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Special Issue on Admissible Rules and Unification

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Admissible rules play a sometimes implicit but fundamental role in many fields of mathematics and computer science. Algorithmically, they can be described as rules that when added to a system produce no new theorems, and semantically, as rules where if the premises of an instance are theorems, then so is the conclusion. The study of admissible rules spans a broad range of topics, including free and projective algebras, equational unification, duality, and proof theory.

The notion of an admissible rule was defined explicitly by Paul Lorenzen in the 1950s in the context of intuitionistic logic, but had appeared already in earlier work of Gerhard Gentzen, Ingebrigt Johansson, and others. The closely related topic of universal sentences valid in free algebras also played a prominent role in the work of Phillip Whitman and Bjarni Jónsson on free lattices. Research on admissible rules initially focused mainly on intuitionistic logic, culminating in Vladimir Rybakov's proof that its set of admissible rules is decidable but not finitely axiomatizable, and independent completeness proofs by Rosalie Iemhoff and Paul Rozière for an elegant axiomatization conjectured by Dick De Jongh and Albert Visser. Iemhoff's proof stems from Silvio Ghilardi's pioneering work on equational unification, which has provided the springboard for a resurgence of interest in admissible rules and their relationship to unification. The papers in this volume bear witness both to this resurgence and its broad scope.

The paper *Admissible rules of the Leibniz hierarchy* by James Raftery presents an analysis of admissibility in the framework of abstract algebraic logic. The key achievement of this work is to extend the reach of the main ideas and theorems for admissible rules, typically formulated algebraically, to nonalgebraizable logics of the "Leibniz hierarchy." These methods are put to work to obtain new results for admissibility in certain substructural logics. Related substructural logics are investigated in the paper *An Abelian rule for* BCI*—and variations* by Tomasz Kowalski and Lloyd Humberstone. The authors provide an ingenious proof-theoretic argument to show the admissibility of certain nonderivable rules and hence the failure of structural completeness for these logics.

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The topic of equational unification is represented in this special issue by two very different papers. An algebraic approach to equational unification is taken in *Unification on subvarieties of pseudocomplemented lattices* by Leonardo Cabrer, who uses natural dualities to obtain a complete classification of the unification types of problems in subvarieties of pseudocomplemented distributive lattices. On the other hand, the paper *Deciding unifiability and computing local unifiers in the description logic* $\mathcal{EL}$ *without top constructor* by Franz Baader, Nguyen Thanh Binh, Stefan Borgwardt, and Barbara Morawska concerns the application of unification in description logics as a tool for detecting redundancy in ontologies. In particular, it is shown that while unification in the description logic $\mathcal{EL}$ (which has been used to define several biomedical ontologies) is NP-complete, removing the top concept from this logic makes unification PSPACE-complete.

Projectivity plays an important role in the context of admissible rules, as illustrated by the papers *Algebraic logic perspective on Prucnal's substitution* by Alex Citkin and *Modal consequence relations extending* S4.3*: An application of projective unification* by Wojciech Dzik and Piotr Wojtylak. The author of the first paper describes a generalization of Prucnal's substitution in an algebraic setting and corresponding results on structural completeness for certain fragments of logics for which such a substitution exists. In the second paper, all finitary consequence relations over S4.3 are characterized both algebraically and in terms of rules. It is shown that they all have the strong finite model property and that they form a countable Heyting algebra.

The origins of this special issue lie with the meeting "Workshop on Admissible Rules and Unification (WARU)" held at Utrecht University, 26–28 May 2011, supported by the Netherlands Organisation for Scientific Research (NWO), which brought together leading researchers on admissibility and unification for three days of stimulating talks and lively discussion.

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