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J. J. Martin, *Bayesian Decision Problems and Markov Chains*. John Wiley and Sons, Inc., New York, 1967. xii + 202 pp. $10.95.

Review by B. R. Bhat

The University of Western Australia and Karnatak University

There has been a spurt of literature on decision problems and Markov processes following the publication of the excellent little book by R. A. Howard in 1960 [1]. The book under review is one of them. It is based on the author's Ph.D. thesis. It contains several new and interesting results based mainly on the work done at M.I.T. on operations research.

The book deals with Bayes decision problems for finite Markov chains (M.C.) with uncertain transition probabilities and rewards discussed by Howard. Chapter 1 introduces the above notions. The book is concerned with families of distributions closed under consecutive sampling rule or ν-step sampling rule and in particular with natural conjugate *a priori* distributions (cf. Raiffa, H. and Schlaiffer, R. (1961)). Some properties of such families are discussed in Chapter 2. Chapters 2 and 6 could have been combined because generalized beta variables are taken up in Chapter 6, while the distribution of a set of independent generalized beta variables, called matrix-beta variable, is introduced in Chapter 2. In Chapter 6, the distribution of transition counts when the M.C. is observed for a fixed length of time, aptly called Whittle distribution, is stated for the case when the transition probabilities (tr. pr.) are known. For the case when they follow the matrix-beta distribution, the first two moments of tr. pr.'s are also derived in this chapter.

Chapter 3 is concerned with adaptive control problems when rewards depend on transitions and alternatives. Chapter 4 deals with the usual M.C., with only one alternative at each state. Expected values of n-step transition probabilities, total reward in n-steps, steady-state probabilities, etc. are derived here, when tr. pr. matrix has a distribution belonging to a family closed under consecutive sampling rule. Chapter 5 discusses terminal control or sequential sampling problems. In both the Chapters 3 and 5, problems of existence, uniqueness and derivation of optimal strategies are discussed. The arguments are lengthy but straightforward. In Chapters 3 and 4 numerical examples involving 2-state M.C. are worked out. The method of deriving the optimal solution is that of dynamic programming. MAD computer programs to solve the recursive equations occurring in Chapters 3 and 4 are given in the Appendix. In Chapter 7, using the results of Chapter 6, prior-posterior and pre-posterior analyses are made in the cases when the initial state is known or random and in the stationary case. The final chapter illustrates, by using a 2-state M.C. the derivation of various results of Chapters 6 and 7, e.g. expected values of n-step transition probabilities, steady-state probabilities, process gain, etc. In conclusion the author says that Bayes treatment is practicable for problems involving 2 or 3 state M.C.'s only.

The book is by and large self-contained. It is readable by those who have a good background of classical analysis, matrix theory and some knowledge of probability and statistics. Even though written primarily for operation researchers, the book may be read with profit by statisticians as well. The glossary at the end of the book is very useful as the notations may baffle any reader. The reviewer has come across only a few minor errors and misprints. The book is a beauty, but priced at $10.95, it is rather expensive.

REFERENCES

[1] Howard, R. A. (1960). *Dynamic Programming and Markov Processes*. Wiley, New York.
[2] Raiffa, H. and R. Schlaifer. (1961). *Applied Statistical Decision Theory*. Harvard University, Cambridge.