *Ergebnisse eines mathematischen Kolloquiums*, vol. 4 (1931/32), p. 40.

Though considerably more accessible than [1932], this reprint has not been cited in earlier bibliographies. The text is identical to the original except for the addition of an opening clause attributing the question to Hahn. (Part of the *Gesammelte Mitteilungen* for 1931/32.)

[1933h] "Bemerkung über projektive Abbildungen," *Ergebnisse eines mathematischen Kolloquiums*, vol. 5 (1932/33), p. 1.

This brief note, part of the 53rd colloquium, November 10, 1932, is devoted to proving that every one-to-one mapping of the real projective plane into itself that carries straight lines into straight lines is a collineation.

[1933i] (With K. Menger and A. Wald), "Diskussion über koordinatenlose Differentialgeometrie," *Ergebnisse eines mathematischen Kolloquiums*, vol. 5 (1932/33), pp. 25-26.

Gödel's only joint paper, previously uncited. A single, mildly technical result is established, whose aim is to show that so-called "volume determinants" are appropriate for giving a coordinate-free characterization of Gaussian surfaces. The paper is intended as a contribution to Menger's program for making precise, in a coordinate-free way, the assertion that Riemannian spaces behave locally like Euclidean spaces. (Part of the 63rd colloquium, May 17, 1933.)

[1933j] "Zum Entscheidungsproblem des logischen Funktionenkalküls," *Monatshefte für Mathematik und Physik*, vol. 40 (1933), pp. 433-443.

Received June 22, 1933, this intricately argued paper extends the result obtained in [1932a] concerning formulas whose prenex forms contain exactly two adjacent universal quantifiers. There a decision procedure for such formulas was described; here it is shown that if such a formula is satisfiable at all, it must be satisfiable in a finite domain. Gödel notes that Bernays, Schönfinkel, and Ackermann had shown this property to be true for formulas whose normal forms involved a single universal quantifier, and he goes on to show that the decision problem in the case of three adjacent universal quantifiers is equivalent to the decision problem for arbitrary formulas.

As in [1932a], Gödel works in the first-order predicate calculus without equality; but he remarks in conclusion that the corresponding result "may also be proved, by the same methods, for formulas that contain the = sign". However, when questioned about this claim by Burton Dreben in 1965, Gödel was unable to justify it. Very recently, Goldfarb has shown that there is no primitive recursive decision procedure for the underlying class of formulas, noting that (full) recursive undecidability would imply the existence of a satisfiable such formula having no finite model. It follows that Gödel's methods do not suffice to establish the claim, which remains an open conjecture. See [Goldfarb, 1981] for further details and commentary.

[1936] Untitled remark following A. Wald, "Über die Produktionsgleichungen der ökonomischen Wertlehre," *Ergebnisse eines mathematischen Kolloquiums*, vol. 7 (1934/35), p. 6.

In this two-sentence remark, Gödel notes that a realistic analysis of demand in economics must take into account a firm's income, which depends upon the cost of production. (Part of the 80th colloquium, November 6, 1934.)

[1936a] "Über die Länge von Beweisen," *Ergebnisse eines mathematischen Kolloquiums*, vol. 7 (1934/35), pp. 23-24.

A short but important paper, largely overlooked until the advent of computational complexity theory. In terms of the latter, [1936a] provides an early example of a speed-up theorem, viz., that in formal number theory, passing to higher types (allowing sets of integers, sets of sets of integers, etc.) not only causes some previously unprovable statements to become provable, but also greatly reduces the length of the shortest proofs of some previous theorems. (Originally presented at the 92nd colloquium, June 19, 1935—misprinted as 1934 in the original. [1965c] is a belated English translation.)

A discussion remark by Gödel on F. Waismann's "Bemerkung von Freges und Russells Definition der Zahl" (90th colloquium, June 5, 1935) is mentioned without quotation on p. 15 of this same volume.

[1938] "The consistency of the axiom of choice and of the generalized continuum hypothesis," *Proceedings of the National Academy of Sciences, U.S.A.*, vol. 24 (1938), pp. 556-557.

[1939] "The consistency of the generalized continuum-hypothesis" (abstract), *Bulletin of the American Mathematical Society*, vol. 45 (1939), p. 93.

[1939a] "Consistency-proof for the generalized continuum-hypothesis," *Proceedings of the National Academy of Sciences, U.S.A.*, vol. 25 (1939), pp. 220-224.

[1940] *The Consistency of the Axiom of Choice and of the Generalized Continuum-Hypothesis with the Axioms of Set Theory*, Annals of Mathematics Studies 3, Princeton University Press, 1940. Second edition, revised with added notes, 1951. Seventh printing, with added notes, 1966.

Aside from the completeness and incompleteness theorems, Gödel's consistency proofs in set theory are his most celebrated results. The relations among the preceding four papers may be briefly characterized as follows: [1938] is an announcement of the consistency results, communicated to the N.A.S. November 9, 1938, together with a nontechnical description of the ideas underlying the proofs; [1939] is an abstract of a talk delivered by Gödel on December 28, 1938 at the 45th annual meeting of the American Mathematical Society, held in Richmond and Williamsburg; [1939a], received for publication February 14, 1939, provides concise technical proofs of the results; [1940] is an extended treatment in monograph form, based on notes by George W. Brown on Gödel's lectures at the I.A.S. during the autumn of 1938. Further chronological details are provided in [Kleene, 1978], its review [Kreisel, 1979], and in [Kreisel, 1980] and [Wang, 1981]: by the time of his visit to Princeton in the autumn of 1935, Gödel had already conceived his notion of constructible sets and had used it to prove the relative consistency of the axiom of choice. He told von Neumann of his results and conjectured that the continuum hypothesis would also hold in his model, but he did not obtain his proof for the *GCH* until the summer of 1938. It is generally agreed that this three-year delay was occasioned partly by Gödel's ill health in 1936, but on the basis of conversations with Gödel and partial access to his unpublished papers, Kreisel asserts that there were technical reasons as well; in particular, that Gödel harbored certain qualms about the axiom of replacement and tried originally to avoid its use in the definition of the constructible sets. (A close reading of [1939a] lends support to Kreisel's contention, since the two models considered there are M_{ω_ω} and M_Ω, where Ω denotes the least inaccessible cardinal; the former does not satisfy replacement, while the second does so at the cost of assuming the existence of an inaccessible.)

There are several notable differences among the various presentations. In [1938], Gödel states his results for von Neumann's system S^*, noting that they hold as well for the system of *Principia Mathematica* and for Fraenkel's set theory; further, in addition to the axiom of choice and the generalized continuum hypothesis, Gödel notes the following two consequences of the axiom of constructibility:

(i) The existence of linear nonmeasurable sets which, together with their complements, are one-to-one projections of two-dimensional co-analytic sets.
(ii) The existence of linear co-analytic sets of the power of the continuum containing no perfect subsets.

Details of (i) and (ii) were later worked out by others.

In [1939a] Gödel works in Zermelo's set theory, with or without the axiom of replacement (called the axiom of substitution there and in [1940]—see Note 2, p. 67 of the latter); no mention is made of (i) or (ii). The constructible hierarchy is defined inductively in terms of definability over previous levels, with the αth level denoted by M_α. The consistency results follow by consideration of the *set* models M_{ω_ω} and M_Ω. (For correction of notational mistakes and misprints in [1939a], see [1947], footnote 23, or [1964a], footnote 24.)

In [1940] Gödel adopts instead the class formalism of Bernays. Correspondingly, the domain of the model is a class L, and the axiom of constructibility is expressed by the class equation $V = L$. Also, since in Bernays's system the (infinite) comprehension schema is replaced by a finite axiomatization, Gödel defines L as the range of a (class) function F defined on the ordinals in terms of eight class operations. The abstract concept of definability is thus eliminated in favor of eight concrete operations, at the expense of complicating the argument and somewhat obscuring the underlying ideas.

