COHOMOLOGY OF INSEPARABLE FIELD EXTENSIONS

BY

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0. Introduction

If k is a field of characteristic $p \neq 0$ and $L = k(a)$ a field extension with a^p in k, then a result of Demazure and Gabriel states that the Amitsur cohomology $H^n(L/k, \boldsymbol{G})$ is zero for $n \geq 3$ when $\boldsymbol{G}$ is a commutative algebraic group. In the case $\boldsymbol{G} = \boldsymbol{G}_m$, the units functor, this result is due to Berkson, and was extended to arbitrary purely inseparable extensions by Rosenberg and Zelinsky, using spectral sequence techniques, which do not seem to generalize. In Section One of this paper we show that Amitsur cohomology of a finite extension with coefficients in a finite group is a local invariant and that one need consider only connected coefficient groups. In the second section we compute the cohomology of extensions with coefficients in $\boldsymbol{\alpha}_{p^s}$ and $\boldsymbol{\mu}_{p^s}$.

1. Group schemes and sheaves

Throughout the paper k is a field of characteristic $p \neq 0$, *algebra* means commutative k-algebra and $\otimes$ means $\otimes_k$. In this section we wish to specialize some well known notions, most of which are collected in [D.-G.].

An *affine group scheme* (*over* k) is a representable group valued functor, $\boldsymbol{G}$, on the category k-*alg* of k-algebras. $\boldsymbol{G}$ is *commutative* if its values are and *finite* if its representing algebra $A = A(\boldsymbol{G})$ is finite dimensional.

$A(\boldsymbol{G})$ has a natural structure as a Hopf algebra (bialgebra with cosymmetric involution in [D.-G.; II, §1, 1.6 et 1.7] and conversely if A is a commutative Hopf algebra, then $\boldsymbol{G}(A) = k\text{-alg}(A, \ \ )$ is an affine group scheme. This gives an anti-equivalence between the category of affine (resp. affine commutative) group schemes and that of commutative (resp. and cocommutative) Hopf algebras.

Henceforth, by group scheme we will always mean *affine commutative group scheme*.

A sequence $0 \to \boldsymbol{F} \to \boldsymbol{G} \to \boldsymbol{H} \to 0$ of (affine commutative) group schemes is exact if and only if it is exact as a sequence of sheaves (in the *fpqc* topology, i.e., "faisceaux durs". cf. III, §1, 3.3 and §3, 7.4 of [D.-G.]). The inclusion

Received November 12, 1971.

[1] The author wishes to acknowledge the opportunities afforded at the 1971 Conference on Rings, Modules and Homological Methods at the Mathematisches Forschungsinstitut, Oberwolfach. In conversations there Rentschler constructed Example 1.3 and R. Rasala pointed out sequence (3.1). Conversations with S. Chase and the hospitality of the Mathematics Department, Cornell University, throughout the summer of 1971, were of great help.

This research was supported by a State University of New York Faculty Fellowship and a National Science Foundation grant.

functor i from the category $\mathcal{S}\mathcal{h}$ of abelian group valued sheaves to the category of k-algebras, is left but not right exact. In particular, if $0 \to \boldsymbol{F} \to \boldsymbol{G} \to \boldsymbol{H} \to 0$ is an exact sequence of group schemes, then $0 \to \boldsymbol{F}(A) \to \boldsymbol{G}(A) \to \boldsymbol{H}(A)$ is exact but $\boldsymbol{G}(A) \to \boldsymbol{H}(A)$ need not be epic. There is however a left adjoint $(\tilde{\ })$ to i called the *associated sheaf* functor. (In [D.-G.; III, §1], the "faisceau dur" associated to $\boldsymbol{F}$ is denoted $\boldsymbol{F}^{\sim\sim}$. $(\tilde{\ })$ there denotes the corresponding construction simply for sheaves, often called sheaves in the *fppf* topology. The remarks below apply to either category of sheaves, and indeed since we will generally be concerned only with finite group schemes, the results of [D.G.; III §3, 5.6] allow one to consider *fppf* sheaves). $(\tilde{\ })$ is an exact functor [D.G.; III, §3, 3.6].

With these observations we can prove a result we need later.

