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NONREVERSIBLE STATIONARY MEASURES FOR EXCHANGE PROCESSES[1]

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We consider nonreversible exchange dynamics in Z^d and prove that the stationary, translation invariant measures satisfy the following property: if one of them is a Gibbs measure with a summable potential $\{J_R, R \subset Z^d\}$, then all of them are convex combinations of Gibbs measures with the same potential, but different chemical potentials $J_{\{0\}}$.

1. Introduction and result. We consider an exclusion process on the infinite lattice $\mathbb{Z}^d$. Each site contains at most one particle and each bond joining adjacent sites is associated with a Poisson process of intensity $c_e(\eta)$, a positive, translation invariant and finite range function of the configuration η in $\Omega = \{0, 1\}^{\mathbb{Z}^d}$. At the jump times of the Poisson process the contents of the two sites are exchanged.

The existence of such a process is well known [see Liggett (1985), Chapter I]. For a simple symmetric exclusion process (i.e., $c_e \equiv 1$), the extremal stationary measures at density α are the product measures with this density [see Liggett (1985), Chapter VIII]. Furthermore, in the translation invariant case, if the process is reversible with respect to a Gibbs measure, then the canonical Gibbs measures are the stationary ones [Holley (1972); Georgii (1979)].

Actually, the same results were obtained first for the stochastic Ising model. More precisely, if the spin-flip dynamics is reversible with respect to a Gibbs measure, then in one and two dimensions all the stationary measures are Gibbs with the same potentials [Holley and Stroock (1977); see Vanheuverzwijn (1981) for the analog for exchange dynamics]. In higher dimensions, the translation invariant stationary measures are Gibbs with the same potentials [Holley (1971)], whereas the problem is still open if one drops translation invariance.

The next question concerns nonreversible dynamics. What are the relations between stationary measures? A first result in this direction is as follows: when, for any $x, y \in \mathbb{Z}^d$, the rate of exchanging site x and y does not depend on the configuration, say, $c_{(x,y)} = p(0, y - x)$, and satisfies for any $t > 0$,

$$\sum_{n \geq 1} \frac{t^n}{n!} \big(p^{(n)}(x, y) + p^{(n)}(y, x) \big) > 0,$$

where $p^{(n)}$ is the nth convolution of p, then Liggett (1976) showed by coupling methods that the extremal translation invariant stationary measures are the

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product measures of density $\alpha \in [0,1]$. Another result concerns spin-flip processes [Künsch (1984)]: if the rates are positive, finite range and if we know a priori that one translation invariant, stationary measure is Gibbs with a summable potential, then all translation invariant measures are Gibbs with the same potential. We generalize the method used by Künsch (1984) to the case of exchange processes. Also, our result extends the result of Liggett (1976) to more general rates, via the relative entropy approach.

THEOREM 1. *Let P and μ be two translation invariant, stationary measures for an exchange dynamics with generator* (2.1). *If P is a Gibbs measure with a potential $\{J_R\}$ satisfying*

$$\sum_{R \ni 0} |J_R| < \infty, \tag{1.1}$$

then μ is a convex combination of Gibbs measures with the same potential $\{J_R,\ |R| > 1\}$, but different chemical potential $J_{\{0\}}$.

This result is of interest in the context of hydrodynamic limits for nonreversible systems [Eyink, Lebowitz and Spohn (1996)]. There, one would like to approximate a slowly varying density by a measure which would look locally like a stationary measure. Thus, it is important to find the relation between stationary measures at different densities. Our result says that in the translation invariant case, one goes from one stationary measure to another by varying the chemical potential.

We follow, as Künsch did, an idea which goes back to Holley (1971): We suppose that P is a stationary measure. Let μ be any measure and μ_t be its image at time t under the dynamics. We form the relative entropy of μ_t with respect to P when both measures are projected over a bounded domain, say Λ. The time derivative of the relative entropy can be divided into a subadditive part, expected to grow like the volume of Λ, and a part growing more slowly (the boundary terms). Thus, if μ is stationary, the subadditive part must vanish. This in turn says that μ is a canonical Gibbs measure. Finally, a theorem of Georgii (1979) identifies extremal canonical measures with grand canonical measures at different chemical potentials.