Overall, Gödel's three principal achievements in set theory may be taken to be the definition of the constructible sets, the introduction of the method of inner-model consistency proofs, and the proof that every constructible subset of M_{ω_α} belongs to $M_{\omega_{\alpha+1}}$. Subsequent developments by others are well known: [Shepherdson, 1951-1953] showed that the inner-model method cannot be applied to prove the independence of $V = L$ (nor hence of the axiom of choice or the generalized continuum hypothesis); [Cohen, 1963-1964] finally established the independence of all three statements via the method of forcing, later recast by Scott and Solovay in terms of Boolean-valued models (cf. Scott's historical introduction to [Bell, 1977]); and [Jensen, 1972] undertook a detailed investigation of the fine structure of L. See also the historical remarks in [Jech, 1973] concerning proofs of the independence of the axiom of choice in various settings, especially the method of permutation models developed by Fraenkel and Mostowski during the years 1922-1939.

[1944] "Russell's mathematical logic," pp. 123-153 in P. A. Schilpp (ed.), *The Philosophy of Bertrand Russell* (*Library of Living Philosophers* series), Northwestern University Press, Evanston, 1944.

Shortly after the discussion recorded in [1931a], Gödel and Arend Heyting were invited by the editors of *Zentralblatt für Mathematik* to contribute a joint report on the foundations of mathematics, stressing logicism and intuitionism, respectively. (See [Troelstra, 1981] as well as Gödel's *curriculum vitae* for his *Habilitation*, reproduced in [Christian, 1980].) But although a correspondence ensued between the collaborators, Gödel never completed his part; a study of his unpublished manuscripts may someday reveal the extent to which that unfinished project may have been incorporated into [1944].

The format of the *Library of Living Philosophers* series provides an opportunity for philosophers to respond to commentaries on their work. Gödel's essay, however, was received late, so that Russell's remarks (p. 741) are confined to an apology for being unable to reply (partially also on grounds of his not having worked in mathematical logic for many years). [1944] has nonetheless been widely reviewed by others—see Appendix A below.

The essay is devoted principally to an epistemological criticism of Russell's "no-class" theory, in particular his vicious circle principle. In Gödel's view (pp. 136-137), "the assumption [that classes and concepts may be conceived as real objects] is quite as legitimate as the assumption of physical bodies, and there is quite as much reason to believe in their existence"; but for "objects that exist independently of our constructions, there is nothing in the least absurd in the existence of totalities containing members which can be described . . . only by reference to [that] totality." [1964] and [1972] are reprints.

[1947] "What is Cantor's continuum problem?," *American Mathematical Monthly*, vol. 54 (1947), pp. 515-525; errata, vol. 55 (1948), p. 151.

Gödel's only popular exposition is notable for its advocacy of mathematical Platonism and for Gödel's conjecture that the continuum hypothesis is not only independent of set theory, but actually false. Of special interest is footnote 26, omitted in the later revision ([1964a]), in which Gödel broaches the notion of ordinal definable sets, notes the relative consistency of $L = OD$, and asserts the independence of the GCH from the assumption that $V = OD$. (Cf. also the discussion of ordinal definable sets in [1965d], a transcript of remarks made by Gödel in 1946.)

Concerning Gödel's views on the continuum hypothesis, [Wang, 1978] states that "in 1943 . . . Gödel arrived at a proof of the independence of the axiom of choice in the framework of (finite) type theory. [His] method looked promising toward proving also the independence of the continuum hypothesis, [but he] did not succeed." (Cf. also [Wang, 1981], p. 657, and [Kreisel, 1980], p. 201.)

[1948] "Sovméstimost' aksiomy vybora i obobščénnoj Kontinuum-gipotézy s aksiomami téorii množestv," Russian translation of [1940] by A. A. Markov, *Uspêhi Matematiceskih Nauk* (n.s.) 3:1 (1948), pp. 96-149.

[1949] "An example of a new type of cosmological solutions of Einstein's field equations of gravitation," *Reviews of Modern Physics*, vol. 21 (1949), pp. 447-450.

[1949a] "A remark about the relationship between relativity theory and idealistic philosophy," pp. 555-562 in P. A. Schilpp (ed.), *Albert Einstein, Philosopher-Scientist* (*Library of Living Philosophers* series), Northwestern University Press, Evanston, 1949.

[1950] "Rotating universes in general relativity theory," *Proceedings of the International Congress of Mathematicians in Cambridge, Massachusetts*, I (1950), pp. 175-181.

Though overshadowed by his results in mathematical logic, Gödel's physical writings are considered significant contributions to general relativity theory. The three papers, directed toward different audiences, reflect a convergence of long-standing intellectual interests: originally a physics major at the University of Vienna, Gödel pursued the study of philosophy throughout his life and was motivated by philosophical considerations in much of his scientific work. Cf. [Kreisel, 1980] or [Wang, 1981]. In particular, both his mathematical and physical investigations are characterized by a persistent concern for nonstandard interpretations of formal theories.

In [1949] Gödel defines a cosmological space S, proves that it satisfies Einstein's field equations, and shows that S is stationary (nonexpanding), spatially homogeneous, and rotationally symmetric, but that there exist within it closed time-like lines, so that "it is theoretically possible in these worlds to travel into the past". Thus, although "a positive direction of time can consistently be introduced" throughout the space, there is "no uniform temporal ordering of all point events" and no concept of absolute time "definable without reference to individual objects such as . . . particular galactic system[s]". Gödel notes without proof that his space and Einstein's static universe are essentially "the only spatially homogeneous cosmological solutions with non-vanishing density of matter and equidistant world lines of matter".

Since Gödel's model is nonexpanding, it is, as noted in [1949], in conflict with the observed red-shift of distant objects. Nonetheless, in [1949a] Gödel takes pains to indicate the possible relevance of his model to our world, noting that "there exist . . . also expanding rotating solutions" in which "an absolute time might also fail to exist". Besides providing a nontechnical description of his discoveries, [1949a] includes a brief discussion of some of the underlying philosophical issues, in agreement with Wang's report that Gödel's work in physics was inspired by "his interest in Kant's philosophy of space and time rather than [by] his talks with Einstein". ([Wang, 1981]; see also the comments by Kreisel and Penrose in [Kreisel, 1980], pp. 214-216.) Einstein's response (pp. 687-688) hails Gödel's discovery as an important advance and acknowledges his own earlier qualms about the concept of absolute time.

[1950] is the text of Gödel's invited address August 31, 1950 to the International Congress of Mathematicians. In it Gödel goes beyond [1949] to consider solutions satisfying the conditions of spatial homogeneity, finiteness, and nonconstancy of the density of matter, observing that these conditions (even without finiteness) rule out rotationally symmetric rotating universes and yield "*a directly observable necessary and sufficient criterion for the rotation of an expanding spatially homogeneous and finite universe; namely, for sufficiently great distances, there must be more galaxies in one half of* *the sky than in the other half*". Furthermore, a "*necessary and sufficient condition for the non-existence of closed time-like lines ... is that the metric in the spaces of constant density be space-like*". For a discussion of later developments stimulated by Gödel's work, including further restrictive hypotheses as well as other "pathological" models subsequently discovered, see Chapter 5 of [Hawking and Ellis, 1973].

[1955] "Eine Bemerkung über die Beziehungen zwischen der Relativitätstheorie und der idealistischen Philosophie," German translation of [1949a] by Hans Hartmann, pp. 406-412 in *Albert Einstein als Philosoph und Naturforscher*, W. Kohlhammer, Stuttgart, 1955. Reprinted 1979, Vieweg & Sohn, Wiesbaden. [This translation includes additions by Gödel to footnotes 11, 13 and 14.]

[1958] "Über eine bisher noch nicht benützte Erweiterung des finiten Standpunktes," *Dialectica*, vol. 12 (1958), pp. 280-287.

[1959] Reprint of [1958], pp. 76-83 in *Logica: Studia Paul Bernays Dedicata*, Editions du Griffon, Neuchatel-Suisse, 1959.

Gödel's consistency proof for arithmetic, published as [1958], was obtained in 1942 and first presented that year in lectures by Gödel at Princeton and Yale; cf. [Wang, 1981], p. 657. It has since proved seminal for developments in constructive mathematics. Though of the same strength as Gentzen's consistency proofs (involving principles equivalent to ordinal induction up to ϵ_o), Gödel's method, invoking the concept of primitive recursive functional of finite type, has proved adaptable in wider contexts; in particular, Gödel's ideas led to Spector's important proof ([Spector, 1962]) of the consistency of analysis (second-order arithmetic). (Cf. [1962a] below.) A partial bibliography of the extensive literature spawned by the *Dialectica* paper is included in [1980].

