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Improving mean estimation in ranked set sampling using the Rao regression-type estimator

Elvira Pelle[a] **and Pier Francesco Perri**[b]

[a] *University of Trieste*
[b] *University of Calabria*

Abstract. Ranked set sampling is a statistical technique usually used for a variable of interest that may be difficult or expensive to measure, but whose units are simple to rank according to a cheap sorting criterion. In this paper, we revisit the Rao regression-type estimator in the context of the ranked set sampling. The expression of the minimum mean squared error is given and a comparative study, based on simulated and real data, is carried out to clearly show that the considered estimator outperforms some competitive estimators discussed in the recent literature.

1 Introduction

In many situations of practical interest, mainly in environmental and ecological studies, the variable of interest, say Y, is not easily observable in the sense that measurements may be expensive, time-consuming, invasive or even destructive. Although data collection may be complex, ranking the potential sampled units can often be relatively simple at no additional cost or for a very little cost.

In those situations where there is enough information to perform ranking without observing the units, the idea of the ranked set sampling (RSS) can be invoked. Accordingly, a set of sampling units drawn from the population is ranked by certain means rather cheaply without the actual measurements of Y. This assumption may appear rather restrictive at a first glance, but there are plenty of situations in practice that meet this requirement (see, e.g., Chen, Bai and Sinha, 2004). The original form of the RSS conceived by McIntyre (1952) requires the selection of k independent samples of size k according to simple random sampling (SRS). The units of each sample are ranked with respect to the variable of interest Y without actually measuring it by using, for instance, personal judgment, visual inspection, or via the use of a concomitant variable correlated with Y. From the ith sample, $i = 1, \dots, k$, the unit with rank i is identified and taken for the measurement on Y, while the remaining $k - 1$ units of the sample are discarded. In doing so, it is obtained a sample of k independent observations distributed, respectively, as the first, the second, up to the kth order statistic from a sample of size k selected according

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to SRS, provided that the ranking in the ith sample has been perfectly performed. This whole process is referred to as a cycle and can be repeated $m \geq 1$ times in such a way as to provide a ranked set sample of final size $n = mk$. We observe that k denotes a design parameter which usually takes a small value in order to facilitate the ranking process.

It is a well-established fact that estimation through the RSS is more efficient than estimation through SRS. Accordingly, literature on RSS has rapidly grown and several estimators, originally conceived for the SRS, have been re-proposed to estimate the mean of the study variable in an RSS framework. On this see, for example, Samawi and Muttlak (1996), Yu and Lam (1997), Bouza (2008), Kadilar, Unyazici and Cingi (2009), Jeelani and Bouza (2015), Mehta and Mandowara (2016).

Motivated by these studies, and in line with many other contributions present in the specialized literature, in this article we investigate the effectiveness of using an alternative estimator, say the Rao regression-type estimator in RSS. We find, by a comparative study based on simulated and real data, that the considered estimator is always more efficient than the aforesaid estimators. Incidentally, the study allows us to cast light on some undisclosed aspects concerning the efficiency of the competitive estimators.

The remainder of the article is organized as follows. In Section 2, we introduce notations and present a brief review of estimators used in RSS when an auxiliary variable is available. For each estimator, the (first-order approximated) expressions of the mean squared error and bias are reported. In Section 3, we adapt the Rao regression-type estimator in RSS and derive the expressions of mean squared error and bias. In Section 4, we perform a simulation study in order to compare, under different scenarios, the efficiency of the proposed estimator with that of other competitive estimators discussed in the literature. In Section 5, the performance of the estimator is investigated by using two real datasets. Finally, Section 6 concludes the article with some remarks and comments.

