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On the modularity of a rigid Calabi-Yau threefold

By

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Abstract

In this paper, we introduce the powerful, new method of Wiles into establishing that a Calabi-Yau threefold defined over the field $\mathbb{Q}$ of rational numbers is modular, answering a question of Saito & Yui [SY].

1. Introduction

In [W], Andrew Wiles proved Fermat's Last Theorem by verifying that every semistable elliptic curve over the field $\mathbb{Q}$ of rational numbers is modular. Elliptic curves are dimension one Calabi-Yau varieties, and the conjecture that they are all modular (i.e. the Taniyama-Shimura Conjecture) has now been established for all elliptic curves over $\mathbb{Q}$ by Wiles, Breuil, Conrad, Diamond, and Taylor [BCDT]. The question arises as to which higher-dimensional Calabi-Yau varieties are modular. Dimension two Calabi-Yau varieties are K3 surfaces and the modularity conjecture which asserts that every singular K3 surface is modular has been verified by Shioda and Inose in [SI]. For dimension three, it has been conjectured that every rigid Calabi-Yau threefold over $\mathbb{Q}$ is modular by Masa-Hiko Saito and Noriko Yui in [SY]. In [V], a certain rigid Calabi-Yau threefold was proved modular by Verrill using the method of Faltings-Serre. Two different geometric proofs were given in [SY] and in [HSGS]. It was asked in [SY] whether one can prove that a Calabi-Yau threefold is modular by using the new, powerful method of Wiles'. In this paper, we answer this question by Saito & Yui and establish the modularity of Verrill's threefold by using Skinner-Wiles [SW].

2. Calabi-Yau threefolds

Definition 2.1. A Calabi-Yau threefold is a smooth projective variety X of dimension 3 such that

(i) $H^i(X, \mathcal{O}_X) = 0$ for $i = 1, 2$, and

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(ii) the canonical bundle is trivial i.e. $K_X := \wedge^3\Omega^1_X \simeq \mathcal{O}_X$.

We denote $X \times_{\mathbb{Q}} \overline{\mathbb{Q}}$ by $\overline{X}$ and define the Hodge number $h^{i,j}(X)$ to be $\dim H^j(\overline{X}, \Omega^i_X)$. The *Hodge diamond* is:

$$\begin{array}{ccccccc}
 & & & h^{0,0} & & & \\
 & & h^{1,0} & & h^{0,1} & & \\
 & h^{2,0} & & h^{1,1} & & h^{0,2} & \\
h^{3,0} & & h^{2,1} & & h^{1,2} & & h^{0,3} \\
 & h^{3,1} & & h^{2,2} & & h^{1,3} & \\
 & & h^{3,2} & & h^{2,3} & & \\
 & & & h^{3,3} & & &
\end{array}$$

The complex conjugation operation and Serre duality on the Hodge cohomology groups imply the symmetry among Hodge numbers:

$$h^{i,j}(X) = h^{j,i}(X) \quad \text{and} \quad h^{i,j}(X) = h^{3-j,3-i}(X).$$

Moreover, condition (i) implies $h^{1,0}(X) = h^{2,0}(X) = 0$ and condition (ii) implies $h^{3,0}(X) = 1$. By the conditions (1), (2) and the symmetry among Hodge numbers the Hodge diamond of Calabi-Yau threefolds are as follows:

$$\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & h^{1,1} & & 0 & \\
1 & & h^{2,1} & & h^{2,1} & & 1 \\
 & 0 & & h^{1,1} & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}$$

The *n-th Betti number*, $B_n(X)$, of X is defined to be the dimension of$H^n(\overline{X} \otimes \mathbb{C}, \mathbb{C})$ over $\mathbb{C}$, which is also equal to the dimension of $H^n_{ét}(\overline{X}, \mathbb{Q}_l)$ over $\mathbb{Q}_l$ for any l.

