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ABSTRACTS OF PAPERS

(*Abstracts of papers presented at the Annual meeting, Madison, Wisconsin, August 26–30, 1968. Additional abstracts appeared in earlier issues.*)

90. Bayesian analysis of growth curves. SEYMOUR GEISSER, State University of New York at Buffalo. (Invited)

From a Bayesian viewpoint, we initiate the study of the generalized growth model of (Potthoff and Roy, *Biometrika* **51** (1964)). The model asserts $E(Y_{p\times N}) = X_{p\times m}\tau_{m\times r}A_{r\times N}$, where X is a known matrix of rank $m \leqq p$, A is a known matrix of rank $r \leqq N$, τ is unknown and the columns of Y are independent p-dimensional multinormal variates having a common unknown arbitrary covariance matrix Σ. A Bayesian justification is presented for the Rao (*Proc. Fifth Berkeley Symp. Math. Statist. Prob.* **1** (1967)) adjusted estimator $\hat{\tau}$ of τ the set of unknown parameters, as well as an estimating region for τ. Using only an augmented location model we also obtain a Bayesian vindication for the unadjusted estimator of τ that is necessarily different in character from the frequentist exculpation since this latter pertains to a restricted covariance structure. The problem of estimating regions for future observations from this model, given a preliminary sample, is also discussed. (Received 12 August 1968.)

91. A unified derivation of tests of goodness of fit based on spacings. B. K. KALE, University of Manitoba.

Let $(x_1, x_2, \cdots, x_n)$ be a random sample from a continuous df F and suppose we wish to test the null hypothesis that $F = F_0$ where F_0 is a completely specified df. Let $x_{(0)} = -\infty < x_{(1)} < x_{(2)} \cdots < x_{(n)} < x_{(n+1)} = +\infty$ denote the order statistics of the sample and let $v_i = F_0[x_{(i)}] - F_0[x_{(i-1)}]$, $i = 1, 2, \cdots, n+1$, be the spacings. Several tests of goodness of fit based on spacings are known in the literature. By using the set of observations and the hypothesized df F_0, we first derive a simple estimate $\Phi_n(x)$ of the underlying df and show that for large n, $\Phi_n(x)$ is close to $F_n(x)$ the usual sample df or empirical df. By giving various definitions of "closeness", essentially using directed distances, we show that $\Phi_n(x)$ is the df which is as "close" as possible to F_0 and therefore most difficult to discriminate from F_0 on the basis of the given sample. Using the rule to reject $F = F_0$ if Φ_n and F_0 are sufficiently apart, we derive several different tests based on spacings v_i corresponding to different definitions of "closeness". (Received 19 August 1968.)

(*An abstract of a paper presented at the European meeting, Amsterdam, Netherlands, September 2–7, 1968. Additional abstracts appeared in earlier issues.*)

14. Nonparametric confidence and tolerance bounds and intervals when sampling from a finite population. H. S. KONIJN, Tel Aviv University.

Nonparametric confidence intervals and bounds for fractiles and tolerance intervals and bounds have been obtained by Wilks for samples of n independent observations from a continuous population. Tukey and others have made some assertions with respect to applicability of these results to discrete populations. Wilks also gives some incomplete results for the case of simple random sampling from a finite discrete population without replacement in which all members of the population take on different values. This paper gives complete and exact results for this case and examines also the case of sampling with replacement. It turns out that the argument is essentially more elementary than that for continuous populations; the latter may then, of course, be obtained from the former by a limiting process. The only quite intractable case is that of tolerance intervals in sampling with replacement; but, for large N, the results for this case can be readily approximated by those for sampling without replacement for N large. (26 August 1968.)

(Abstracts of papers to be presented at the Central Regional meeting, Iowa City, Iowa, April 23–25, 1969. Additional abstracts will appear in future issues.)

1. Υ-minimax selection procedures—Treatment vs. control. RONALD H. RANDLES and MYLES HOLLANDER, The Florida State University.

The Υ-minimax principle for the use of incomplete prior information has been investigated by Blum and Rosenblatt (*Ann. Math. Statist.* **38** (1967) 1671–78) and Menges (*Unternehmensforschung* **1** (1966) 81–91), among others. If Υ is a class of distributions over the parameter space Θ, a Υ-minimax procedure is a rule δ_0 which minimizes $\sup_{\tau \varepsilon \Upsilon} r(\tau, \delta)$ where $r(\iota, \delta)$ denotes the expected risk. Let $T_0, T_1, \cdots, T_k$ be independent random variables having respective probability densities $f(t_i - \theta_i)$ where f is a Pólya frequency function of order 2. The ith distribution is defined to be positive if $\theta_i \geqq \theta_0 + \Delta$ and negative if $\theta_i \leqq \theta_0$ for $i = 1, \cdots, k$ and $\Delta > 0$. The approach here is similar to Lehmann (*Ann. Math. Statist.* **32** (1961) 990–1012). The loss is $L_1R + L_2S$ where L_1, L_2 are positive constants, R is the number of positive distributions rejected and S is the number of negative distributions selected. Υ-minimax procedures are found for θ_0 known and unknown when Υ is the set of all distributions over Θ for which $P[\theta_i \geqq \theta_0 + \Delta] = \pi_i$ and $P[\theta_i \leqq \theta_0] = \pi_i'$, $i = 1, \cdots, k$. Comparisons of the Υ-minimax procedure with Bayes competitors based on independent normal priors are presented for the case where $f(t)$ is the density of a normal variate with known variance. (Received 1 October 1968.)

(Abstracts of papers not connected with any meeting of the Institute.)