The main difficulty, when using this approach, is to estimate the boundary terms, which are here of a different nature than those encountered by Künsch. However, we found it useful to introduce, like Künsch did, the rates $\{\hat{c}_e\}$ of the dual process: for instance, one special (to these models) and quite simple observation is (3.6),

$$E\left[\sum_{e \in [\Lambda_n]} \hat{c}_e || F_{\Lambda_n}\right] - \sum_{e \in [\Lambda_n]} c_e = o(|\Lambda_n|).$$

In Section 3, we prove Theorem 1, while in Section 4, we give an example where our result brings some new information.

2. Definitions and notations. Two sites x and y in Z^d define a bond $e = (x, y)$ if they are one unit apart, that is, if $\sum_{i=1}^{d} |x_i - y_i| = 1$. If $\{\tilde{e}_i,\ i = 1, \dots, d\}$ is a unit basis of Z^d, then for $i = 1, \dots, d$, e_i denotes the bond $(0, \tilde{e}_i)$. For a subset X of Z^d, $[X]$ denotes the set of bonds with vertices in X, while $|X|$ denotes the number of sites in X.

The space of configurations Ω is equipped with the product topology and its Borel σ-algebra. Let

$$\begin{aligned} \Lambda_n &= [-2^n + 1, 2^n - 1]^d, \\ \tilde{\Lambda}_n &= [-2^n + n, 2^n - n]^d, \\ \Lambda_n^+ &= \Lambda_n + [-1, 1]^d, \end{aligned}$$

and let F_n be the σ-algebra generated by $\{\eta_i,\ i \in \Lambda_n\}$. Also, the set of cylinders in Λ_n is denoted by $\Re_n \equiv \{0, 1\}^{\Lambda_n}$.

Now, we define three operations on Ω: T_e exchanges values of the pair of sites e in η, that is, if $e = (i, j)$,

$$T_e(\eta)(i) = \eta(j), \qquad T_e(\eta)(j) = \eta(i)$$

and

$$\forall k \neq i, j \qquad T_e(\eta)(k) = \eta(k);$$

σ_i flips only the ith coordinate of η, that is, $\sigma_i(\eta)(i) = 1 - \eta(i)$; τ_i shifts by $i \in Z^d$, that is, $\tau_i \eta(j) = \eta(j + i)$.

A Gibbs measure μ associated with the potential $\{J_R\}$ is a probability measure such that for any finite subset Λ, for $\zeta \in \Omega$ and $\eta \in \{0, 1\}^{\Lambda}$,

$$\mu(\eta \| F_{\Lambda^c})(\zeta) = \frac{1}{Z_{\Lambda, \zeta}} \exp\biggl(-\sum_{R \cap \Lambda \neq \varnothing} J_R \prod_{i \in R \cap \Lambda} \eta_i \prod_{i \in R \cap \Lambda^c} \zeta_i\biggr),$$

where $Z_{\Lambda, \zeta}$ is a normalizing factor. Furthermore, we assume that the potential is translational invariant, that is, $J_R = J_{\tau_i R}$. In this case, $J_{\{0\}}$ is called the chemical potential.

A canonical Gibbs measure μ is a probability measure such that for any finite subset Λ, for $\zeta \in \Omega$ and $\eta \in \{0, 1\}^{\Lambda}$,

$$\mu\biggl(\eta \| F_{\Lambda^c}, \sum_{\Lambda} \eta_i = n\biggr)(\zeta) = \frac{I_{\{\sum_\Lambda \eta_i = n\}}}{Z_{\Lambda, \zeta, n}} \exp\biggl(-\sum_{R \cap \Lambda \neq \varnothing} J_R \prod_{i \in R \cap \Lambda} \eta_i \prod_{i \in R \cap \Lambda^c} \zeta_i\biggr).$$

The generator of our exchange process is

$$Lf(\eta) = \sum_{i=1}^{d} \sum_{j \in Z^d} c_{e_i}(\tau_j \eta)(f(T_{e_i + j}\eta) - f(\eta)), \tag{2.1}$$

where $e_i + j$ is the translate of j units of the pair e_i, and $\{c_{e_i},\ i = 1, \dots, d\}$ are finite range functions.