2 Estimation of population mean using the RSS

Let $U = \{1, \dots, N\}$ be a finite population of N units, Y the variable under study, and X an auxiliary variable correlated with Y. Let μ_y and μ_x denote the population means of Y and X, respectively, S_y^2 and S_x^2 the variances, C_y and C_x the coefficients of variation, ρ_{xy} the correlation coefficient between X and Y, and $C_{xy} = \rho_{xy} C_x C_y$. Let us suppose that μ_y is unknown and has to be estimated when the RSS is used and auxiliary information is available. For the aim of this paper, we follow Stokes (1997), who proposed ordering the sample units on the basis of the value of the auxiliary variable X instead of a subjective judgment on Y. We assume that this ranking is perfect with respect to X although errors may occur in the ordering with respect to Y. Moreover, we suppose that the population mean of X is known in advance and, hence, X may be also used at the estimation stage to build efficient estimators of μ_y. In this framework, to carry out the ranking, k bivariate random samples, each having k units, are drawn from the population. Then, the values of Y associated with the ith smallest X from the ith sample, $i = 1, \ldots, k$, are quantified. The whole process is repeated for m cycles so that $n = mk$ measurements of Y are obtained. For the rth cycle, $r = 1, \ldots, m$, let the couple $(x_{(i)r}, y_{[i]r})$ denote, respectively, the ith smallest X and the concomitant value of Y observed on the ith sample, $i = 1, \ldots, k$.

2.1 Overview of some estimators

In sampling from finite population, it is a well-established fact that improvements upon the estimates of the target parameter μ_y can be obtained using the auxiliary information on X, for instance, through the ratio, the product and regression estimation methods. Along these lines, many authors have extended a part of these results to RSS. In the following, we present a brief review of some contributions.

Samawi and Muttlak (1996) proposed the use of the ratio estimator

$$\hat{\bar{Y}}_{\mathrm{R}} = \frac{\bar{Y}_{[n]}}{\bar{X}_{(n)}} \mu_x,$$

where $\bar{Y}_{[n]} = \sum_{r=1}^{m} \sum_{i=1}^{k} y_{[i]r}/mk$ and $\bar{X}_{(n)} = \sum_{r=1}^{m} \sum_{i=1}^{k} x_{(i)r}/mk$ are two unbiased estimators of μ_y and μ_x, respectively. The expressions for the bias (B) and the mean squared error (MSE) of $\hat{\bar{Y}}_{\mathrm{R}}$ to the first-order of approximation are given by

$$\mathrm{B}(\hat{\bar{Y}}_{\mathrm{R}}) = \mu_y\big[\theta\big(C_x^2 - C_{xy}\big) - \big(W_{x(i)}^2 - W_{yx(i)}\big)\big]$$

and

$$\mathrm{MSE}(\hat{\bar{Y}}_{\mathrm{R}}) = \mu_y^2\big[\theta\big(C_y^2 + C_x^2 - 2C_{xy}\big) - \big(W_{y[i]}^2 + W_{x(i)}^2 - 2W_{yx(i)}\big)\big],$$

where

$$\theta = \frac{1}{mk} - \frac{1}{N} \cong \frac{1}{mk}$$

and

$$W_{yx(i)} = \frac{1}{mk^2\mu_x\mu_y} \sum_{i=1}^{k} \tau_{yx(i)},$$

$$W_{x(i)}^2 = \frac{1}{mk^2\mu_x^2} \sum_{i=1}^{k} \tau_{x(i)}^2,$$

$$W_{y[i]}^2 = \frac{1}{mk^2\mu_y^2} \sum_{i=1}^{k} \tau_{y[i]}^2,$$

with

$$\tau_{y[i]} = \mu_{y[i]} - \mu_y,$$
$$\tau_{x(i)} = \mu_{x(i)} - \mu_x,$$
$$\tau_{yx(i)} = (\mu_{y[i]} - \mu_y)(\mu_{x(i)} - \mu_x).$$

Here, $\mu_{x(i)}$ and $\mu_{y[i]}$ denote, respectively, the expected value of the ith ordered statistic from the distribution of X and the expected value for the concomitant statistic from the distribution of Y. It is worth observing that the computation of $\mu_{x(i)}$ and $\mu_{y[i]}$ requires some assumptions on the distribution law of X and Y.