PROPOSITION 1.1. *Let $\boldsymbol{F} \to \boldsymbol{G}$ be a monomorphism of sheaves and let $\boldsymbol{H}$ be the functor quotient. Then the natural map $\boldsymbol{H} \to \boldsymbol{H}^{\sim}$ is a monomorphism of functors.*

Proof. For any algebra T the diagram of abelian groups

$$\begin{array}{ccccccccc} 0 & \to & \boldsymbol{F}(T) & \to & \boldsymbol{G}(T) & \to & \boldsymbol{H}(T) & \to & 0 \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 0 & \to & \boldsymbol{F}^{\sim}(T) & \to & \boldsymbol{G}^{\sim}(T) & \to & \boldsymbol{H}^{\sim}(T) & & \end{array}$$

is exact and commutative. The left two vertical maps are isomorphisms since $\boldsymbol{F}$ and $\boldsymbol{G}$ are sheaves, and the result follows.

There is a fairly extensive decomposition theory of finite group schemes which will be of use.

A (finite) group scheme is *connected* if its representing algebra is connected (i.e. has no non-trivial idempotents or, equivalently, is local) and *étale* if its algebra is separable. Curiously, in this connection the notions of separable and reduced (no non-zero nilpotents) coincide. This can be proved Hopf algebraically or from the following important result.

PROPOSITION 1.2. (cf. [D.-G.; II, §5, 1.8] *If $\boldsymbol{G}$ is a finite commutative group scheme and $\boldsymbol{G}_0$ its connected component of identity, there is an exact sequence of group schemes*

$$0 \to \boldsymbol{G}_0 \to \boldsymbol{G} \to \boldsymbol{G}_s \to 0 \tag{1.1}$$

with $\boldsymbol{G}_s$ étale.

On the algebra level this is described as follows: the maximal separable subalgebra A_s of $A = A(\boldsymbol{G})$ is a sub-Hopf-algebra. The Hopf algebra quotient [Sw 1] is the projection of A onto the unique component A_0 (A is finite dimensional) whose idempotent is not annihilated by the augmentation of A. The exact sequence

$$0 \to A_s \to A \to A_0 \to 0$$

of Hopf algebras induces the exact sequence (1.1) where $G_0 = k\text{-alg}(A_0, \ )$ and $G_s = k\text{-alg}(A_s, \ )$.

In fact G_0 can be decomposed further (cf. [Sh.], [O., I.2] and [D.-G., IV] and the entire theory is even more satisfactory if k is perfect. In that case the sequence (1.1) splits and in particular $G \to G_s$ is an epimorphism of functors.

Unfortunately, this is not the case if k is not perfect as the following example due to R. Rentschler shows.

Example 1.3. Let k have characteristic two, set $L = k(a)$ with a^2 in k but a not in k, and let $A = k[x]/(x^2) \times L$. Write $e_1 = (1, 0)$, $e_2 = (0, 1)$, $y = (x, 0)$ and $z = (0, a)$ and regard A as a k-algebra diagonally, so that $\{e_i, y, z\}$ is a basis. The Hopf algebra structure is given by

$$\Delta(y) = y \otimes e_1 + e_1 \otimes y + z \otimes e_2 + e_2 \otimes z, \qquad \varepsilon(y) = 0,$$
$$\Delta(z) = z \otimes e_1 + e_1 \otimes z + y \otimes e_2 + e_2 \otimes y, \qquad \varepsilon(z) = 0,$$
$$\Delta(e_1) = e_1 \otimes e_1 + e_2 \otimes e_2, \qquad \varepsilon(e_1) = 1,$$
$$\Delta(e_2) = e_2 \otimes e_1 + e_1 \otimes e_2, \qquad \varepsilon(e_2) = 0.$$

The identity is taken as the antipode (involution) of A.

The maximal separable subalgebra A_s is $\{ce_1 + de_2 \mid c, d \text{ in } k\} \cong k \times k$ and one easily sees that $k\text{-alg}(A_s, k)$ has two elements, whereas $k\text{-alg}(A, k)$ has only one, so that $k\text{-alg}(A, \ ) \to k\text{-alg}(A_s, \ )$ is not a surjection of functors.

2. Reduction to the connected cohomology

As in [C.-R.], if S is a commutative algebra over a commutative ring R and F is an Abelian group valued functor on a (suitably large) category of R-algebras, the *Amitsur complex* $C^*(S/R, F)$ has *n-th* cochain group $F(S^{n+1}) = F(S \otimes_R \cdots \otimes_R S)$ the tensor product taken $n+1$ times. The coboundary d^n is $\sum_{i=0}^{n+1} (-1)^i F\varepsilon_i$, where $\varepsilon_i : S^{n+1} \to S^{n+2}$ is the R-algebra map obtained by inserting 1_S after the i-*th* entry of a homogeneous tensor (ε_0 inserts 1_S before such an element). The *n-th* cohomology group is denoted $H^n(S/R, F)$. $H^0(S/R, F)$ is by definition $\operatorname{Ker}(F\varepsilon_0 - F\varepsilon_1) \subseteq F(S)$. A short exact sequence of functors induces a short exact sequence of complexes, and so, in the usual way, a long exact sequence of cohomology.