[1961] "Preposizioni formalmente indecidibili dei Principia Mathematica e di sistemi affini," Italian translation of [1931] by Evandro Agazzi, pp. 203-228 in *Introduzione ai problemi dell'assiomatica*, Pubblicazioni dell'Università Cattolica del Sacro Cuore, Serie terza, Scienze filosofiche no. 4, Società Editrice Vita e Pensiero, Milan, 1961.

[1962] *On Formally Undecidable Propositions of Principia Mathematica and Related Systems*, English translation of [1931] by B. Meltzer, Oliver and Boyd, Edinburgh, 1962.

The earliest of the three available English translations of [1931], [1962] was prepared without Gödel's approval and is seriously deficient in many respects; for corrections, see the devastating review by Bauer-Mengelberg. (Cf. Appendix A.)

[1962a] Postscript to [Spector, 1962], p. 27.

Spector's paper was published posthumously, and Gödel's postscript briefly describes the circumstances under which the paper was written, noting especially Kreisel's influence. Gödel himself revised Spector's original title.

[1964] Reprint of [1944], pp. 211-232 in [Benacerraf and Putnam, 1964].

As noted by Gödel in a prefatory footnote, the text of [1944] is here reproduced unaltered, no attempt having been made to update it to take account of later developments.

[1964a] Revised and expanded version of [1947], pp. 258-273 in [Benacerraf and Putnam, 1964].

The revisions to the text of [1947] consist largely of accommodations to the finer points of English syntax; in addition, the text is updated to recognize the development during the interim of category theory (footnote 12) and of new axioms of infinity (footnotes 16 and 20). The original article is expanded by the addition of a supplement and postscript. The former is primarily a reply to critics, against whom Gödel maintains that the status of the continuum hypothesis is comparable to that of Euclid's fifth postulate: the question of its truth, he argues, would not be rendered meaningless by a proof of its undecidability. The postscript, announcing Cohen's forthcoming results, then provides a dramatic and unexpectedly timely climax.

[1965] "On formally undecidable propositions of Principia Mathematica and related systems, I," English translation of [1931] by Elliott Mendelson, pp. 4-38 in [Davis, 1965].

[1965a] "On undecidable propositions of formal mathematical systems," pp. 39-74 in [Davis, 1965].

[1965b] "On intuitionistic arithmetic and number theory," English translation of [1933e] by Martin Davis, pp. 75-81 in [Davis, 1965].

[1965c] "On the length of proofs," English translation of [1936a] by Martin Davis, pp. 82-83 in [Davis, 1965].

[1965d] "Remarks before the Princeton Bicentennial Conference on Problems in Mathematics," (December 17, 1946), pp. 84-88 in [Davis, 1965].

Bauer-Mengelberg has given a detailed critical review of Davis's anthology which includes an extensive list of corrections to the papers therein. (Cf. Appendix A.) In addition to providing an improved English translation of [1931], the anthology includes the only English translations of [1933e] and [1936a], as well as the notes of Gödel's 1934 lectures and the text of his 1946 address.

A brief commentary by Davis preceding [1965a] indicates the main differences between the treatments there and in [1931]; in addition, the notes are followed by a Postscriptum by Gödel dated June 3, 1964. The latter notes the later contributions of Turing and Rosser, cites [Hilbert and Bernays, 1939] and the *Dialectica* paper, and mentions Gödel's unpublished result that the undecidable sentence may be taken to be a Π_2 statement with polynomial matrix of degree 4.

[1965d] appears without introduction or annotation. It is noteworthy for the directions outlined by Gödel for further investigations in set theory. On the one hand, Gödel foresaw that axioms asserting the largeness of the universe of sets might be used to replace arguments involving passages to higher types; on the other, he suggested that the class of ordinal definable sets might yield a simpler proof of the consistency of the axiom of choice

(though not of the continuum hypothesis). Both ideas have since proven fruitful, although the concept of ordinal definable sets lay dormant for nearly twenty years before its independent rediscovery by several others; see for example [Myhill and Scott, 1971]. For an overview of the vast literature of large-cardinal theory, see [Kanamori and Magidor, 1978].

[1967] "The completeness of the axioms of the functional calculus of logic," English translation of [1930] by Stefan Bauer-Mengelberg, pp. 582-591 in [van Heijenoort, 1967].

[1967a] "Some metamathematical results on completeness and consistency," English translation of [1930b] by Stefan Bauer-Mengelberg, pp. 595-596 in [van Heijenoort, 1967].

[1967b] "On formally undecidable propositions of Principia Mathematica and related systems I," English translation of [1931] by Jean van Heijenoort, pp. 596-616 in [van Heijenoort, 1967]. [With added note by Gödel, August 28, 1963.)

[1967c] "On completeness and consistency," English translation of [1932b] by Jean van Heijenoort, pp. 616-617 in [van Heijenoort, 1967]. [With addition by Gödel to footnote 1, May 18, 1966.]

Van Heijenoort's source book is a superb anthology that places Gödel's completeness and incompleteness papers in historical perspective with papers of Frege, Russell, Skolem, Herbrand, Finsler, and others. The translations, of very high quality, are accompanied by detailed critical commentary on both historical and technical aspects of the papers.

[1967d] "La logica matematica di Russell," Italian translation of [1944] by Francesco Gana, pp. 81-112 in [Cellucci, 1967].

[1967e] "Che cos'è il problema del continuo di Cantor?," Italian translation of [1947] by Carlo Cellucci, pp. 113-136 in [Cellucci, 1967].

[1967f] "Osservazioni al Convegno su Problemi di Matematica per il Secondo Centenario di Princeton," Italian translation of [1965d] by Carlo Cellucci, pp. 137-141 in [Cellucci, 1967].

[1967g] "Ob odnom éščé né ispol'zovannom rasširénii finitnoj točki zréniá," Russian translation of [1958] by G. E. Minc (with added appendix), pp. 299-310 in A. V. Idél'son and G. E. Minc (eds.), *Matematičéskaá téoriá logičéskogo vyvoda*, Matematičéskaá logika i osnovaniá matématiki, Izdatel'stvo "Nauka", Moscow, 1967.

[1968] Reprint of [1965d], with a few minor changes in wording, pp. 250-253 in R. Klibansky (ed.), *Contemporary Philosophy, A Survey, I, Logic and Foundations of Mathematics*, La Nuova Italia Editrice, 1968.

[1969] "An interpretation of the intuitionistic sentential logic," English translation of [1933f] by J. Hintikka and L. Rossi, pp. 128-129 in J. Hintikka, *The Philosophy of Mathematics*, Oxford University Press, London, 1969.

[1969a] "La logique mathematique de Russell," French translation of [1944] by J. A. Miller and J. C. Milner, pp. 87-107 in *La Formalization, Cahiers pour l'Analyse*, vol. 10, Editions du seuil, Paris, 1969.