Moving from Prasad (1989), Kadilar, Unyazici and Cingi (2009) introduced the ratio-type estimator

$$\hat{\hat{\bar{Y}}}_\xi = \frac{\xi \bar{Y}_{[n]}}{\bar{X}_{(n)}}\mu_x,$$

where the constant ξ is determined in order to maximize the efficiency of the estimator. The estimator is biased, with bias to the first-order of approximation given by

$$\mathrm{B}(\hat{\hat{\bar{Y}}}_\xi) = (\xi - 1)\mu_y - \xi\mu_y\left[\theta\left(C_x^2 - C_{xy}\right) - \left(W_{x(i)}^2 - W_{yx(i)}\right)\right].$$

To the first-order of approximation, the MSE is given by

$$\begin{aligned}\mathrm{MSE}(\hat{\hat{\bar{Y}}}_\xi) = \mu_y^2\big[&\theta\left(\xi^2 C_y^2 + C_x^2 - 2\xi C_{xy}\right)\\ &- \left(\xi^2 W_{y[i]}^2 + W_{x(i)}^2 - 2\xi W_{yx(i)}\right) + \theta(\xi - 1)^2\big],\end{aligned}$$

which is minimized for

$$\xi = \frac{1 + \theta C_{xy} - W_{yx(i)}}{1 + \theta C_y^2 - W_{y[i]}^2}.$$

When the correlation between the auxiliary variable and the variable of interest is negative, Bouza (2008) introduced the product estimator

$$\hat{\hat{\bar{Y}}}_\mathrm{P} = \frac{\bar{Y}_{[n]}}{\mu_x}\bar{X}_{(n)},$$

with bias

$$\mathrm{B}(\hat{\hat{\bar{Y}}}_\mathrm{P}) = [\theta C_{xy} - W_{yx(i)}]$$

and approximate first-order MSE

$$\mathrm{MSE}(\hat{\hat{\bar{Y}}}_\mathrm{P}) = \mu_y^2\left[\theta\left(C_y^2 + C_x^2 + 2C_{xy}\right) - \left(W_{y[i]}^2 + W_{x(i)}^2 + 2W_{yx(i)}\right)\right].$$

Jeelani and Bouza (2015), motivated by Samawi and Muttlak (1996), proposed a new ratio-type estimator defined as

$$\hat{\bar{Y}}_\nu = \nu \frac{\bar{Y}_{[n]}}{\bar{X}_{(n)}} \mu_x,$$

where

$$\nu = \frac{\mu_x M_d + Q_d}{\bar{X}_{(n)} M_d + Q_d},$$

with M_d denoting the population median of X, $Q_d = Q_3 - Q_1$ being Q_3 and Q_1 the first and third quartiles, respectively. To the first-order of approximation, the bias and the MSE are given by

$$\mathrm{B}(\hat{\bar{Y}}_\nu) = (\nu - 1)\mu_y - \nu\mu_y\big[\theta\big(C_x^2 - C_{xy}\big) - \big(W_{x(i)}^2 - W_{yx(i)}\big)\big]$$

and

$$\begin{aligned}\mathrm{MSE}(\hat{\bar{Y}}_\nu) = \mu_y^2\big[&\theta\big(\nu^2 C_y^2 + C_x^2 - 2\nu C_{xy}\big)\\ &- \big(\nu^2 W_{y[i]}^2 + W_{x(i)}^2 - 2\nu W_{yx(i)}\big) + \theta(\nu - 1)^2\big].\end{aligned}$$

