Murray Rosenblatt, *Random Processes*. Oxford University Press, New York, 1962. $6.00, x + 208 pp.

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One of the important problems in teaching probability theory is the paucity of good text-books which give an excellent, logical, and interesting, account of the subject, but do not burden the reader with bulky and cumbrous presentations and voluminous size. A lack of such good literature is keenly felt, especially in the field of random processes. In spite of the fact that there exists a series of excellent books by Bartlett, Doob, Dynkin, Takács and many others, all original expositions of the fundamentals of the Theory of Probability, we cannot admit that the situation is satisfactory. Often one experiences great difficulty in recommending a concise, yet well laid-out, book on modern lines about random processes. Therefore the book of M. Rosenblatt deserves great attention. It is a small book, well written and at the same time containing rich material and well selected problems for a fundamental text-book. The author does not feel the necessity of stating the connection between the theory laid down by him and practical problems. Only in very particular instances does he write a few words about such relations. Such a book would be of constant use to mathematicians, physicists, technicians, biologists, geophysicists, and to the representatives of other concrete sciences.

Let us say a few words about the contents of the book under review. It contains six chapters, a few appendices to the main body of the text (Chap VIII), and an introduction containing the ideas of the author (Chap I). A bibliography (77 references) and an index are at the end of the book. At the end of each chapter there are problems for solution by the reader, and a short historical note by the author. Problems are presented in an interesting manner. Undoubtedly the reader who solves all these problems not only grasps the ideas and the results of the theory of random processes, but appreciates the methods of proof.

The second chapter, "Basic Notions for Finite and Denumerable State Models," acquaints the reader with the fundamental ideas and formulas of probability theory. Previous knowledge of the subject on the part of the reader is not presumed. At the same time the reader is presented, at the end of the chapter, with the law of large numbers and, based on these ideas, the proof of S. Bernstein for the theorem of Weierstrass on the uniform approximation of continuous functions by polynomials in the interval [a, b]. Further, Central Limit Theorems are proved by the methods of differential equations, and the ideas of entropy are introduced. The only special deficiency, which we can point out to the author, is that, in the proof of the law of large numbers, he limits himself to the case of identically and equally distributed components.

In the third chapter, "Markoff Chains," are presented the elements of very important discrete schemes: definition of Markoff chains, notions of birth and death processes, a short exposition of the theory of matrices with non-negative elements, classification of the states, limit theorems, and some information about functions of Markoff chains.

The fourth, and at the same time the last, introductory chapter, "Probability Spaces with an Infinite Number of Sample Points," acquaints the reader with the axiomatic definition of probability, distributions of probability, different types of convergence of sequences of random variables, ideas of characteristic functions, probability measures, conditional probability, and, at the end, general notions of random processes.

The fifth chapter, "Stationary Processes," starts with the definition of strictly stationary processes, giving three examples adopted from physics, and the theory of Markoff chains. The ergodic theorem is formulated and proved, and the general results are applied to Markoff chains. The chapter ends with the formulation and the proof of the well known theorem of Macmillan in Information Theory.

The sixth chapter, "Markoff Processes," contains the elements of Markoff processes. Integral-differential equations of jump processes with continuous time, and differential equations of diffusion processes, are discussed. In somewhat greater detail are discussed the theory of Brownian processes and examples of jump processes.

The seventh chapter, "Weakly Stationary Processes and Random Harmonic Analysis," gives a fairly good account of the theory of stationary processes based on the properties of second moments. Then the author discusses harmonic analysis of stationary processes, questions of linear prediction, and properties of spectral functions of normal stationary processes with discrete parameters.

In the eighth chapter, "Additional Topics," are collected a few results and examples of general interest: a zero-one law; the construction of Markoff chains by means of a series of independent random variables, each uniformly distributed over (0, 1); the study of a representation of a class of random processes; and conditions of uniform mixing.

The book, although small, contains a lot of information, and the presentation is an example of clarity and logic. I noticed only a few slips: for example, on page 148 a reference is made to Chapter six, whereas it ought to be to Chapter four. I know of only one minor mistake: on page 194, an appeal is made to Dini's theorem for the uniform convergence part of a theorem stated on page 187. But Dini's theorem is insufficient for the intended purpose.

The book obviously carries a tinge of the author's subjective tastes; we find that minimum attention is paid to Markoff processes in spite of their importance both in theoretical and in practical questions. The style of presentation is analytic; geometric and many theoretical concepts are very weakly presented. How ever, the individuality of the author and his method of presentation add a charm to the book.

I read the entire book with great interest, and I hope that it will be of immense help to many, especially to young mathematicians.