1. Asymptotic relative efficiencies of some tests for trend and autocorrelation (preliminary report). R. J. AIYAR, University of California, Berkeley. (Introduced by Peter J. Bickel)

Many distribution free tests have been proposed in the literature to test the hypothesis of randomness. Against alternatives specifying a linear trend in location, many of these test statistics are shown to have asympotically normal distributions, both under the hypothesis and under alternatives converging to the hypothesis at suitable rates. The Pitman efficiency of these procedures is computed and, in particular, the records test, proposed by Stuart, is shown to have efficiency zero against most competitors. It is shown, further, that the efficiencies are unchanged if, under the alternative, in addition to the trend we have an autoregressive process of order one, provided the alternatives are contiguous to the hypothesis. Finally the rank serial correlation and normal scores serial correlation of order one have been investigated for testing the hypothesis of randomness against the alternative that we have an autoregressive process of order one. These statistics have been shown to have asympotically normal distributions under the hypothesis and for "near" alternatives. The Pitman efficiency of these tests relative to the classical test based on serial correlation between successive observations is computed and lower bounds on these efficiencies are now being sought. (Received 6 August 1968.)

2. Probabilities of moderate deviations of U-statistics (preliminary report). GERALD M. FUNK, Michigan State University. (Introduced by Herman Rubin)

Let $X_1, X_2, \cdots$ be independently and identically distributed random variables. The U-statistic, $U(X_1, \cdots, X_n)$, generated by a function symmetric in its k arguments based on $X_1, \cdots, X_n$ was defined by Hoeffding (*Ann. Math. Statist.* (1948)). Under certain moment conditions Rubin and Sethuraman (*Sankhyā Ser. A* **27** (1965) 325–346) obtained order results for probabilities of moderate deviations of these statistics. These results are extended to obtain asymptotic expressions of the form $P(U(X_1, \cdots, X_n) - EU(X_1, \cdots, X_n) > ck\sigma(\log n/n)^{\frac{1}{2}}) \sim (2\pi c^2 \log n)^{-\frac{1}{2}}(n^{-\frac{1}{2}c^2})$ $(c > 0)$ where σ is a constant not dependent on n. Similar results are obtained for the generalized U-statistics of Lehmann (*Ann. Math. Statist.* (1951)), regular functionals of the sample df and some functions of U-statistics. (Received 12 August 1968).

3. Bayesian estimation of means of a two-way classification with random effect model. N. Giri, Forschungsinstitut füı Mathematik, Eidenossische Technische Hochschule.

The problem of estimating the means in a two-way classification with random effect model $Y_{ij} = \alpha_i + \beta_j + 1_{ij}$, where 1_{ij} are independently normally distributed with zero mean and variance σ_1^2, has been considered here. Posterior distributions of α_i, β_j have been obtained under the assumption that α_i, β_j are independently normally distributed as $N(\alpha, \sigma_2^2)$, $N(\beta, \sigma_3^2)$ and are compared with the corresponding distributions for a fixed effect model. (Received 12 August 1968.)

4. Admissibility of the usual confidence sets for the mean of a univariate or bivariate normal population. V. M. Joshi, Government of Maharashtra, Bombay.

Let X be an m dimensional random vector distributed normally with mean vector θ and covariance matrix equal to the $m \times m$ identity matrix. The usual confidence sets for θ are spheres of fixed volume centered at the sample mean. They have the property that for given lower confidence level $(1 - \alpha)$ they minimize the maximum expected volume of the confidence sets. Stein (*J. Roy Statist. Soc. Ser. B* **24** 265–296) raised the question whether the usual procedure is unique in having this property and conjectured that it is probably unique for $m = 1$, probably not so for $m \geqq 3$, the case $m = 2$ being doubtful. For $m \geqq 3$, the conjecture was shown to be true in a previous paper of the author (*Ann. Math. Statist.* **38** (1967), 1868–1875). The cases $m = 1$ and $m = 2$ are here investigated and it is shown that provided a certain class of procedure is treated as equivalent, the usual procedure is unique and admissible. The uniqueness and admissibility are proved for the extended class of randomized confidence procedures. (Received 12 August 1968.)

5. A generalized estimator for the mean of a finite population using multi-auxiliary information. Surendra K. Srivastava, Lucknow University.

For estimating the mean $\bar{Y}$ of a character y of a finite population with the help of information on p auxiliary characters $x_1, \cdots, x_p$, some of which may be positively and others negatively correlated with y, the estimator $\tilde{y} = \bar{y} \sum_{i=1}^{p} (\bar{x}_i/\bar{x}_i)^{\theta_i}$ is considered. Here $\bar{y}$, $\bar{x}_i$, and $\bar{X}_i$ denote respectively the means of y and x_i based on a simple random sample of size n and the known population mean of x_i, and θ_i's are determined to minimize the variance of y. The optimum variance, up to order $O(n^{-1})$, of $\tilde{y}$ comes out to be $(n^{-1} - N^{-1})S_y^2(1 - R^2)$ where R is the multiple correlation coefficient of y on $x_1, \cdots, x_p$. The estimator $\tilde{y}$ has been shown to have a smaller variance than that of Olkin's multivariate ratio estimator [*Biometrika* **45** (1958) 154–165] always and than that of Srivastava's multivariate estimator [*Calcutta Statist. Assoc. Bull.* **16** (1967) 121–132] for a particular case. Singh's ratio cum product estimator [*Metrika* **12** (1967) 34–42] is a particular case of $\tilde{y}$ with θ_i's equal to -1 and $+1$. (Received 24 August 1968.)