Mehta and Mandowara (2016), similarly to Kadilar, Unyazici and Cingi (2009) and moving from Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999), Singh et al. (2004) and Tailor and Sharma (2009), recently investigated the efficiency of some ratio-type estimators based on the knowledge of the coefficient of variation and kurtosis of X. In particular, when the population coefficient of variation C_x is known, the following ratio-type estimator is defined

$$\hat{\bar{Y}}_{\mathrm{MM1}} = \bar{Y}_{[n]}\bigg[\frac{\mu_x + C_x}{\bar{X}_{(n)} + C_x}\bigg],$$

while, when the coefficient of kurtosis $\beta_2(x)$ is known, the ratio-type estimator takes the form

$$\hat{\bar{Y}}_{\mathrm{MM2}} = \bar{Y}_{[n]}\bigg[\frac{\mu_x + \beta_2(x)}{\bar{X}_{(n)} + \beta_2(x)}\bigg].$$

To the first-order of approximation, the bias and the MSE of $\hat{\bar{Y}}_{\mathrm{MM1}}$ and $\hat{\bar{Y}}_{\mathrm{MM2}}$ are given, respectively, by

$$\mathrm{B}(\hat{\bar{Y}}_{\mathrm{MM1}}) = \mu_y\big[\theta\big(\varphi_1^2 C_x^2 - \varphi_1 C_{xy}\big) - \big(\varphi_1^2 W_{x(i)}^2 - \varphi_1 W_{yx(i)}\big)\big]$$

and

$$\mathrm{MSE}(\hat{\bar{Y}}_{\mathrm{MM1}}) = \mu_y^2\big[\theta\big(C_y^2 + \varphi_1^2 C_x^2 - 2\varphi_1 C_{xy}\big) - \big(W_{y[i]} - \varphi_1 W_{x(i)}\big)^2\big];$$

$$\mathrm{B}(\hat{\bar{Y}}_{\mathrm{MM2}}) = \mu_y\big[\theta\big(\varphi_2^2 C_x^2 - \varphi_2 C_{xy}\big) - \big(\varphi_2^2 W_{x(i)}^2 - \varphi_2 W_{yx(i)}\big)\big]$$

and

$$\mathrm{MSE}(\hat{\hat{\bar{Y}}}_{\mathrm{MM2}}) = \mu_y^2[\theta(C_y^2 + \varphi_2^2 C_x^2 - 2\varphi_2 C_{xy}) - (W_{y[i]} - \varphi_2 W_{x(i)})^2],$$

where

$$\varphi_1 = \frac{\mu_x}{\mu_x + C_x} \quad \text{and} \quad \varphi_2 = \frac{\mu_x}{\mu_x + \beta_2(x)}.$$

Additionally, when both C_x and $\beta_2(x)$ are known, Mehta and Mandowara (2016) proposed the following ratio-type ($\hat{\hat{\bar{Y}}}_{\mathrm{MM3}}$), product-type ($\hat{\hat{\bar{Y}}}_{\mathrm{MM4}}$) and ratio-cum-product ($\hat{\hat{\bar{Y}}}_{\mathrm{MM5}}$) estimators

$$\hat{\hat{\bar{Y}}}_{\mathrm{MM3}} = \bar{Y}_{[n]}\left[\frac{\mu_x C_x + \beta_2(x)}{\bar{X}_{(n)} C_x + \beta_2(x)}\right],$$

$$\hat{\hat{\bar{Y}}}_{\mathrm{MM4}} = \bar{Y}_{[n]}\left[\frac{\bar{X}_{(n)} C_x + \beta_2(x)}{\mu_x C_x + \beta_2(x)}\right],$$

and

$$\hat{\hat{\bar{Y}}}_{\mathrm{MM5}} = \bar{Y}_{[n]}\left[\eta\left(\frac{\mu_x C_x + \beta_2(x)}{\bar{X}_{(n)} C_x + \beta_2(x)}\right) + (1-\eta)\left(\frac{\bar{X}_{(n)} C_x + \beta_2(x)}{\mu_x C_x + \beta_2(x)}\right)\right],$$

where η is a constant to be properly chosen.