[1971] Reprint of [1932], p. 186 in [Berka and Kreiser, 1971].
[1971a] Reprint of [1933f], pp. 187-188 in [Berka and Kreiser, 1971].
[1971b] Reprint of [1930], pp. 283-294 in [Berka and Kreiser, 1971].
[1971c] Reprint of [1930b], pp. 320-321 in [Berka and Kreiser, 1971].
[1972] Reprint of [1944], pp. 192-226 in D. F. Pears (ed.), Bertrand Russell, *A Collection of Critical Essays*, Anchor Books, Garden City, New York, 1972. In comparison with [1964] the prefatory footnote has been expanded and minor changes made in footnotes 7, 17, and 45; footnote 50 has been deleted.
[1973] "Appendice agli 'Atti del secondo Convegno di Epistemologia delle scienze esatte' di Königsberg (1931)," Italian translation of the *Nachtrag* to [1931a], pp. 55-57 in Ettore Casari, *La Filosofia della Matematica del 1900*, Sansoni, Firenze, 1973.
[1979] "A lógica matemática de Russell," Portuguese translation of [1944] by Manuel Lourenço, pp. 183-216 in [Lourenço, 1979].
[1979a] "O que é o problema do contínuo de Cantor?," Portuguese translation of [1947] by Manuel Lourenço, pp. 217-244 in [Lourenço, 1979].
[1979b] "Acerca de proposições formalmente indecidíveis nos *Principia Mathematica* e sistemas replacionados," Portuguese translation of [1931] by Manuel Lourenço, pp. 245-290 in [Lourenço, 1979].
[1979c] "Acerca de proposições indecidíveis de sistemas matemáticos formais," Portuguese translation of [1965a] by Manuel Lourenço, pp. 291-358 in [Lourenço, 1979].
[1979d] "Acerca da aritmética e da teoria intuicionista dos números," Portuguese translation of [1933e] by Manuel Lourenço, pp. 359-369 in [Lourenço, 1979].
[1979e] "Acerca do comprimento das demonstrações," Portuguese translation of [1936a] by Manuel Lourenço, pp. 371-375 in [Lourenço, 1979].
[1979f] "Comunicação ao bicentenário sobre problemas de matemática," Portuguese translation of [1965d] by Manuel Lourenço, pp. 377-383 in [Lourenço, 1979].
[1979g] Reprint of [1939a], in *Mengenlehre*, Wissenschaftliche Buchgesellschaft, Darmstadt, 1979.
[1980] "On a hitherto unexploited extension of the finitary standpoint," English translation of [1958] by Wilfrid Hodges and Bruce Watson, *Journal of Philosophical Logic*, vol. 9 (1980), pp. 133-142.

Earlier bibliographies have cited a revised English translation of [1958] as forthcoming in *Dialectica*. Indeed, a revision by Gödel of a translation by Leo F. Boron survives in galley proof in the *Nachlass*, but was never published. Cf. Feferman's recent review of [1980] cited in Appendix A. Thus the Hodges-Watson translation, published after Gödel's death with his wife's permission, is so far the only version of [1958] to be published in English. Despite its availability, I believe that the German original should always be consulted, as the philosophical character of [1958] poses special difficulties for any would-be translator. [1980] does include a useful bibliography, compiled by J. R. Hindley, of work resulting from [1958].

[1981] *Obras Completas*, Spanish translations edited by Jesús Mosterín, Alianza Editorial, Madrid, 1981.

Despite its title, this collection is incomplete, omitting [1933b], [1933c], and [1933i]. The translations are by Mosterín, with the following exceptions: [1938], [1939a], [1940], [1944], [1947], [1965], and [1965d] are by Enrique Casanovas; [1949], [1949a], and [1950] are by Ulises Moulines. Each paper is preceded by an introduction by Mosterín. The complete contents, listed in order of appearance in the volume with cross-references to this bibliography, are as follows: "La sufficienza de los axiomas del cálculo lógico de primer orden," ([1930]), pp. 20-34; "Un caso especial del probleme de la decisión en la lógica teórica," ([1932a]), pp. 37-39; "Algunos resultados metamatemáticos sobre completud y consistencia," ([1930b]), pp. 42-43; "Sobre sentencias formalmente indecidibles de *Principia Mathematica* y sistemas afines," ([1931]), pp. 55-89; "Sobre completud y consistencia," ([1932b]), pp. 92-94; "Discusión sobre la fundamentación de la matemática," ([1931a]), pp. 99-100; "Una propiedad de los modelos del cálculo conectivo," ([1932c]), pp. 103-104; "Sobre pruebas de independencia en el cálculo conectivo," ([1933a]), pp. 107-108; "Sobre el cálculo conectivo intuicionista," ([1932]), pp. 110-111; "Una interpretación del cálculo conectivo intuicionista," ([1933f]), pp. 115-116; "Sobre la teoriá de números y la aritmética intuicionista," ([1933e]), pp. 120-126; "Sobre el problema de la decisión la lógica de primer orden," ([1933j]), pp. 130-144; "Observaciones sobre aplicaciones proyectivas," ([1933h]), p. 146; "Sobre sentencias indecidibles de sistemas formales matemáticos," ([1965]), pp. 151-182; "Recensión de Th. Skolem [1933]," ([R1934b]), pp. 185-186; "Sobre la longitud de las deducciones," ([1936a]), pp. 189-190; "La consistencia del axioma de elección y la hipótesis generalizada del continuo," ([1938]), pp. 192-194; "Prueba de la consistencia de la hipótesis generalizada del continuo," ([1939a]), pp. 197-203; "La consistencia del axioma de elección y de la hipótesis generalizada del continuo con los axiomas de la teoría de conjunctos," ([1940]), pp. 214-293; "La lógica matemática de Russell," ([1944]), pp. 297-327; "Observaciones presentadas ante la conferencia del bicentenario de Princeton sobre problemas de las matemáticas," ([1965d]), pp. 331-335; "¿Qué es el problema del continuo de Cantor?," ([1947]), pp. 340-362; "Un ejemplo de un nuevo tipo de soluciones cosmológicas a las ecuaciones einsteinianas del campo gravitatorio," ([1949]), pp. 365-376; "Una observación sobre la relación entre la teoría de la relatividad y la filosofía idealista," ([1949a]), pp. 379-385; "Universos rotatorios en la teoría general de la relatividad," ([1950]), pp. 389-400; "Sobre una ampliación todavía no utilizada del punto de vista finitario," ([1958]), pp. 404-411; "Declaración sobre el análisis no-estándar," ([Robinson, 1974], p. x), pp. 415-416.

[1981a] "Su un'estensione fino a oggi non ancora utilizzata del punto di vista finitista," Italian translation of [1958] by Angelo Odone, pp. 117-123 in Donatella Cagnoni (ed.), *Teoria della dimostrazione*, Feltrinelli, Milano, 1981.

II Reviews by Gödel During the years 1931-1936 Gödel was an active reviewer for *Zentralblatt für Mathematik und ihre Grenzgebiete*, but after volume 12 (1936) his contributions abruptly ceased. He is not listed in the index of reviewers for *The Journal of Symbolic Logic* (vol. 26, pp. 143-148), and records of the American Mathematical Society show that Gödel never responded to a rather belated invitation (in 1962) to contribute to *Mathematical Reviews*. Neither was he ever a reviewer for *Jahrbuch über die Fortschritte der Mathematik*. No other reviews have been found, but some widely scattered reviews may have been overlooked.

Citations below list volume and page of the *Zentralblatt* review, followed by author and title of article reviewed and publication information as cited in the review. Reviews are reportorial unless otherwise noted.

[R1931]	Z1, 5-6	Neder, Ludwig: "Über den Aufbau der Arithmetik," *Jber. dtsch. Math. ver. igg.* 40, 22-37 (1931).
[R1931a]	Z1, 260	Hilbert, David: "Die Grundlegung der elementaren Zahlenlehre," *Math. Annalen* 104, 485-494 (1931).
[R1932]	Z2, 3	Skolem, Th.: "Über einige Satzfunktionen in der Arithmetik," *Skr. norske Vidensk. Akad. Oslo, Math.-naturwiss. Kl.* Nr 7, 1-28 (1931).
[R1932a]	Z2, 321	Carnap, Rudolf: "Die logistische Grundlegung der Mathematik," *Erkenntnis* 2, 91-105 and 135-155 (1931).
[R1932b]	Z2, 321-322	Heyting, Arend: "Die intuitionistische Grundlegung der Mathematik," *Erkenntnis* 2, 106-115 and 135-155 (1931).
[R1932c]	Z2, 322	Neumann, Johann v.: "Die formalistische Grundlegung der Mathematik," *Erkenntnis* 2, 116-121 and 135-155 (1931).

The last two sentences of this review confirm that the second incompleteness theorem had not been obtained at the time of the Königsberg conference. Cf. the commentary to [1931] above.