A crucial assumption is that there are $\underline{c}$ and $\bar{c}$ positive constants such that

$$\forall i = 1, \dots, d, \qquad \underline{c} \le c_{e_i} \le \bar{c}. \tag{2.2}$$

For simplicity, we have stated our result in the case of nearest neighbor exchanges; however, the whole argument goes almost unchanged for a process where particles can jump within a finite range.

The relative entropy of the projection of any measure μ in the volume Λ_n, with respect to P, is well defined,

$$H_n(\mu) = \sum_{\eta\in\Re_n} \mu(\eta)\log\left(\frac{\mu(\eta)}{P(\eta)}\right), \tag{2.3}$$

with the convention that $0\log(0) = 0$. The measure μ_t is defined on $\Re_n$ by

$$\forall \eta \in \Re_n, \qquad \mu_t(\eta) = \int S_t(I_\eta)\,d\mu,$$

where S_t is the semigroup associated with L, and I_η is the indicator of the cylinder η. Now,

$$\frac{d\mu_t(\eta)}{dt} = \int_\Omega S_t L(I_\eta)\,d\mu = \sum_{e\in[\Lambda_n^+]} \int_\Omega c_e I_{T_e^{-1}(\eta)}\,d\mu_t - \int_\Omega c_e I_\eta\,d\mu_t.$$

It is convenient to define

$$\Gamma_e(\eta) = \int_\Omega c_e I_{\{\eta\}}\,d\mu$$

and rewrite

$$\left.\frac{dH_n(\mu_t)}{dt}\right|_{t=0} = \sum_{\eta\in\Re_n}\sum_{e\in[\Lambda_n^+]}\left[(T_e^{-1}\Gamma_e - \Gamma_e)\log\left(\frac{\mu}{P}\right)\right](\eta). \tag{2.4}$$

We note that, on Ω, $T_e^{-1} = T_e$. However, this is not so on $\Re_n$: if $j \notin \Lambda_n$ and $i \in \Lambda_n$, then

$$\begin{aligned}\forall \eta \in \Re_n, \qquad T_e^{-1}(\eta) = {}& \big(\eta \cap \{\xi\in\Omega\colon \xi(i) = \xi(j)\}\big)\\ &\cup \big(\sigma_i\eta \cap \{\xi\in\Omega\colon \xi(i) = 1-\xi(j)\}\big).\end{aligned} \tag{2.5}$$

3. Proof of Theorem 1. We first rewrite (2.4) so as to separate boundary terms from the subadditive part,

$$\left.\frac{dH_n(\mu_t)}{dt}\right|_{t=0} = \sum_{\eta\in\Re_n}\sum_{e\in[\tilde\Lambda_n]}\left[(T_e^{-1}\Gamma_e - \Gamma_e)\log\left(\frac{\mu}{P}\right)\right](\eta) + R_1 + R_2 \tag{3.1}$$

with

$$\begin{aligned} R_1 &= \sum_{\Re_n}\sum_{[\Lambda_n^+]\setminus[\Lambda_n]} (T_e^{-1}\Gamma_e - \Gamma_e)\log\left(\frac{\mu}{P}\right)(\eta),\\ R_2 &= \sum_{\Re_n}\sum_{[\Lambda_n]\setminus[\tilde\Lambda_n]} (T_e^{-1}\Gamma_e - \Gamma_e)\log\left(\frac{\mu}{P}\right)(\eta).\end{aligned} \tag{3.2}$$

Furthermore, if we define

$$g_n(\eta) \equiv \frac{\mu(\eta)}{P(\eta)}, \qquad \forall \eta \in \Re_n \quad \text{and} \quad F(x) = x(\log(x) - 1) + 1,$$

then

$$\sum_{\Re_n} \sum_{[\tilde{\Lambda}_n]} \left[T_e^{-1} \Gamma_e \log\left(\frac{\mu}{P} \frac{T_e P}{T_e \mu} \right) \right](\eta) = -\sum_{[\tilde{\Lambda}_n]} \int c_e\, T_e g_n F\left(\frac{g_n}{T_e g_n} \right) dP + R, \tag{3.3}$$

where

$$R = \sum_{\eta \in \Re_n} \sum_{e \in [\tilde{\Lambda}_n]} \left[\frac{T_e \Gamma_e}{T_e \mu} \frac{T_e P}{P} \mu - T_e \Gamma_e \right](\eta). \tag{3.4}$$

Now, we can rewrite (3.1) as

$$0 \le \sum_{e \in [\tilde{\Lambda}_n]} \int c_e\, T_e g_n F\left(\frac{g_n}{T_e g_n} \right) dP \le -\frac{dH_n(\mu_t)}{dt}\bigg|_{t=0} + R + R_1 + R_2. \tag{3.5}$$

We now estimate R, R_1 and R_2. Then, Lemma 1 concludes the argument by taking advantage of the convexity of the integrals which appear in (3.5).