The expressions of the bias and MSE to the first-order of approximation of these estimators are given by

$$\mathrm{B}(\hat{\hat{\bar{Y}}}_{\mathrm{MM3}}) = \mu_y[\theta(\gamma_1^2 C_x^2 - \gamma_1 C_{xy}) - (\gamma_1^2 W_{x(i)}^2 - \gamma_1 W_{yx(i)})]$$

and

$$\mathrm{MSE}(\hat{\hat{\bar{Y}}}_{\mathrm{MM3}}) = \mu_y^2[\theta(C_y^2 + \gamma_1^2 C_x^2 - 2\gamma_1 C_{xy}) - (W_{y[i]} - \gamma_1 W_{x(i)})^2];$$

$$\mathrm{B}(\hat{\hat{\bar{Y}}}_{\mathrm{MM4}}) = \mu_y \gamma_1[\theta C_{xy} - W_{yx(i)}]$$

and

$$\mathrm{MSE}(\hat{\hat{\bar{Y}}}_{\mathrm{MM4}}) = \mu_y^2[\theta(C_y^2 + \gamma_1^2 C_x^2 + 2\gamma_1 C_{xy}) - (W_{y[i]} + \gamma_1 W_{x(i)})^2];$$

$$\begin{aligned}\mathrm{B}(\hat{\hat{\bar{Y}}}_{\mathrm{MM5}}) = \mu_y\{&\theta\gamma_1 C_x^2[T + \alpha\gamma_1(\gamma_1 - 2T)] \\ &- [\alpha\gamma_1^2 W_{x(i)}^2 + (1-2\alpha)\gamma_1 W_{yx(i)}]\}\end{aligned}$$

and

$$\begin{aligned}\mathrm{MSE}(\hat{\hat{\bar{Y}}}_{\mathrm{MM5}}) = \mu_y^2\{&\theta[C_y^2 + (1-2\eta)\gamma_1^2 C_x^2((1-2\eta)\gamma_1 + 2T)] \\ &- [W_{y[i]} + (1-2\eta)\gamma_1 W_{x(i)}]^2\},\end{aligned}$$

where

$$\gamma_1 = \frac{\mu_x C_x}{\mu_x C_x + \beta_2(x)} \quad \text{and} \quad T = \rho_{xy}\frac{C_y}{C_x}.$$

The mean squared error of $\hat{\hat{Y}}_{\mathrm{MM5}}$ attains its minimum value bound for

$$\eta = \frac{\gamma_1 + T}{2\gamma_1}.$$

As for the estimator $\hat{\hat{Y}}_{\mathrm{P}}$, we notice that the expression for $\mathrm{B}(\hat{\hat{Y}}_{\mathrm{MM4}})$ is exact and not approximated to the first-order.

Yu and Lam (1997) introduced the regression estimator

$$\hat{\hat{Y}}_{\mathrm{Reg}} = \bar{Y}_{[n]} + \beta(\mu_x - \bar{X}_{(n)}),$$

where $\beta = \rho_{xy} S_y / S_x$. The estimator is unbiased with variance

$$\mathrm{Var}(\hat{\hat{Y}}_{\mathrm{Reg}}) = \mu_x^2 \mu_y^2 \Bigg[\theta\left(\frac{1}{\mu_x^2}C_y^2 + \frac{\beta^2}{\mu_y^2}C_x^2 - \frac{2\beta}{\mu_x\mu_y}C_{xy}\right) - \left(\frac{1}{\mu_x^2}W_{y[i]}^2 + \frac{\beta^2}{\mu_y^2}W_{x(i)}^2 - \frac{2\beta}{\mu_x\mu_y}W_{yx(i)}\right)\Bigg]$$

which, after simple algebra, can be alternatively rephrased in the more appealing form

$$\mathrm{Var}(\hat{\hat{Y}}_{\mathrm{Reg}}) = \mu_y^2[\theta C_y^2 - W_{y[i]}^2](1 - \rho_{xy}^2). \tag{1}$$

3 The suggested estimator

Mainly motivated by the contributions discussed in the previous section, we now introduce in RSS the Rao regression-type estimator and investigate its efficiency. For the SRS framework, Rao (1991) defined the estimator as

$$\hat{\hat{Y}}_{\mathrm{Rao\ (SRS)}} = \omega_1 \hat{\hat{Y}} + \omega_2(\mu_x - \hat{\hat{X}}), \tag{2}$$

where $\hat{\hat{Y}}$ and $\hat{\hat{X}}$ are the two unbiased estimators of μ_y and μ_x, and ω_1 and ω_2 are two constants to be wisely chosen.