Throughout this section we continue the conventions and notation of the previous section. L will be a fixed purely inseparable field extension finite dimensional over k.

Since sequence (1.1) is not exact as a sequence of functors, it does not induce a long exact sequence of cohomology. This problem can be overcome by replacing G_s by the functor cokernel and computing the resulting cohomology. We require first a more or less well-known result (cf. [Sh.; Prop. 17]).

PROPOSITION 2.1. *If F is an étale group scheme, then $H^n(L/k, F) = 0$ for $n \geq 1$.*

Proof. $A = A(\boldsymbol{F})$ is the product of separable field extensions of k and L^{n+1} is local with residue class field L. Therefore any algebra map $A \to L^{n+1}$ factors through k. Equivalently, $k \to L^{n+1}$ induces an isomorphism $\boldsymbol{F}(k) \to \boldsymbol{F}(L^{n+1})$. Any algebra map $L^{n+1} \to L^{n+2}$ induces, via this isomorphism, the identity on $\boldsymbol{F}(k)$, so the Amitsur complex in this case becomes

$$0 \longrightarrow \boldsymbol{F}(k) \xrightarrow{0} \boldsymbol{F}(k) \xrightarrow{1} \boldsymbol{F}(k) \longrightarrow \cdots$$

giving the result.

PROPOSITION 2.2. *If $\boldsymbol{F}$ is a finite connected group scheme, then $k \to L$ induces an isomorphism $\boldsymbol{F}(k) \to \boldsymbol{F}(L)$. (Here L may be an arbitrary field extension.)*

Proof. $A = A(\boldsymbol{F})$ has residue class field k, so every algebra map $A \to L$ factors through k.

Our main result in this section is

THEOREM 2.3. *Let L be a finite dimensional purely inseparable field extension of k and let $\boldsymbol{G}$ be a finite affine commutative group scheme with connected component of the identity $\boldsymbol{G}_0$. Then the natural map $H^n(L/k, \boldsymbol{G}_0) \to H^n(L/k, \boldsymbol{G})$ is an isomorphism for $n \geq 2$. If $n = 1$ this map has kernel naturally isomorphic to $\boldsymbol{G}(k)/\boldsymbol{G}_0(k)$.*

Proof. Let $\boldsymbol{G}'$ be the functor quotient $\boldsymbol{G}/\boldsymbol{G}_0$. By Proposition 1.1, the commutative diagram

$$\begin{array}{ccccc} 0 \to & \boldsymbol{G}'(L^{n+1}) & \to & \boldsymbol{G}_s(L^{n+1}) \\ & \Big\downarrow \boldsymbol{G}'(m) & & \Big\downarrow \boldsymbol{G}_s(m) \\ 0 \to & \boldsymbol{G}'(L) & \longrightarrow & \boldsymbol{G}_s(L) \end{array}$$

has exact rows where $m : L^{n+1} \to L$ is induced by $x_0 \otimes x_1 \otimes \cdots \otimes x_n \to x_1 x_2 \cdots x_n$. Now m is a retract of $j : x_0 \to x_0 \otimes 1 \otimes \cdots \otimes 1$, hence $\boldsymbol{G}'(m)$ is an epimorphism. It follows from the proof of Proposition 2.1 that $\boldsymbol{G}_s(j)$ is an isomorphism, whence $\boldsymbol{G}'(m)$ is also an isomorphism. Using the corresponding fact for $\boldsymbol{G}_s$, routine diagram chasing shows that all the coface maps $\boldsymbol{G}'\varepsilon_i$ in $C^*(L/k, \boldsymbol{G}')$ induce the identity on $\boldsymbol{G}'(L)$, so, similarly to Proposition 1.1, $H^n(L/k, \boldsymbol{G}') = 0$ for $n \geq 1$. The cohomology sequence arising from $0 \to \boldsymbol{G}_0 \to \boldsymbol{G} \to \boldsymbol{G}' \to 0$ gives the result for $n \geq 2$ and gives an exact sequence