Estimate for R. If R is given by (3.4), then $R = O(|\partial \Lambda_n|)$.
We first rewrite (3.4) as

$$R = \sum_{\eta \in \Re_n} \sum_{e \in [\tilde{\Lambda}_n]} \left[\frac{T_e \Gamma_e}{T_e \mu} \frac{T_e P}{P} \mu - \Gamma_e \right](\eta).$$

Choose n larger than the range of the rates. Then, thinking of the following quantities as functions on $\Re_n$, we have

$$\frac{T_e \Gamma_e}{T_e \mu} = T_e c_e, \quad \text{and} \quad \frac{T_e \Gamma_e}{T_e \mu} \frac{T_e P}{P} \mu = T_e c_e \mu \frac{T_e P}{P} = E\big[\hat{c}_e || F_n\big] \mu, \quad \text{for } e \in [\tilde{\Lambda}_n],$$

where $\{\hat{c}_e\}$ are the rates of the dual process in $L^2(dP)$,

$$\hat{c}_e = T_e c_e \frac{dT_e P}{dP} \qquad \forall e \in [Z^d].$$

Thus,

$$\sum_{[\tilde{\Lambda}_n]} \frac{T_e \Gamma_e}{T_e \mu} \frac{T_e P}{P} \mu - \Gamma_e = \left(E\left[\sum_{[\tilde{\Lambda}_n]} \hat{c}_e || F_n \right] - \sum_{[\tilde{\Lambda}_n]} c_e \right) \mu.$$

To obtain that $R = O(|\partial \Lambda_n|)$, we only need to show that

$$E\left[\sum_{e \in [\tilde{\Lambda}_n]} \hat{c}_e || F_n \right] - \sum_{e \in [\tilde{\Lambda}_n]} c_e = o(|\Lambda_n|), \tag{3.6}$$

where the conditional expectation is taken relative to P.

Note that $\{\hat{c}_e\}$ are uniformly bounded, positive and continuous. Now, for $f \in F_n$,

$$\int LfdP = \int \sum_{[e\in\Lambda_n^+]} c_e(T_e f - f)\,dP = \int \sum_{[\Lambda_n^+]} (\hat{c}_e - c_e) f\,dP.$$

Now, because P is invariant and nonnull, $E[\sum_{e\in[\Lambda_n^+]} \hat{c}_e - c_e || F_n] = 0$. Thus,

$$E\bigg[\sum_{e\in[\tilde{\Lambda}_n]} \hat{c}_e - c_e || F_n\bigg] = E\bigg[\sum_{e\in[\Lambda_n^+]\setminus[\tilde{\Lambda}_n]} \hat{c}_e - c_e || F_n\bigg]$$

and

$$\bigg|E\bigg[\sum_{e\in[\tilde{\Lambda}_n]} \hat{c}_e || F_n\bigg] - \sum_{e\in[\tilde{\Lambda}_n]} c_e\bigg| \le 2d(|\hat{c}|_\infty + |c|_\infty)|\Lambda_n \setminus \tilde{\Lambda}_n|,$$

which yields (3.6) at once.