[R1932d]	Z2, 323	Klein, Fritz: "Zur Theorie der abstrakten Verknüpfungen," *Math. Annalen* 105, 308-323 (1931).
[R1932e]	Z3, 289	Hoensbroech, Franz Graf: "Beziehungen zwischen Inhalt und Umfang von Begriffen," *Erkenntnis* 2, 291-300 (1931).
[R1932f]	Z3, 291	Klein, Fritz: "Über einen Zerlegungssatz in der Theorie der abstrakten Verknüpfungen," *Math. Ann.* 106, 114-130 (1932).
[R1932g]	Z4, 145-146	Church, Alonzo: "A set of postulates for the foundation of logic," *Ann. of Math.*, 11.s.33, 346-366 (1932).

Church's original system was subsequently shown to be inconsistent. Cf. [R1934d] and [R1936] below.

[R1932h]	Z4, 146	Kalmár, László: "Ein Beitrag zum Entscheidungsproblem," *Acta Litt. Sci. Szeged* 5, 222-236 (1932).
[R1932i]	Z4, 146	Huntington, Edward V.: "A new set of independent postulates for the algebra of logic with special reference to Whitehead and Russell's Principia Mathematica," *Proc. Nat. Acad. Sci. U.S.A.* 18, 179-180 (1932).
[R1932j]	Z4, 385	Skolem, Th.: "Über die symmetrisch allgemeinen Lösungen im identischen Kalkul," *Skr. norske Vid.-Akad., Oslo* Nr 6, 1-32 and *Fundam. Math.* 18, 61-76 (1932).
[R1933]	Z5, 146	Kaczmarz, S.: "Axioms for arithmetic," *J. London Math. Soc.* 7, 179-182 (1932).
[R1933a]	Z5, 337-338	Lewis, C. I.: "Alternative systems of logic," *Monist* 42, 481-507 (1932).
[R1933b]	Z6, 385-386	Kalmár, László: "Über die Erfüllbarkeit derjenigen Zählausdrücke, welche in der Normalform zwei benachbarte Allzeichen enthalten," *Math. Ann.* 108, 466-484 (1933).

Gödel notes that Kalmár's solution of this case of the decision problem does not differ essentially from that outlined in [1932a].

[R1934]	Z7, 97-98	Skolem, Th.: "Ein kombinatorischer Satz mit Anwendung auf ein logisches Entscheidungsproblem," *Fundam. Math.* 20, 254-261 (1933).
[R1934a]	Z7, 98	Quine, W. V.: "A theorem in the calculus of classes," *J. London Math. Soc.* 8, 89-95 (1933).
[R1934b]	Z7, 193-194	Skolem, Th.: "Über die Unmöglichkeit einer vollständigen Charakterisierung der Zahlenreihe mittels eines endlichen Axiomensystems," *Norsk Mat. Forenings Skr.*, 11.s. Nr 1/12, 73-82 (1933).

In the middle of this review, Gödel notes that the existence of nonstandard models of arithmetic follows easily from his own work in [1931]. This statement is misleading, however, since Skolem considers models of the full theory of **N**, to which the incompleteness theorem is irrelevant. What is true, in the words of [Kleene, 1976], p. 768, is that "It has been little noticed that the incompleteness of first-order formal systems for number theory (Gödel [1931]) and the completeness of the first-order logical calculus (Gödel [1930]) taken together (without using compactness . . .) imply the existence of nonstandard models of effectively given axiom systems of arithmetic. I have found only a two-line allusion to this in Gödel's *Zentralblatt* review . . . of Skolem [1933], and three lines in Henkin [1950, p. 91] prior to my own independent discussion of it in [1952, p. 430]." Skolem's result does, of course, follow easily from the compactness theorem of [1930].

[R1934c]	Z7, 385	Chen, Kien-Kwong: "Axioms for real numbers," *Tohoku Math. J.* 37, 94-99 (1933).

A very brief, negative review. Gödel notes that the proposed axiom system actually involves only the $<$ relation and, despite the author's claim, does not suffice to prove the existence of countable dense subsets. This and the following two reviews were written at Princeton rather than Vienna.

[R1934d] Z8, 289 Church, Alonzo: "A set of postulates for the foundation of logic. II," *Ann. of Math.* II.s.34, 839-864 (1933).

Gödel notes the contradiction, found after the appearance of [R1932g], in Church's earlier system and describes the modifications introduced to avoid it. Unfortunately, the revised system also turned out to be inconsistent; [R1936] describes Church's ultimate resolution of the difficulties.

[R1934e] Z9, 3 Notcutt, Bernard: "A set of axioms for the theory of deduction," *Mind* 43, 63-77 (1934).

Gödel treats with obvious dubiety this attempt to remove from the usual formalizations of mathematical systems what its author regards as tacit assumptions, e.g., that formulas are read from left to right.

[R1935] Z10, 49 Skolem, Th.: "Über die Nicht-charakterisierbarkeit der Zahlenreihe mittels endlich oder abzählbar unendlich vieler Aussagen mit ausschließlich Zahlenvariablen," *Fundam. Math.* 23, 150-161 (1934).

[R1935a] Z10, 49 Huntington, Edward V.: "Independent postulates related to C. I. Lewis's theory of strict implication," *Mind* 43, 181-198 (1934).

[R1935b] Z11, 1 Carnap, Rudolf: "Die Antinomien und die Vollständigkeit der Mathematik," *Mh. Math. Phys.* 41, 263-284 (1934).

The concluding sentence of this review cites the related ideas of Tarski, helping to place the latter in historical perspective.

[R1935c] Z11, 3-4 Kalmár, László: "Über einen Löwenheimschen Satz," *Acta Litt. Sci. Szeged* 7, 112 bis 121 (1934).

[R1936] Z12, 241-242 Church, Alonzo: "A proof of freedom from contradiction," *Proc. Nat. Acad. Sci. U.S.A.* 21, 275-281 (1935).

In this essentially reportorial review, Gödel seems impressed by the economy of means by which Church at last achieved a consistent reformulation of his earlier systems.

III Correspondence, extracts, and quoted remarks Very little of Gödel's correspondence has yet been published, and most of what has appeared is fragmentary. It seems that no uniform cataloguing has yet been attempted.

The only scholarly study to date of any of the correspondence is [Grattan-Guinness, 1979], which includes the complete text of Gödel's reply to Zermelo's criticism of the incompleteness results. [Christian, 1980] reproduces the text of Gödel's *Habilitation* application, which mentions the proposed article with Heyting (cf. the annotation to [1944]. above). [Troelstra, 1981] also refers to that ill-fated collaboration and to the ensuing correspondence, preserved among Heyting's papers.

Extracts of letters from Gödel to Hao Wang, dated December 7, 1967, and March 7, 1968, are quoted in [Wang, 1974], pp. 8-11; further remarks by Gödel, touching on a number of philosophical issues, are quoted on pp. 187 and 325-326 and paraphrased on pp. 84-87, 186-190, and 324-326. An extract from a letter of Gödel to A. W. Burks, clarifying certain of von Neumann's references to Gödel's work, is quoted in [von Neumann, 1966], pp. 55-56, and a remark of Gödel's concerning one of von Neumann's axioms for set theory appears in [Ulam, 1958], footnote 5, p. 13. Further references to Gödel's letters, including some brief quotations and paraphrases, are to be found in [Kreisel, 1980], [Kleene, 1976 and 1978], and in the editorial commentary to the translations of Gödel's papers in [van Heijenoort, 1967]. (A few paraphrases are also to be found in a report of a journalist's popular interview with Gödel, which appeared in the *New Yorker*, August 23, 1952, pp. 13-15, under the title "Inexhaustible.")

A brief statement of Gödel's views on geometric intuition, quoted from a letter of October 1973 to Marvin Greenberg, appears in [Greenberg, 1980], p. 250, following an excerpt from [1964a]. Remarks by Gödel on nonstandard analysis are quoted on page x of the preface to [Robinson, 1974], and a tribute to Robinson, quoted from a letter of Gödel to Robinson's widow, appears opposite the frontispiece portrait of Robinson in [Saracino and Weispfenning, 1975]. Gödel's response to rumors of his having anticipated Cohen's independence proofs, quoted from a letter to Wolfgang Rautenberg of June 30, 1967, appears in [Rautenberg, 1965].