$$\begin{aligned} 0 \to H^0(L/k, \boldsymbol{G}_0) \to H^0(L/k, \boldsymbol{G}) \to H^0(L/k, \boldsymbol{G}') \xrightarrow{\partial} H^1(L/k, \boldsymbol{G}_0) \\ \to H^1(L/k, \boldsymbol{G}) \to 0 \end{aligned} \tag{2.2}$$

where ∂ is the connecting homomorphism. Since $\boldsymbol{G}_0$ and $\boldsymbol{G}$ are sheaves, the natural maps $\boldsymbol{G}_0(k) \to H^0(L/k, \boldsymbol{G}_0)$ and $\boldsymbol{G}(k) \to H^0(L/k, \boldsymbol{G})$ are isomorphisms. It follows that the kernel of ∂ is isomorphic to $\boldsymbol{G}'(k) = \boldsymbol{G}(k)/\boldsymbol{G}_0(k)$, completing the proof.

We remark that all the maps in the above theorem are natural in $\boldsymbol{G}$.

3. Cohomology in $\boldsymbol{\alpha}_{p^s}$ and $\boldsymbol{\mu}_{p^s}$

The previous section showed how the cohomology of purely inseparable extensions is determined by connected group schemes. In this section we study two such group schemes which are fundamental in decomposition theory (cf. [D.-G.; IV, §3]). The conventions of the previous sections apply except that L is a finite dimensional but not necessarily purely inseparable field extension. The group schemes $\boldsymbol{G}_a$, $\boldsymbol{G}_m$, $\boldsymbol{\alpha}_{p^s}$, and $\boldsymbol{\mu}_{p^s}$ have, respectively, the following values on an algebra A: the additive group, the multiplicative group of units, the additive group of p^s-nilpotent elements, and the multiplicative group of p^s-*th* roots of unity of A. When no confusion will arise we may write simply A for $\boldsymbol{G}_a(A)$ and A^* for $\boldsymbol{G}_m(A)$.

PROPOSITION 3.1. *Let B and C be fields linearly disjoint over a subfield A. Then the natural map $B \otimes_A \cdots \otimes_A B \rightarrow BC \otimes_C \cdots \otimes_C BC$ is a monomorphism.*

Proof. $B \otimes_A \cdots \otimes_A B \rightarrow (B \otimes_A C) \otimes_C \cdots \otimes_C (B \otimes_A C) \cong BC \otimes_C \cdots \otimes_C BC$ by linear disjointness, and the result follows.

PROPOSITION 3.2. *Let L be a finite dimensional extension of k and fix an integer $s > 0$. Let $M = \{x \text{ in } L \mid x^{p^s} \text{ is in } k\}$. If L^{p^s} and k are linearly disjoint over their intersection, then the following are exact sequences of abelian groups*:

$$(3.1)\quad 0 \rightarrow \boldsymbol{\alpha}_{p^s}(L \otimes_k \cdots \otimes_k L) \subset L \otimes_k \cdots \otimes_k L \xrightarrow{\rho} L \otimes_M \cdots \otimes_M L \rightarrow 0$$

$$(3.2)\quad \begin{aligned} 0 \rightarrow \boldsymbol{\mu}_{p^s}(L \otimes_k \cdots \otimes_k L) &\rightarrow \boldsymbol{G}_m(L \otimes_k \cdots \otimes_k L) \\ &\xrightarrow{\rho} G_m(L \otimes_M \cdots \otimes_M L) \rightarrow 0. \end{aligned}$$

(The maps ρ are induced by $a_1 \otimes_k \cdots \otimes_k a_n \rightarrow a_1 \otimes_M \cdots \otimes_M a_n$. The remaining maps are the obvious inclusions.)

Proof. Consider the following sequence of maps

$$(3.3)\quad \begin{aligned} L \otimes_k \cdots \otimes_k L \xrightarrow{\rho} L \otimes_M \cdots \otimes_M L &\xrightarrow{f} L^{p^s} \otimes_{M^{p^s}} \cdots \otimes_{M^{p^s}} L^{p^s} \\ &\xrightarrow{g} k \cdot L^{p^s} \otimes_k \cdots \otimes_k k \cdot L^{p^s} \xrightarrow{h} L \otimes_k \cdots \otimes_k L \end{aligned}$$

where f is induced by raising to the p^s-*th* power, g is the natural map of Prop. 3.1 with $A = M^{p^s}$, $C = k$, and $B = L^{p^s}$

$$(\text{thus } g(x_1 \otimes_{M^{p^s}} \cdots \otimes_{M^{p^s}} x_n) = x_1 \otimes_k \cdots \otimes_k x_n)$$

and h is the natural inclusion. f is a ring map and all other maps are algebra maps. The composition $hgf\rho$ is raising to the p^s-*th* power map. Since $M^{p^s} = L^{p^s} \cap k$, Prop. 3.1 implies g is monic. f is an isomorphism of rings and ρ is clearly epic, so the exactness of (3.1) easily follows.