The Hodge decomposition asserts that

$$H^n(X \otimes \mathbb{C}, \mathbb{C}) = \oplus_{i+j=n} H^j(X \otimes \mathbb{C}, \Omega^i_{X \otimes \mathbb{C}}).$$

It follows that

$$B_n(X) = \sum_{i+j=n} h^{i,j}(X) = \sum_{i=0}^{n} h^{i,n-i}(X).$$

By the Poincaré duality $B_n(X) = B_{2dim(X)-n}$, the Betti numbers of Calabi-Yau threefolds are as follows:

$$\begin{aligned}
B_0 = B_6 &= 1 \\
B_1 = B_5 &= 0 \\
B_2 = B_4 &= h^{1,1} \\
B_3 &= 2(1 + h^{2,1})
\end{aligned}$$

Define the *Euler characteristic* $E(X)$ of a Calabi-Yau threefold X to be

$$E(X) = \sum_{i=0}^{6} (-1)^i B_i = 2(h^{1,1} - h^{2,1}),$$

where B_i is the *i-th* Betti number of X.

Definition 2.2. A smooth projective Calabi-Yau variety over $\mathbb{Q}$ is *rigid* if $h^{2,1}(X) = 0$ and so $B_3(X) = 2$.

Let X be a rigid Calabi-Yau threefold over $\mathbb{Q}$ with a suitable integral model. The action of $G_{\mathbb{Q}}$ on $H^3_{\acute{e}t}(\overline{X}, \mathbb{Q}_l)$ yields a two dimensional l-adic Galois representation for any l. X is called *modular* if the L-series of this Galois representation coincides with L-series of a modular (cusp) form f, necessary of weight 4, on some $\Gamma_0(N)$, where N is a positive integer divisible by the primes of bad reduction. In other words, up to finitely many Euler factors,

$$L(H^3_{\acute{e}t}(\overline{X}, \mathbb{Q}_l), s) = L(f, s) \text{ for some } f \in S_4(\Gamma_0(N)).$$

Conjecture 2.1 (The Modularity Conjecture). *Any rigid Calabi-Yau threefold X defined over $\mathbb{Q}$ is modular in the sense that, up to a finite Euler factors,*

$$L(H^3_{\acute{e}t}(\overline{X}, \mathbb{Q}_l), s) = L(f, s) \text{ for some } f \in S_4(\Gamma_0(N)).$$

Conjecture 2.1 is due to [SY] and also a more general form was given by Serre [S].

Given a prime of good reduction p, $p \neq l$, for X, let $X(\mathbb{F}_p)$ denote the set of points of X which are rational over $\mathbb{F}_p$. The Lefschetz fixed point formula tell us that

$$\#X(\mathbb{F}_p) = \sum_{j=0}^{6} (-1)^j \mathrm{Tr}(\mathrm{Frob}_p; H^j_{\acute{e}t}(\overline{X}, \mathbb{Q}_l)),$$

where Frob_p is induced from the geometry Frobenius morphism of X at p. Define $t_j(p) = \mathrm{Tr}(\mathrm{Frob}_p; H^j_{\acute{e}t}(\overline{X}, \mathbb{Q}_l))$. In view of the Hodge diamond of X and by using various dualities, we have

$$t_1(p) = t_5(p) = 0,\ t_0(p) = 1,\ t_6(p) = p^3 \text{ and } t_4(p) = pt_2(p).$$

Thus

$$\#X(\mathbb{F}_p) = 1 + p^3 + (1 + p)t_2(p) - t_3(p),$$

whence

$$t_3(p) = 1 + p^3 + (1 + p)t_2(p) - \#X(\mathbb{F}_p).$$

Let $a_p = \mathrm{Tr}(\rho(\mathrm{Frob}_p))$, where ρ is the Galois representation induced by the action of $G_{\mathbb{Q}}$ on $H^3_{\acute{e}t}(\overline{X}, \mathbb{Q}_l)$. Thus, we have $a_p = t_3(p)$. It implies that

$$a_p = 1 + p^3 + (1 + p)t_2(p) - \#X(\mathbb{F}_p).$$

We consider the perfect pairing

$$H^3_{ét}(\overline{X},\mathbb{Q}_l)\times H^3_{ét}(\overline{X},\mathbb{Q}_l)\to H^6_{ét}(\overline{X},\mathbb{Q}_l)\cong\mathbb{Q}_l$$

induced by Poincaré duality. It follows that

$$\wedge^2 H^3_{ét}(\overline{X},\mathbb{Q}_l)\cong H^6_{ét}(\overline{X},\mathbb{Q}_l)\cong\mathbb{Q}_l.$$

The action of Frob_p on $H^6_{ét}(\overline{X},\mathbb{Q}_l)$ is multiplication by p^3.