Appendix A: Index to reviews of Gödel's publications Each reference to an entry from Part I is followed by a list, by author and source, of its reviews. *Z*, *MR*, *JSL*, and *JFM* abbreviate, respectively, *Zentralblatt für Mathematik und ihre Grenzgebiete, Mathematical Reviews, The Journal of Symbolic Logic*, and *Jahrbuch über die Fortschritte der Mathematik.*

[1930]	Lilly Buchhorn, *JFM*, vol. 56 (1930), pp. 46-47.
[1931]	Walter Dubislav, *Z*, vol. 2 (1932), p. 1.
[1931a]	G. Feigl, *JFM*, vol. 57 (1931), p. 54 (*Nachtrag*). C. G. Hempel, *JFM*, vol. 57 (1931), p. 54 (*Anhang*).
[1932]	Th. Skolem, *JFM*, vol. 59 (1933), pp. 866-867.
[1932a]	W. Ackermann, *JFM*, vol. 57 (1931), p. 1321.
[1932b]	W. Ackermann, *JFM*, vol. 57 (1931), p. 1318. H. Busemann, *Z*, vol. 4 (1932), p. 266.
[1932c]	W. Ackermann, *JFM*, vol. 57 (1931), pp. 1319-1320.
[1933a]	Haskell Curry, *Z*, vol. 7 (1934), p. 13. Thoralf Skolem, *JFM*, vol. 59 (1933), p. 865.

[1933c] H. Busemann, *Z*, vol. 4 (1932), p. 266.
Erika Pannwitz, *JFM*, vol. 59 (1933), p. 1261.

[1933e] Haskell Curry, *Z*, vol. 7 (1934), p. 193.
Thoralf Skolem, *JFM*, vol. 59 (1933), pp. 865-866.

[1933f] Haskell Curry, *Z*, vol. 7 (1934), p. 13.
Thoralf Skolem, *JFM*, vol. 59 (1933), p. 866.

[1933h] Erika Pannwitz, *JFM*, vol. 59 (1933), p. 1297. (Misprint: "R. Gödel" for "K. Gödel.")

[1933j] Haskell Curry, *Z*, vol. 8 (1934), p. 289.
Abraham A. Fraenkel, *JFM*, vol. 59 (1933), p. 865.

[1936a] F. Bachmann, *JFM*, vol. 62 (1936), p. 43.
Haskell Curry, *Z*, vol. 14 (1936), p. 241.
J. Barkley Rosser, *JSL*, vol. 1 (1936), p. 116.

[1938] G. Aumann, *JFM*, vol. 64 (1938), p. 35.
Paul Bernays, *JSL*, vol. 5 (1940), pp. 116-117.
L. Egyed, *Z*, vol. 20 (1939), p. 297.

[1939a] Paul Bernays, *JSL*, vol. 5 (1940), pp. 117-118.
Andrzej Mostowski, *Z*, vol. 21 (1939), p. 1.
Thoralf Skolem, *JFM*, vol. 65 (1939), p. 185.

[1940] Paul Bernays, *JSL*, vol. 6 (1941), pp. 112-114.
Leon Henkin, *JSL*, vol. 17 (1952), pp. 207-208 (of 2nd edition).
Hans Hermes, *Z*, vol. 61 (1961), pp. 9-10.
Stephen C. Kleene, *MR*, vol. 2 (1941), pp. 66-67.
C. C. Torrance, *Bulletin of the American Mathematical Society*, vol. 47 (1941), pp. 191-192.

[1944] Virgil Charles Aldrich, *Journal of Philosophy*, vol. 42 (1945), pp. 594-607.
Paul Bernays, *JSL*, vol. 11 (1946), pp. 75-79.
James Collins, *Modern Schoolman*, vol. 22 (1945), pp. 107-112.
Robert Eisler, *Hibbert Journal*, vol. 5 (1945), pp. 281-284.
L. J. Russell, *Philosophy*, vol. 20 (1945), pp. 281-284.
Paul Weiss, *Philosophy and Phenomenological Research*, vol. 5 (1945), pp. 594-599.
Herman Weyl, *American Mathematical Monthly*, vol. 53 (1946), pp. 208-214.

(Bernays's review is restricted to Gödel's article; the others are reviews of all of Schilpp's anthology.)

[1947] G. Hajos, *Z*, vol. 38 (1951), p. 30.
Bjarni Jónsson, *MR*, vol. 9 (1948), p. 403.
Stephen C. Kleene, *JSL*, vol. 13 (1948), pp. 116-117.

[1949] J. Haantjes, *Z*, vol. 41 (1952), p. 567.
M. Wyman, *MR*, vol. 11 (1950), p. 216.

[1950]	P. Jordan, *Z*, vol. 49 (1959), pp. 272-273. H. S. Ruse, *MR*, vol. 13 (1952), p. 500.
[1958]	Wilhelm Ackermann, *Z*, vol. 90 (1961), p. 10. Evert W. Beth, *MR*, vol. 21 (1960), pp. 235-236. Gisbert F. R. Hasenjaeger, *JSL*, vol. 25 (1960), p. 351.
[1962]	Stefan Bauer-Mengelberg, *JSL*, vol. 30 (1965), pp. 359-362. J. Tucker, *MR*, vol. 27 (1964), p. 275.
[1965-1965d]	Stefan Bauer-Mengelberg, *JSL*, vol. 31 (1966), pp. 484-494.
[1967d-1967f]	Ruggero Ferro, *JSL*, vol. 34 (1969), p. 313.
[1967g]	Jean van Heijenoort, *JSL*, vol. 35 (1970), p. 323.
[1980]	Solomon Feferman, *MR*, vol. 81i (1981), pp. 3410-3411.

Appendix B: Published photographs of Kurt Gödel Although Gödel's reclusive nature has been somewhat exaggerated, he was not a public figure and relatively few photographs of him, especially as a young man, have been published. Those known to the author are listed below, by photographer where known. Within the work of a given photographer, different photographs are separately numbered, while multiple listings following a single number indicate reproductions of the same photograph in different sources.

The author wishes to thank all those who responded to his query in the A.M.S. *Notices*, February 1981, p. 167; their efforts contributed several items to the list below.

Anonymous

1. [Kreisel, 1980], p. 155.
2. Chicago *Herald and Examiner*, January (?), 1939.

    1. According to Kreisel, this is a snapshot taken at the time of Gödel's discovery of the incompleteness results. 2. A youthful portrait accompanying an announcement of Gödel's visiting appointment at Notre Dame, spring term 1939; the date written on a photocopy obtained from the Notre Dame Archives has proved incorrect, and I have not been able to determine the precise reference.
3. Boston *Herald*, June 20, 1952, p. 42.

    A group photo of those receiving honorary degrees from Harvard.
4. [Schimanovich and Weibel, 1980], p. 49.

    A color photo of Gödel and Einstein together outdoors. The interview [Popper, 1980] on the following two pages is also of interest.

Alfred Eisenstaedt

1. [Bergamini et al., 1963], p. 52; [Davis and Hersh, 1981], p. 228.
2. [Higham, 1973], p. 432; [Boffa, van Dalen and McAloon, 1979], p. vi.
3. [Adian, Boone and Higman, 1980], p. vi.
4. [Kreisel, 1980], p. 148.

    These are part of a series of some 95 photographs of Gödel taken by Eisenstaedt in 1963 and available through *Life* Picture Service. Each shows Gödel seated (2 and 4) or standing (1 and 3) before a bookcase.

A portrait sketch, apparently based on 1, appears in [McGraw-Hill, 1980], p. 438.

Tom Herde

1. Newark, N.J., *Star-Ledger*, September 21, 1975, section 1, p. 24.

This portrait accompanies an article about the award to Gödel of the National Medal of Science.

Stephen C. Kleene

1. [Kleene, 1981], p. 59.

A snapshot of Kurt and his wife Adele with Kleene's parents, taken in 1941 in Hope, Maine. Perhaps the only published photograph of Adele.

Arnold Newman

1. [Nagel and Newman, n.d.], p. 222; *Scientific American*, June 1956, p. 72.
2. [Newman, 1974], #100; *Science 82*, October 1982, p. 57.
3. [Christian, 1980], p. 260.