Now if A and B denote, respectively, $L \otimes_k \cdots \otimes_k L$ and $L \otimes_M \cdots \otimes_M L$, then for $x = \rho(a)$ in B^* we have $x^{-1} = \rho(a')$, so $1 = \rho(aa')$. From (3.1) we see $aa' - 1$ is in $\boldsymbol{\alpha}_{p^s}(k)$, whence $(aa')^{p^s} = 1$ so that a is a unit, i.e., ρ is epic in sequence 3.2. Applying $\boldsymbol{G}_m$ to (3.3) then gives exactness of (3.2)

Proposition 3.2 may be regarded as giving short exact sequences of Amitsur complexes, at least in higher dimensions, and we next exploit the resulting exact sequence

$$(3.4) \quad H^1(L/k,\, \boldsymbol{G}) \to H^1(L/k,\, \boldsymbol{G}') \to H^1(L/M,\, \boldsymbol{G}') \to H^2(L/k,\, \boldsymbol{G}) \to \cdots$$

where $\boldsymbol{G}$ is $\boldsymbol{\alpha}_{p^s}$ (resp. $\boldsymbol{\mu}_{p^s}$) and $\boldsymbol{G}'$ is $\boldsymbol{G}_a$ (resp. $\boldsymbol{G}_m$). Note that H^0 does not appear because the proposition applies only in degree one ($L \otimes L$ etc.) or higher. The lost information about $H^1(L/k,\, \boldsymbol{G})$ will be recovered by direct computation.

THEOREM 3.3. *Let k, L, M be as in* Prop. 3.2. *Then $H^n(L/k,\, \boldsymbol{\alpha}_{p^s}) = 0$ for $n \geq 2$ and $H^1(L/k,\, \boldsymbol{\alpha}_{p^s}) \cong M/k$.*

Proof. The first assertion follows from the cohomology sequence (3.4) and the well-known fact that for $n \geq 1$, $H^n(B/A,\, \boldsymbol{G}_a) = 0$ for any fields $A \subseteq B$ (e.g. [Am; Th. 3.8]).

Direct computation (cf. Lemma 3.8, [C.-R.]) shows that $m \to \mathrm{cl}(m \otimes 1 - 1 \otimes m)$ gives an epimorphism $M \to H^1(L/k,\, \boldsymbol{\alpha}_{p^s})$ with kernel k. (See the similar computation in Theorem 3.4 below.)

Remark. When our linear disjointness and finite dimensionality conditions hold, some of the computations of Dobbs [Do; p. 83 ff.] can be recovered. For example, if $k^{1/p^s} \subseteq L$, then raising to the p^s-*th* power gives an isomorphism $M/k \cong k/k^{p^s}$, so that, where the hypotheses coincide, our result is his Cor. 2.9. If L is finite separable over k, then the linear disjointness conditions hold for each s, and so the finite dimensional case of Dobbs' Cor. 2.13 follows from our theorem since a finite perfect field extension of k is necessarily separable. Note however that neither this case of his corollary, nor the whole of its consequence in the second paragraph of the remark following it, require either our or Dobbs' machinery, since L/k finite separable implies that $C^n(L/k,\, \boldsymbol{\alpha}^{p^s}) = 0$ for $n > 0$.

THEOREM 3.4. *Let k, L, M be as in Prop. 3.2. Suppose further that L is purely inseparable of finite exponent over k. Then $H^n(L/k,\, \boldsymbol{\mu}_{p^s}) = 0$ for $n \geq 3$, there is an exact sequence*

$$0 \to H^2(L/k,\, \boldsymbol{\mu}_{p^s}) \to H^2(L/k,\, \boldsymbol{G}_m) \to H^2(L/M,\, \boldsymbol{G}_m) \to 0$$

and $H^1(L/k,\, \boldsymbol{\mu}_{p^s}) \cong M^*/k^*$.