The action of Frob_p on $H^3_{ét}(\overline{X},\mathbb{Q}_l)$ therefore satisfies $\det(\rho(\mathrm{Frob}_p))=p^3$. Therefore,

$$\det(\rho)=\epsilon^3, \tag{2.1}$$

where ϵ is the cyclotomic character. Thus, if ρ is modular, it is associated to a form of weight 4. Henceforth, we focus on the Calabi-Yau variety studied by Verrill [V]. Verrill's Calabi-Yau variety $\mathcal{Z}_{A_3}$ is the smooth model of the variety V given in inhomogeneous coordinates by

$$V:(1+x+xy+xyz)(1+z+zy+zyx)=\frac{(t+1)^2}{t}xyz,$$

where 2 and 3 are bad primes.

Theorem 2.1 ([V, Prop. 3.7]). *The Calabi-Yau variety $\mathcal{Z}_{A_3}$ has Hodge numbers $h^{p,q}=0$ except the following*:

$$h^{0,0}=h^{3,0}=h^{0,3}=h^{3,3}=1,\ \ h^{1,1}=h^{2,2}=50.$$

In particular, $\mathcal{Z}_{A_3}$ is rigid.

Theorem 2.2 ([V, Lemma 3.8]). *Let $p\geq 5$ be a prime. For $k=0,2,4$, 6, the action of* Frob_p *on $H^k_{ét}(\mathcal{Z}_{A_3},\mathbb{Q}_l)$ is multiplication by $p^{\frac{k}{2}}$.*

Consider the Barth-Nieto quintic given by the equations

$$N=\left\{\sum_{i=0}^{5}x_i=\sum_{i=0}^{5}\frac{1}{x_i}=0\right\}\subset\mathbb{P}_5.$$

It has a smooth Calabi-Yau model, denoted by Y.

Lemma 2.1. *Let $p\geq 5$ be a prime. The reduction of Y modulo is smooth over $\mathbb{F}_p$.*

Proof. By an easy calculation(cf. [BN, (3.1), (9.1) and (9.3)]). ☐

Theorem 2.3 ([HSGS, Thm. 2.1]). *The Calabi-Yau variety Y has Hodge numbers $h^{p,q}=0$ except the following*:

$$h^{0,0}=h^{3,0}=h^{0,3}=h^{3,3}=1,\ \ h^{1,1}=h^{2,2}=50.$$

In particular, Y is rigid.

Lemma 2.2 ([HSGS, Thm. 4.1 and Thm. 4.3]). *Y and $\mathcal{Z}_{A_3}$ are birational equivalent over $\mathbb{Q}$.*

Theorem 2.4 ([HSGS, Prop. 2.4]). *Let $p \geq 5$ be a prime. For $k = 0, 2, 4, 6$, the action of Frob_p on $H^k_{\acute{e}t}(Y, \mathbb{Q}_l)$ is multiplication by $p^{\frac{k}{2}}$.*

For $X = Y$ or $\mathcal{Z}_{A_3}$, the Lefschetz fixed point formula and the above theorems imply that

$$a_p = 1 + 50p + 50p^2 + p^3 - \#X(\mathbb{F}_p).$$

The following table gives the first few a_p by computer calculation.

p	$\#X(\mathbb{F}_p)$	a_p
5	1620	6
7	3160	-16
11	7920	12
13	11260	38
17	20340	-126
19	25840	20
23	39600	168

3. Techniques of Skinner-Wiles

Suppose that $\rho : \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \longrightarrow \mathrm{GL}_2(E)$ where E is a finite extension of $\mathbb{Q}_l$. We denote by $\bar{\rho}$ the residual representation and write $\bar{\rho}^{ss}$ for the semisimplification of $\bar{\rho}$.

Theorem 3.1 (Skinner-Wiles [SW, p. 6]). *Suppose that $\rho : \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \longrightarrow \mathrm{GL}_2(E)$ is a continuous representation, irreducible and unramified outside a finite set of primes, where E is a finite extension of $\mathbb{Q}_l$. Suppose also $\bar{\rho}^{ss} \simeq 1 \oplus \chi$ and that*

(i) $\chi|_{D_l} \neq 1$, *where D_l is a decomposition group at l.*

(ii) $\rho|_{I_l} \simeq \begin{pmatrix} * & * \\ 0 & 1 \end{pmatrix}$.