1. A rather grim study of Gödel seated by the blackboard in his office at the Institute for Advanced Study. 2. A similar study, dated 1956. 3 A fine portrait.

Alan Windsor Richards

1. [Laurence, 1951], p. 31.
2. Trenton, N.J., *Evening Times*, March 15, 1951, p. 2.
3. [Richards, 1979], n.p.

Richards was formerly official Princeton photographer. The photos show Gödel and Julian Schwinger accepting the first Einstein awards from Einstein at ceremonies March 14, 1951, at the Princeton Inn; Lewis S. Strauss looks on in 1 and 2. (Neither 1 nor 2 is credited; they are attributed to Richards on the basis of their similarity to 3.)

William E. Sauro

1. [Rensberger, 1972], p. 43.
2. [Jones, 1974], p. 43.

These two photographs are so similar that I have attributed 2 to Sauro, although it appears uncredited. Both show Gödel and Carl Kaysen standing outside the I.A.S. They are believed to have been taken in 1972 at a symposium held at the I.A.S. in honor of von Neumann's contributions to the development of electronic computers.

Howard Schrader

1. [Hoffman, 1972], p. 254.

A group photo of Gödel, Einstein, Wigner and others, taken March 19, 1949, at an exclusive 70th birthday celebration for Einstein. Unfortunately, Gödel is caught with partly closed eyes and "wringing" hands.

Orren J. Turner

1. [Princeton University, 1946], p. 33.
2. [Bulloff, Holyoke and Hahn, 1969], frontispiece; [Monna and van Dalen, 1972], p. 58; [Springer-Verlag, 1977]; [Wang, 1978], p. 182; [Hofstadter, 1979], p. 16; [Open University, 1971], p. 21.

   1. A group photograph of participants in the Princeton Bicentennial Conference on Problems of Mathematics, December 1946. 2. Although this is the most widely reproduced portrait of Gödel, it is credited to Turner only in the first of the sources cited. A sketch of Gödel by Christopher Munoz, apparently based on 2, appears in [Lenz, 1980], p. 612.

Veli Valpola

1. [Wedberg, 1966], between pp. 192 and 193.

   Gödel is photographed in front of a blackboard containing logical symbols. (I thank Gunnar Berg of Uppsala Universitet for this citation. A letter from Valpola to Gödel in Gödel's *Nachlass* dates this photograph to 1958.)

In addition to the photographs listed above, a portrait sketch of Gödel by Henry Benson, dated 1973, appears on p. 243 of [Greenberg, 1980].

Appendix C: Cross reference index to translations of Gödel's works This index is divided into two sections. The first lists the translations by language. The second lists all translations of a given paper, indicating the language by a parenthetical initial letter.

Language Index

English:	[1962], [1965], [1965b], [1965c], [1967], [1967a], [1967b], [1967c], [1969], [1980], [Hahn, 1980].
French:	[1969a]
German:	[1955]
Italian:	[1961], [1967d], [1967e], [1967f], [1973]
Portuguese:	[1979] through [1979f]
Russian:	[1948], [1967g]
Spanish:	[1981]

Index by Paper Translated

[1930]:	[1967](E); [1981](S).
[1930b]:	[1967a](E); [1981](S).
[1931]:	[1961](I); [1962](E); [1965](E); [1967b](E); [1979b](P); [1981](S).
[1931a]:	[1973](I, *Nachtrag* only); [Hahn, 1980](E, Hahn's remarks only); [1981](S, Gödel's remarks only).
[1932]:	[1981](S).
[1932a]:	[1981](S).
[1932b]:	[1967c](E); [1981](S).

[1932c]: [1981](S).
[1933a]: [1981](S).
[1933e]: [1965b](E); [1979d](P); [1981](S).
[1933f]: [1969](E); [1981](S).
[1933h]: [1981](S).
[1933j]: [1981](S).
[1936a]: [1965c](E); [1979e](P); [1981](S).
[1938]: [1981](S).
[1939a]: [1981](S).
[1940]: [1948](R); [1981](S).
[1944]: [1967d](I); [1969a](F); [1979](P); [1981](S).
[1947]: [1967e](I); [1979a](P); [1981](S).
[1949]: [1981](S).
[1949a]: [1955](G); [1981](S).
[1950]: [1981](S).
[1958]: [1967g](R); [1980](E); [1981](S).
[1965]: [1981](S).
[1965a]: [1979c](P).
[1965d]: [1967f](I); [1979f](P); [1981](S).
[R1934b]: [1981](S).
Remark in preface to [Robinson, 1974]: [1981](S).

NOTE

1. English translations here and in the annotations to [1933d] and [1933j] are my own.

REFERENCES

Adian, S. I., W. W. Boone and G. Higman (eds.), *Word Problems II, Studies in Logic 95*, North-Holland, Amsterdam-London, 1980.

Bell, John Lane, *Boolean-Valued Models and Independence Proofs in Set Theory*, Clarendon Press, Oxford, 1977.

Benacerraf, Paul and Hilary Putnam (eds.), *Philosophy of Mathematics*, Prentice-Hall, Englewood Cliffs, New Jersey; Basil Blackwell, Oxford, 1964.

Bergamini, David and the editors of *Life*, *Mathematics*, in *Life Science Library*, Time, Inc., New York, 1963.

Berka, Karel and Lothar Kreiser, *Logik-Texte*, Akademie-Verlag, Berlin, 1971.

Boffa, M., D. van Dalen and K. McAloon (eds.), *Logic Colloquium '78*, North-Holland, Amsterdam-London, 1979.

Boolos, George, *The Unprovability of Consistency, An Essay in Modal Logic*, Cambridge University Press, Cambridge-London-New York-Melbourne, 1979.

Bulloff, Jack J., Thomas C. Holyoke and S. W. Hahn (eds.), *Foundations of Mathematics: Symposium Papers Commemorating the Sixtieth Birthday of Kurt Gödel*, Springer-Verlag, New York-Heidelberg-Berlin, 1969.

Cellucci, Carlo (ed.), *La Filosofia della matematica*, Editori Laterza, Bari, 1967.

Christian, Curt, "Das Lebenswerk Kurt Gödels," *Zeitschrift für Wissenschaftsforschung*, vol. 1 (1978), pp. 71-92.

Christian, Curt, "Leben und Wirken Kurt Gödels," *Monatshefte für Mathematik*, vol. 89 (1980), pp. 261-273.

Cohen, Paul J., "The independence of the continuum hypothesis," *Proceedings of the National Academy of Sciences, U.S.A.*, vol. 50, pp. 1143-1148; vol. 51, pp. 105-110 (1963-64).

Davis, Martin (ed.), *The Undecidable*, Raven Press, New York, 1965.

Davis, Philip J. and Reuben Hersh, *The Mathematical Experience*, Birkhäuser, Boston, 1981.

Gentzen, Gerhard, *The Collected Papers of Gerhard Gentzen*, ed., M. E. Szabo, North-Holland, Amsterdam-London, 1969.

Goldfarb, Warren D., "On the Gödel class with identity," *The Journal of Symbolic Logic*, vol. 46 (1981), pp. 354-364.

Grattan-Guinness, Ivor, "In memoriam Kurt Gödel: His 1931 correspondence with Zermelo on his incompletability theorem," *Historia Mathematica*, vol. 6 (1979), pp. 294-304.

Greenberg, Marvin Jay, *Euclidean and Non-Euclidean Geometries, Development and History*, 2nd Ed., W. H. Freeman, San Francisco, 1980.

Hahn, Hans, *Empiricism, Logic, and Mathematics* (Philosophical Papers, ed., Brian McGuinness), D. Reidel, Dordrecht-Boston-London, 1980.

Hawking, S. W. and G. F. R. Ellis, *The Large-Scale Structure of Space-Time*, Cambridge University Press, Cambridge, 1973.

Henkin, Leon, "The completeness of the first-order functional calculus," *The Journal of Symbolic Logic*, vol. 14 (1949), pp. 159-166.

Henkin, Leon, "Completeness in the theory of types," *The Journal of Symbolic Logic*, vol. 15 (1950), pp. 81-91.