Estimate of R_1. Fix a bond $e = (i, j)$ with $j \notin \Lambda_n$ and $i \in \Lambda_n$. Define $\eta_e = \sigma_i \eta \cap \{\xi \in \Omega\colon T_e(\xi) \neq \xi\}$ and

$$R_e = \sum_{\eta\in\Re_n} (T_e^{-1}\Gamma_e - \Gamma_e) \log\bigg(\frac{\mu}{P}\bigg)(\eta).$$

Thus, recalling (2.5),

$$R_e = \frac{1}{2} \sum_{\eta\in\Re_n} (\Gamma_e(\eta_e) - T_e^{-1}\Gamma_e(\eta_e)) \log\bigg(\frac{\mu}{\sigma_i\mu}\,\frac{\sigma_i P}{P}(\eta)\bigg).$$

Now, $\sigma_i P/P$ is uniformly bounded. Indeed, if N_Λ stands for the projection from Ω to $\{0, 1\}^\Lambda$,

$$\frac{\sigma_i P}{P} \circ N_\Lambda = E\bigg[\frac{d\sigma_i P}{dP} || F_\Lambda\bigg], \quad P\text{-a.s.}$$

Also,

$$\frac{d\sigma_i P}{dP} = \exp\bigg((1 - 2\eta_i) \sum_{R\ni i} J_R \prod_{a\in R\setminus i} \eta_a\bigg)$$

implies that

$$\bigg|\log\bigg(\frac{\sigma_i P}{P}(\eta)\bigg)\bigg| \le \sum_{R\ni 0} |J_R| \equiv K$$

and

$$\sum_{\eta\in\Re_n} \bigg|(T_e^{-1}\Gamma_e(\eta_e) - \Gamma_e(\eta_e))||\log\bigg(\frac{\sigma_i P}{P}\bigg)\bigg|(\eta) \le 2\bar{c}K \sum_{\eta\in\Re_n} \mu(\eta_e) \le 2\bar{c}K.$$

Now, it is easy to see that

$$T_e^{-1}\eta_e = (\sigma_i \eta)_e \subset \eta.$$

Thus, by using symmetry,

$$R_e \le \bar{c}K + \sum_{\mu(\eta)\le\mu(\sigma_i(\eta))} \log\left(\frac{\mu(\sigma_i(\eta))}{\mu(\eta)}\right) \int c_e I_\eta \, d\mu.$$

Therefore,

$$R_e \le \bar{c}K + \bar{c} \sum_{\mu(\eta)\le\mu(\sigma_i(\eta))} \mu(\eta) \log\left(\frac{\mu(\sigma_i(\eta))}{\mu(\eta)}\right).$$

Now, the basic inequality

$$x \log\left(\frac{A}{x}\right) \le \frac{A}{e} \quad \text{for } x \ge 0$$

implies that

$$R_e \le \bar{c}\left(K + \frac{1}{e}\right),$$

and by summing e over $[\Lambda_n^+]/[\Lambda_n]$, we obtain $R_1 = O(|\partial\Lambda|)$.

Estimate of R_2. Rewrite R_2 as

$$\begin{aligned} R_2 &= \sum_{\eta\in\Re_n} \sum_{e\in[\Lambda_n]/[\tilde{\Lambda}_n]} \left[(T_e\Gamma_e - \Gamma_e) \log \frac{\Gamma_e}{T_e\Gamma_e}\right](\eta) \\ &\quad + \frac{1}{2} \sum_{\eta\in\Re_n} \sum_{e\in[\Lambda_n]/[\tilde{\Lambda}_n]} \left[(T_e\Gamma_e - \Gamma_e) \log\left(\frac{T_e\Gamma_e}{T_e\mu} \frac{\mu}{\Gamma_e} \frac{T_e P}{P}\right)\right](\eta) \\ &\le \sum_{\eta\in\Re_n} \sum_{e\in[\Lambda_n]/[\tilde{\Lambda}_n]} T_e\Gamma_e \log\left(\frac{T_e\Gamma_e}{T_e\mu} \frac{\mu}{\Gamma_e} \frac{T_e P}{P}\right). \end{aligned} \tag{3.7}$$

Now,

$$\sup_{\eta\in\Re_n} \left|\log \frac{\mu}{\Gamma_e}(\eta)\right| \le \max\big(|\log(\bar{c})|, |\log(\underline{c})|\big) \equiv K'.$$

Also, if $R\Delta\{x, y\}$ denotes $(R\backslash\{x, y\}) \cup (\{x, y\}\backslash R)$,

$$\frac{T_e P}{P} \circ N_\Lambda = E\left[\frac{dT_e P}{dP} || F_n\right]$$

and

$$\frac{dT_{x,y}P}{dP} = \exp\left[(\eta_x - \eta_y) \sum_{|R\cap\{x,y\}|=1} J_R \eta_{R\Delta\{x,y\}}\right]$$

imply that

$$\left|\log\left(\frac{dT_{x,y}P}{dP}\right)\right| \le 2K \quad \text{and} \quad \sup_{\eta\in\Re_n} \left|\log\left(\frac{T_e P}{P}\right)\right| \le 2K.$$