Proof. By a result of Berkson-Rosenberg-Zelinsky [R.-Z.2; Th. 6.2],

$$H^n(L/k,\, \boldsymbol{G}_m) = H^n(L/M,\, \boldsymbol{G}_m) = 0 \quad \text{for } n \neq 2.$$

By [R.-Z.2; Cor. 6.2] (or [R.-Z.1; Th. 3] and [*H.*; Th. 5]), $H^2(L/k,\, \boldsymbol{G}_m) \to$

$H^2(L/M, \mathbf{G}_m)$ is surjective and the result follows from the cohomology sequence (3.4) except for the assertion about H^1.

Now regarded as a sequence of complexes, Propn. 3.2 yields an exact commutative diagram

$$\begin{array}{ccccccccc}
 & & & & \mathbf{G}_m(L) & \xrightarrow{\mathrm{id}} & \mathbf{G}_m(L) & & \\
 & & & & \downarrow{\scriptstyle d^0} & & \downarrow{\scriptstyle d^0} & & \\
0 \to & \boldsymbol{\mu}_{p^s}(L \otimes L) & \to & & \mathbf{G}_m(L \otimes L) & \longrightarrow & \mathbf{G}_m(L \otimes_M L) & \to 0 & \\
 & \downarrow{\scriptstyle d^1} & & & \downarrow{\scriptstyle d^1} & & \downarrow{\scriptstyle d^1} & & \\
0 \to & \boldsymbol{\mu}_{p^s}(L \otimes L \otimes L) & \to & & \mathbf{G}_m(L \otimes L \otimes L) & \to & \mathbf{G}_m(L \otimes_M L \otimes_M L) & \to 0. &
\end{array}$$

If x is a 1-cocycle in $\boldsymbol{\mu}_{p^s}(L \otimes L)$, it is in particular a cocycle in $\mathbf{G}_m(L \otimes L)$. Since $H^1(L/k, \mathbf{G}_m) = 0$ [R.-Z.2; Cor. 6.1] we have $x = a \otimes a^{-1}$ for some a in L^*. Since $x^{p^s} = 1$ we conclude from Lemma 3.8 of [C.-R.] that a^{p^s} is in k, i.e., a is in M. Thus $a \to \mathrm{cl}(a \otimes a^{-1})$ is a surjection $M^* \to H^1(L/k, \boldsymbol{\mu}_{p^s})$. Its kernel is k^*, for if $a \otimes a^{-1}$ is a coboundary from $\boldsymbol{\mu}_{p^s}(L) = \{1\}$ then Lemma 3.8 of [C.-R.] again implies a is in k.

COROLLARY 3.5. *With k, L, M as in Theorem* 3.4, $H^2(L/k, \boldsymbol{\mu}_{p^s}) \cong H^2(M/k, \mathbf{G}_m)$.

Proof. By [R.-Z.2, Cor. 6.2], the kernel of $H^2(L/M, \mathbf{G}_m) \to H^2(L/M, \mathbf{G}_m)$ also coincides with $H^2(M/k, \mathbf{G}_m)$ giving the result.

Applications. (1) If L has exponent m then L^{p^s} and k are linearly disjoint (over their intersection) for $s \geq m$ and so the results of this section apply for all such s. In particular, this generalizes III, §6, 9.9 of [D.-G.] for $\boldsymbol{\alpha}_{p^s}$ and $\boldsymbol{\mu}_{p^s}$. The inflation techniques of [R.-Z.2] do not seem to generalize to arbitrary groups, making it difficult to extend the result of [D.-G.] to more general field extensions.

(2) If L is *modular* (i.e. the tensor product of primitive extensions), then L^{p^s} and k are always disjoint [Sw 2; Lemma 4] and so for modular extensions the results of this section apply for all positive integers s.

(3) The characterization of $H^1(L/k, \boldsymbol{\mu}_{p^s})$ does not use the pure inseparability in its proof, but only that $H^1(L/k, \mathbf{G}_m) = 0$.

(4) Note that if $k^{1/p^s} \subseteq L$ then raising to p^s*-th* powers gives an isomorphism $M^*/k^* \cong k^*/k^{*p^s}$ yielding results analogous to those of Dobbs mentioned after Theorem 3.3.

(5) In view of the exactness of inflation-restriction for H^2 [R.-Z.2; Cor. 6.2] and the usual identification [R.-Z.1; Th. 3] of the relative Brauer group $B(M/k)$ with $H^2(M/k, \mathbf{G}_m)$, our Theorem 3.4 gives an isomorphism $H^2(L/k, \boldsymbol{\mu}_{p^s}) \cong B(M/k)$. Results of Hochschild [H, especially p. 140] may be cast in this light [cf. R.-Z. 2, Cor 6.5 ff.].

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