(iii) $\det; \rho = \psi\epsilon^{k-1}$ *for some $k \geq 2$ and is odd, where ϵ is the cyclotomic character and ψ is of finite order. Then ρ comes from a modular form.*

Consider the continuous Galois representation $\rho : \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \longrightarrow \mathrm{GL}_2(\mathbb{Q}_5)$ induced by the action of $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ on $H^3_{\acute{e}t}(\mathcal{Z}_{A_3}, \mathbb{Q}_5)$. To say $\mathcal{Z}_{A_3}$ is modular is equivalent to ρ being modular. To apply the theorem of Skinner-Wiles to verify that ρ is modular, we consider the following:

A : $a_p \equiv 1 + p^3 \pmod 5$.

B : $\rho|_{I_5} \simeq \begin{pmatrix} * & * \\ 0 & 1 \end{pmatrix}$.

C : $\rho : \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \longrightarrow \mathrm{GL}_2(\mathbb{Q}_5)$ is irreducible.

Condition **C** is a hypothesis to Theorem 3.1. As for its other hypotheses, first condition **A** says that the trace of Frobenius at p, $Tr(\rho(Frob_p)) \equiv 1 +$

$p^3 \pmod 5) = 1 + \epsilon(Frob_p)^3 \pmod 5)$. By the Tchebotarev theorem, which says the Frobenius elements are dense, **A** implies $\bar{\rho}^{ss} \simeq 1 \oplus \epsilon^3$. Moreover, $\chi = \epsilon^3$ satisfies hypothesis (i) of Theorem 3.1. Hypothesis (ii) is just condition **B**, and (iii) was established on (2.1). Therefore, to verify ρ is modular by Theorem 3.1 it suffices to prove $\mathbf{A}, \mathbf{B}$ and **C**.

First, we quote some lemmas to establish **A**.

Let $D_0 = \{x_0 \cdots x_5 = 0\}$. Set $U := N \setminus D_0$.

Lemma 3.1 ([HSGS, Prop. 2.13]). *Let $p \geq 5$ be a prime. $\#Y(\mathbf{F_p}) = \#U(\mathbf{F_p}) + 50p^2 + 50p + 20$.*

Let $\rho_1 : \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \to \mathrm{GL}_2(\mathbb{Q}_5)$ be the representation induced by the action of $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ on $H^3_{\acute{e}t}(Y, \mathbb{Q}_5)$. The birational equivalence between Y and $\mathcal{Z}_{A_3}$ implies that $\mathrm{Tr}(\rho(\mathrm{Frob}_p))$=$\mathrm{Tr}(\rho_1(\mathrm{Frob}_p))$. Hence, by the formula $a_p = 1 + 50p + 50p^2 + p^3 - \#Y(\mathbb{F}_p)$, to prove **A** is equivalent to prove that $\#Y(\mathbf{F}_p)$ is divisible by 5. Theorem 3.3 tell us that it suffice to prove that 5 divide $\#U(\mathbf{F}_p)$.

Theorem 3.2. $\#U(\mathbf{F}_p) \equiv 0 \pmod 5$.

Proof. $U = \left\{(x_0, \ldots, x_5) \in \mathbb{P}_5 | \sum_{i=0}^5 x_i = \sum_{i=0}^5 \frac{1}{x_i} = 0 \text{and} x_0 \cdots x_5 \neq 0\right\}$ Consider the action of S_6 on U. U is the union of orbits of the action. To prove that each orbit is divisible by 5 is to prove that the order of the stabilizer of certain element in the orbit is not divisible by 5. Suppose otherwise. Then the stabilizer has a cyclic subgroup of order 5. Without loss of generality suppose this subgroup is generated by (1 2 3 4 5). Other cases are proved similarly. The defining equations of N gives us the following system:

$$\begin{cases} x_0 + 5x_1 = 0, \\ \dfrac{1}{x_0} + \dfrac{5}{x_1} = 0, \end{cases}$$

whence

$$\begin{cases} x_0 + 5x_1 = 0, \\ 5x_0 + x_1 = 0. \end{cases}$$

The only solution is trivial, which is a contradiction. □

To get the result of **B**, we quote the following theorem from [CSS, p. 388, Thm. 1.8].