In the RSS setting, the estimator can be reformulated as

$$\hat{\hat{Y}}_{\mathrm{Rao}} = \omega_1 \bar{Y}_{[n]} + \omega_2(\mu_x - \bar{X}_{(n)}).$$

It is straightforward to observe that the estimator is biased, with bias equal to $(\omega_1 - 1)\mu_y$. Moreover, using the usual algebra given in the articles previously quoted (see, e.g., Mehta and Mandowara, 2016), we get the following expression for the MSE

$$\begin{aligned}\mathrm{MSE}(\hat{\hat{Y}}_{\mathrm{Rao}}) = \mu_x^2\mu_y^2\Big[\theta\Big(&\frac{\omega_1^2}{\mu_x^2}C_y^2 + \frac{\omega_2^2}{\mu_y^2}C_x^2 - \frac{2\omega_1\omega_2}{\mu_x\mu_y}C_{xy} \\ &- \Big(\frac{\omega_1^2}{\mu_x^2}W_{y[i]}^2 + \frac{\omega_2^2}{\mu_y^2}W_{x(i)}^2 - \frac{2\omega_1\omega_2}{\mu_x\mu_y}W_{yx(i)}\Big)\Big)\Big] \\ &+ (\omega_1 - 1)^2\mu_y^2,\end{aligned} \tag{3}$$

which is minimized for

$$\omega_1 = \frac{1}{C_y^2[1-\rho_{xy}^2]+1} \quad \text{and} \quad \omega_2 = \omega_1\beta. \tag{4}$$

Substituting (4) into (3), after some algebra, we obtain

$$\min \mathrm{MSE}(\hat{\hat{Y}}_{\mathrm{Rao}}) = \mu_y^2[\theta C_y^2 - W_{y[i]}^2]\frac{1-\rho_{xy}^2}{C_y^2(1-\rho_{xy}^2)+1}. \tag{5}$$

Finally, comparing (5) with (1), we immediately observe that

$$\mathrm{MSE}(\hat{\hat{Y}}_{\mathrm{Rao}}) \le \mathrm{MSE}(\hat{\hat{Y}}_{\mathrm{Reg}}).$$

4 Comparing estimators by a simulation study

When evaluating the performance of new proposals, it is common in the literature to analytically derive the conditions under which an estimator is more efficient than others. Nonetheless, these conditions are usually difficult to verify and, hence, their use appear of questionable utility in the practice, unless a uniform superiority of an estimator is definitely ascertained. For this reason, up to now, we have intentionally avoided deriving tedious theoretical comparisons in RSS based on the mean squared error. For comparison purposes, in this section we therefore prefer giving and discussing the results of a number of numerical simulation experiments. In Section 5, we will instead provide some results based on real data.

The simulation study is conceived to gain insight into the efficiency of the Rao estimator upon the competitive estimators $\bar{Y}_{[n]}$, $\hat{\hat{Y}}_{\mathrm{R}}$, $\hat{\hat{Y}}_{\xi}$, $\hat{\hat{Y}}_{\mathrm{P}}$, $\hat{\hat{Y}}_{\nu}$, $\hat{\hat{Y}}_{\mathrm{MM1}} - \hat{\hat{Y}}_{\mathrm{MM5}}$ and $\hat{\hat{Y}}_{\mathrm{Reg}}$ in RSS. Throughout the simulation, different populations for (X, Y) are generated from a bivariate Normal distribution, with $\mu_x = \mu_y = 10$, $C_x = 0.25, 1$, $C_y = 0.25, 1, 1.5, 3$ and $\rho_{xy} = 0.7$.