Higham, Norman, "Gödel," pp. 432-433 in *McGraw-Hill Encyclopedia of World Biography*, Vol. 4, McGraw-Hill, New York et al., 1973.

Hilbert, David and Wilhelm Ackermann, *Grundzüge der theoretischen Logik*, J. Springer, Berlin, 1928. (English translation: *Principles of Mathematical Logic*, Chelsea Publishing Co., New York, 1950.)

Hilbert, David and Paul Bernays, *Grundlagen der Mathematik*, Vol. 2, J. Springer, Berlin, 1939.

Hoffman, Banesh, *Albert Einstein, Creator and Rebel*, Plume Books, New York, 1972.

Hofstadter, Douglas R., *Gödel, Escher, Bach: An Eternal Golden Braid*, Basic Books, New York, 1979.

Jech, Thomas J., *The Axiom of Choice*, North-Holland, Amsterdam-London, 1973.

Jensen, Ronald Bjorn, "The fine structure of the constructible hierarchy," *Annals of Mathematical Logic*, vol. 4 (1972), pp. 229-308.

Jones, Landon Y., Jr., "Bad days on Mount Olympus," *Atlantic Monthly* (February 1974), pp. 37-53.

Kalmár, László, "Über die Erfüllbarkeit derjenigen Zählausdrücke, welche in der Normalform zwei benachbarte Allzeichen enthalten," *Mathematische Annalen*, vol. 108 (1933), pp. 466-484.

Kanamori, A. and M. Magidor, "The evolution of large cardinal axioms in set theory," pp. 99-275 in *Higher Set Theory* (Proceedings, Oberwolfach, Germany, 1977), Springer-Verlag, New York-Heidelberg-Berlin, 1978.

Kleene, Stephen C., *Introduction to Metamathematics*, North-Holland, Amsterdam, 1952.

Kleene, Stephen C., "The work of Kurt Gödel," *The Journal of Symbolic Logic*, vol. 41 (1976), pp. 761-778.

Kleene, Stephen C., "An addendum to 'The work of Kurt Gödel'," *The Journal of Symbolic Logic*, vol. 43 (1978), p. 613.

Kleene, Stephen C., "Origins of recursive function theory," *Annals of the History of Computing*, vol. 3 (1981), pp. 52-67.

Kreisel, Georg, Review of [Kleene, 1978], *Zentralblatt für mathematik und ihre Gronzgebiiete*, vol. 366 (1979), pp. 6-7.

Kreisel, Georg, "Kurt Gödel, 1906-1978, Elected For. Mem. R. S. 1968," *Biographical Memoirs of Fellows of the Royal Society*, vol. 26 (1980), pp. 148-224; Corrigenda, vol. 27 (1981), p. 697.

Laurence, William L., "Einstein shares birthday honors," *New York Times*, March 15, 1951, p. 31.

Lenz, Gerald E., "Kurt Gödel, mathematician and logician," *Mathematics Teacher*, vol. 73 (1980), pp. 612-614.

Lourenço, Manuel (ed. and transl.), *O teorema de Gödel e a hipotese do continuo*, Fundação Calouste Gulbenkian, Lisbon, 1979.

McGraw-Hill Book Company, *McGraw-Hill Modern Scientists and Engineers*, McGraw-Hill, New York, 1980.

Monna, A. F. and Dirk van Dalen, *Sets and Integration, An Outline of the Development*, Woolters-Noordhoof, Groningen, 1972.

Myhill, John and Dana S. Scott, "Ordinal definability," pp. 271-278 in *Axiomatic Set Theory, Proceedings of Symposia in Pure Mathematics*, 13 (1), American Mathematical Society, Providence, 1971.

Nagel, Ernest and James R. Newman, "Gödel's proof," in *Mathematics in the Modern World* (Readings from *Scientific American*), W. H. Freeman, San Francisco-London.

Newman, Arnold, *One Mind's Eye*, David R. Godine, Boston, 1974.

Open University, *Logic II–Proof*, Mathematics Foundation Course Unit 17, Open University Press, Bletchley, 1971.

Popper, Karl R., "Der wichtigste Beitrag seit Aristoteles" (interview), *Wissenschaft aktuell*, 4/80 (September 1980), pp. 50-51.

Princeton University, *Problems of Mathematics*, Princeton University Bicentennial Conferences, Series 2, Conference 2, Princeton University Press, Princeton, 1946.

Rautenberg, Wolfgang, "Die Unabhängigkeit der Kontinuumhypothese–Problematik und Diskussion," *Mathematik in der Schule*, vol. 6 (1968), pp. 18-37.

Rensberger, Boyce, "Man and computer: Uneasy allies of 25 years," *New York Times*, June 27, 1972, p. 43.

Richards, Alan Windsor, *Einstein As I Knew Him*, Harvest House Press, Princeton, 1979.

Robinson, Abraham, *Non-Standard Analysis*, 2nd Ed., North-Holland, Amsterdam, 1974.

Rosser, B., "Extensions of some theorems of Gödel and Church," *The Journal of Symbolic Logic*, vol. 1 (1936), pp. 87-91.

Saracino, D. H. and V. B. Weispfenning (eds.), *Model Theory and Algebra. A Memorial Tribute to Abraham Robinson* (Lecture Notes in Mathematics, 498), Springer-Verlag, Berlin-Heidelberg-New York, 1975.

Schimanovich, Werner and Peter Weibel, "Die Unerschöpflichkeit des Geistes," *Wissenschaft aktuell*, 4/80 (September 1980), pp. 48-49.

Schütte, Kurt, "Untersuchungen zum Entscheidungsproblem der mathematischen Logik," *Mathematische Annalen*, vol. 109 (1934), pp. 572-603.

Shepherdson, J. C., "Inner models for set theory," *The Journal of Symbolic Logic*, vol. 16, pp. 161-190; vol. 17, pp. 225-237; vol. 18, pp. 145-167 (1951-53).

Skolem, Thoralf, "Einige Bemerkungen zur axiomatischen Begründung der Mengenlehre," pp. 217-232 in *Matematikerkongressen i Helsingfors den 4-7 Juli 1922, Den femte skandinaviska matematikerkongressen, Redogörelse*, Akademiska Bokhandeln, Helsinki, 1923.

Smorýnski, C., "The incompleteness theorems," pp. 821-866 in *Handbook of Mathematical Logic*, ed., J. Barwise, North-Holland, Amsterdam-New York-Oxford, 1977.

Spector, Clifford, "Provably recursive functionals of analysis: A consistency proof of analysis by an extension of principles formulated in current intuitionistic mathematics," pp. 1-27 in *Recursive Function Theory, Proceedings of Symposia in Pure Mathematics*, 5, American Mathematical Society, Providence, 1962.

Springer-Verlag, *Mathematics Calendar 1977*, Springer-Verlag, New York-Heidelberg-Berlin, 1977.

Troelstra, A. S., "Arend Heyting and his contribution to intuitionism," *Nieuw Archief voor Wiskunde* (3), XXIX (1981), pp. 1-23.

Ulam, S., "John von Neumann, 1903-1957," *Bulletin of the American Mathematical Society*, vol. 64, no. 3, pt. 2 (1958), pp. 1-49.

van Heijenoort, Jean (ed.), *From Frege to Gödel, A Source Book in Mathematical Logic, 1879-1931*, Harvard University Press, Cambridge, Massachusetts, 1967.

von Neumann, John, *Theory of Self-Reproducing Automata*, edited and completed by A. Burks, University of Illinois Press, Urbana, Illinois, 1966.

Wang, Hao, *From Mathematics to Philosophy*, Routledge and Kegan Paul, London; Humanities Press, New York, 1974.

Wang, Hao, "Kurt Gödel's intellectual development," *Mathematical Intelligencer*, vol. 1 (1978), pp. 182-184.

Wang, Hao, "Some facts about Kurt Gödel," *The Journal of Symbolic Logic*, vol. 46 (1981), pp. 653-659.

Wedberg, A., *Filosofins historia, från Bolzano till Wittgenstein*, Bonniers, Stockholm, 1966.

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