Finally,

$$R_2 \le 2\bar{c}\ (K + K')|[\Lambda_n]\backslash[\tilde{\Lambda}_n]|. \tag{3.8}$$

LEMMA 1. *If μ is stationary and translation invariant, then for any m,*

$$\forall i = 1, \ldots, d,\ \forall \eta \in \Re_m, \qquad \frac{\mu(T_{e_i}\eta)}{\mu(\eta)} = \frac{P(T_{e_i}\eta)}{P(\eta)}. \tag{3.9}$$

PROOF. By translation invariance we only need to show that for $i = 1, \ldots, d$,

$$\mathscr{D}_{e_i}(g_m) \equiv \int c_{e_i}\big(T_{e_i}\sqrt{g_m} - \sqrt{g_m}\big)^2\, dP = 0.$$

The reason to introduce $\mathscr{D}_{e_i}$ is its convexity, because if $m < n$, $g_m = E[g_n||F_m]$, we have

$$\mathscr{D}_e(g_m) \le \mathscr{D}_e(g_n).$$

Using (3.5), the estimates for R, R_1 and R_2, the inequality $F(x) \ge (1-\sqrt{x})^2$ for $x \ge 0$ and translation invariance,

$$\begin{aligned} O(|\partial\Lambda_n|) &\ge \sum_{e\in[\tilde{\Lambda}_n]} \int c_e\ T_e g_n F\bigg(\frac{g_n}{T_e g_n}\bigg)\, dP \ge \sum_{e\in[\tilde{\Lambda}_n]} \mathscr{D}_e(g_n) \\ &\ge \sum_{j\in\tilde{\Lambda}_n} \sum_{i=1}^{d} \mathscr{D}_{j+e_i}(\tau_j g_m) \ge \sum_{i=1}^{d} \frac{|\tilde{\Lambda}_n|}{2} \mathscr{D}_{e_i}(g_m), \end{aligned} \tag{3.10}$$

and the result follows easily. □

This tells us that μ is a canonical measure. A well known theorem of Georgii (1979) implies that μ is a convex combination of grand canonical measures with the same potential, but different chemical potential.

4. One example. There is a simple one-dimensional example where our result applies. It appeared in Katz, Lebowitz and Spohn (1984). Let the potential $J_R = -J$ if $|R| = 2$ and $J_R = 0$ otherwise. Let G be the corresponding set of Gibbs measures. Any rates which satisfy $c_{i,i+1} = \tau_i c$ with $c = c(\eta_{-1}, \eta_0, \eta_1, \eta_2)$ and

$$\begin{gathered} c(0,1,0,1) = e^{J/2} c(1,1,0,0), \\ c(1,1,0,1) + c(0,1,0,0) = c(0,1,0,1) + c(1,1,0,0), \\ c(\eta^{0,1}) = c(\eta)\exp(J(\eta_0 - \eta_1)(\eta_2 - \eta_{-1}) + E(\eta_1 - \eta_0)) \end{gathered} \tag{4.1}$$

would make the generator invariant for any $\nu \in G$. For some definite examples, take

$$c(\eta_{-1}, \eta_0, \eta_1, \eta_2) = \exp\bigg(-\bigg[J(\eta_0\eta_2 + \eta_{-1}\eta_1) + \frac{E}{2}(\eta_1 - \eta_0)\bigg]\bigg)$$

or

$$c(\eta_{-1}, \eta_0, \eta_1, \eta_2) = \exp\left((\eta_0 - \eta_1)\left[\frac{J}{2}(\eta_2 - \eta_{-1}) + \frac{E}{2}\right]\right).$$

Then, Theorem 1 would say that this system has no other invariant measures.

REMARK. A straightforward corollary of our proof and observations of Holley (1972) is that for any translation invariant initial measure μ and convergent subsequence μ_{t_n}, the limiting measure is necessarily a convex combination of Gibbs measures with the same potential as P; however, we do not know if this limiting measure is stationary for the dynamics.

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