We consider two simulation situations: one based on the theoretical expressions of the bias and MSE as given in the previous sections, and one based on estimated bias and MSE.

4.1 Simulation with true population parameters

In order to compute the bias and the MSE of the above-mentioned estimators, we remind that the sampled units are ranked with respect to the values of the auxiliary variable X and, hence, the quantities $\tau_{y[i]}$, $\tau_{x(i)}$ and $\tau_{yx(i)}$ (see Section 2.1) are to be computed. Under the bivariate Normal distribution, the expression of these quantities may be obtained using the results for $\mu_{y[i]}$ and $\mu_{x(i)}$ given, for instance, in David and Nagaraja (2003), p. 145. Besides the bias and the MSE, we also compute for each estimator the Percent Relative Bias (PRB) and the Percent Relative Efficiency (PRE) defined as

$$\mathrm{PRB}(\hat{\hat{Y}}_*) = \frac{\mathrm{B}(\hat{\hat{Y}}_*)}{\mu_y} \times 100$$

and

$$\mathrm{PRE}(\hat{\hat{Y}}_*) = \frac{\mathrm{Var}(\bar{Y}_{[n]})}{\mathrm{MSE}(\hat{\hat{Y}}_*)} \times 100,$$

where $\hat{\hat{Y}}_* = \hat{\hat{Y}}_{\mathrm{R}}, \hat{\hat{Y}}_{\xi}, \hat{\hat{Y}}_{\mathrm{P}}, \hat{\hat{Y}}_{\nu}, \hat{\hat{Y}}_{\mathrm{MM1}} - \hat{\hat{Y}}_{\mathrm{MM5}}, \hat{\hat{Y}}_{\mathrm{Reg}}, \hat{\hat{Y}}_{\mathrm{Rao}}$. In this study, we consider populations of size $N = 10{,}000$, random sets of size $k = 3, 5, 7, 10$ and $m = 3$ cycles. For space saving purposes, we show and discuss in the following only the results for the PRB and the PRE, while those for the bias and the MSE are furnished in the Appendix. From Table 1, the PRB appears in general of modest entity and, however, nearly negligible in many cases although it tendentially increases (in absolute value) when C_y increases. For $C_y = 3$, the bias mostly affects the estimators $\hat{\hat{Y}}_{\mathrm{MM5}}$ and $\hat{\hat{Y}}_{\mathrm{Rao}}$ and, to a lesser extent, $\hat{\hat{Y}}_{\mathrm{R}}$ and $\hat{\hat{Y}}_{\xi}$. We also observe that, except for $\hat{\hat{Y}}_{\nu}$, the PRB always decreases when k increases, *ceteris paribus*.

Looking at the PRE, we observe that the Rao estimator always outperforms the other estimators with an efficiency gain that becomes considerably striking when C_y increases. This aspect assumes major relevance if referred in particular to the estimators $\hat{\hat{Y}}_{\mathrm{MM3}} - \hat{\hat{Y}}_{\mathrm{MM5}}$ which employ much more auxiliary information than $\hat{\hat{Y}}_{\mathrm{Rao}}$. For small C_y, the most competitive estimators with $\hat{\hat{Y}}_{\mathrm{Rao}}$ are $\hat{\hat{Y}}_{\mathrm{Reg}}$ and $\hat{\hat{Y}}_{\mathrm{MM5}}$: in this case the improvement of $\hat{\hat{Y}}_{\mathrm{Rao}}$ upon $\hat{\hat{Y}}_{\mathrm{Reg}}$ and $\hat{\hat{Y}}_{\mathrm{MM5}}$ is nearly negligible. Moreover, we note that, except for $\hat{\hat{Y}}_{\nu}$, the PRE of the estimators decreases as k increases. This could be wrongly interpreted in terms of inconsistent estimates. Indeed, a thorough analysis (see Tables 7 and 8 in the Appendix) points out that, as expected, the MSE of all the estimators decreases as k increases but the decreasing is slower than that of the variance of the sample mean $\bar{Y}_{[n]}$.