Theorem 3.3. *Let R be a complete local noetherian ring with finite residue field k of characteristic $l \neq 2$. Choose a flat representation*

$$\bar{\rho} : D_l \to \mathrm{GL}_2(k)$$

with $\det\bar{\rho}|_{I_l} = \omega|_{I_l}$ *and a continuous lift* $\rho : D_l \to \mathrm{GL}_2(R)$ *which give rise to an element of* $\mathcal{D}^{fl}_{\bar{\rho}}(R)$.

Let

$$\phi_2 : I_l \overset{\psi_2}{\to} \mathbf{F}_{l^2}^{\times} \hookrightarrow \mathrm{GL}_2(\mathbf{F}_l) \hookrightarrow \mathrm{GL}_2(k)$$

be the map arising from a choice of an $\mathbf{F}_l$*-basis of* $\mathbf{F}_{l^2}$.

(i) *If* $\bar{\rho}$ *is reducible, then there exist continuous unramified characters* $\overline{\chi}_i : D_l \to k^{\times}$ *such that*

$$\bar{\rho} \simeq \begin{pmatrix} \omega\overline{\chi}_1 & * \\ 0 & \overline{\chi}_2 \end{pmatrix}.$$

Otherwise $\bar{\rho}$ *is absolutely irreducible and* $\bar{\rho}|_{I_l} \simeq \phi_2$ *(and so* $\bar{\rho}|_{I_l} \otimes \overline{\mathbb{F}}_l \simeq \psi_2 \oplus \psi_2^l$*).*

(ii) *If* $\bar{\rho}$ *is reducible, then there exist continuous unramified characters* $\chi_i : D_l \to R^{\times}$ *such that*

$$\rho \simeq \begin{pmatrix} \epsilon\chi_1 & * \\ 0 & \chi_2 \end{pmatrix}.$$

In particular, $\det \rho|_{I_l} = \epsilon|_{I_l}$ *and* $\chi_i \mathrm{mod}\ \boldsymbol{m}_R = \overline{\chi}_i$.

(iii) *If* $\bar{\rho}$ *is irreducible, then* $\det \rho|_{I_l} = \epsilon|_{I_l}$.

Since the variety $\mathcal{Z}_{A_3}$ has good reduction at 5, the representation $\bar{\rho}$ is flat. Therefore, $\bar{\rho}$ is a reducible flat representation with $\det\bar{\rho} = \omega^3$ where ω is the reduction of 5-adic cyclotomic character ϵ modulo 5. Hence $\det\bar{\rho}^{-1} = \omega$. Applying theorem 3.4 (ii) to $\bar{\rho}^{-1}$ give us $\rho^{-1}|_{D_5} \simeq \begin{pmatrix} \epsilon\chi_1 & * \\ 0 & \chi_2 \end{pmatrix}$, where $\chi_i : D_5 \to R^{\times}$ are unramified characters. In particular, $\chi_i|_{I_5}$ are trivial. Hence $\rho|_{I_5}$ has the form $\begin{pmatrix} * & * \\ 0 & 1 \end{pmatrix}$.

Finally, in order to apply Theorem 3.1, we have to show that ρ is irreducible. Suppose that ρ were reducible, which means $\rho \simeq \begin{pmatrix} \phi_1 & * \\ 0 & \phi_2 \end{pmatrix}$, where the ϕ_i's are characters. The fact that $det(\rho) = \epsilon^3$ implies $\phi_1 = \epsilon^a\psi_1$ and $\phi_2 = \epsilon^b\psi_2$, where $a + b = 3$ and $\psi_i : \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \to \mathbb{Q}_5^*$ are characters of finite order. Then, $\mathrm{Tr}\rho = \epsilon^a\psi_1 + \epsilon^{3-a}\psi_2$ where $a = 0$ or 1. Consider the prime $p = 7$. $a_7 = -16 = 7^a\psi_1(Frob_7) + 7^{3-a}\psi_2(Frob_7)$. Thus $a_7 = -16 = 7^a\psi_1(Frob_7) + 7^{3-a}\psi_1^{-1}(Frob_7)$. Because the only finite subgroup of $\mathbb{Q}_5^*$ is C_4, $\psi_1(Frob_7) \in \{\pm 1, \pm i\}$, which is a contradiction to the former equality. This complete the proof of **C**.

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