The simulation findings also contribute to shed light on the performance of other estimators. First, through the simulation study, we observe that the two estimators $\hat{\hat{Y}}_{\mathrm{Reg}}$ and $\hat{\hat{Y}}_{\mathrm{MM5}}$ are substantially equivalent, while $\hat{\hat{Y}}_{\mathrm{Reg}}$ always outperforms

Table 1 *Results for the theoretical PRB and PRE for $C_x = 0.25$ and $\rho_{xy} = 0.7$. The PRB of $\bar{Y}_{[n]}$ and $\hat{\hat{Y}}_{\mathrm{Reg}}$ is omitted since the estimators are unbiased*

	$C_y = 0.25$				$C_y = 1$				$C_y = 1.5$				$C_y = 3$			
k	3	5	7	10	3	5	7	10	3	5	7	10	3	5	7	10
								PRB								
$\hat{\hat{Y}}_{\mathrm{R}}$	1.06	0.44	0.24	0.13	−6.44	−2.67	−1.47	−0.77	−11.41	−4.73	−2.60	−1.37	−26.64	−11.04	−6.08	−3.19
$\hat{\hat{Y}}_{\xi}$	−0.21	−0.11	−0.07	−0.05	1.48	1.33	1.09	0.84	3.77	3.59	2.98	2.31	17.27	17.55	14.50	11.06
$\hat{\hat{Y}}_{\mathrm{P}}$	0.02	0.01	0.01	0.00	0.10	0.04	0.02	0.01	0.16	0.06	0.04	0.02	0.31	0.13	0.07	0.04
$\hat{\hat{Y}}_{\nu}$	−1.02	−0.70	−0.70	−1.43	2.92	1.74	0.73	0.39	2.02	2.03	1.81	1.48	−0.53	−3.58	−1.28	−1.59
$\hat{\hat{Y}}_{\mathrm{MM1}}$	0.10	0.04	0.02	0.01	−0.64	−0.26	−0.15	−0.08	−1.15	−0.48	−0.26	−0.14	−2.63	−1.09	−0.60	−0.31
$\hat{\hat{Y}}_{\mathrm{MM2}}$	0.02	0.01	0.00	0.00	−0.56	−0.23	−0.13	−0.07	−0.96	−0.40	−0.22	−0.12	−2.14	−0.89	−0.49	−0.26
$\hat{\hat{Y}}_{\mathrm{MM3}}$	−0.04	−0.02	−0.01	0.00	−0.38	−0.16	−0.09	−0.05	−0.62	−0.26	−0.14	−0.07	−1.33	−0.55	−0.30	−0.16
$\hat{\hat{Y}}_{\mathrm{MM4}}$	0.11	0.05	0.03	0.01	0.46	0.19	0.11	0.06	0.69	0.29	0.16	0.08	1.40	0.58	0.32	0.17
$\hat{\hat{Y}}_{\mathrm{MM5}}$	−0.20	−0.17	−0.14	−0.11	−4.56	−3.30	−2.57	−1.92	−10.86	−7.79	−6.03	−4.50	−44.11	−31.31	−24.14	−17.94
$\hat{\hat{Y}}_{\mathrm{Rao}}$	−0.36	−0.21	−0.15	−0.11	−5.38	−3.30	−2.38	−1.68	−12.03	−7.58	−5.54	−3.94	−34.27	−23.83	−18.26	−13.53

Table 1 *Continued*

	$C_y = 0.25$				$C_y = 1$				$C_y = 1.5$				$C_y = 3$			
k	3	5	7	10	3	5	7	10	3	5	7	10	3	5	7	10
								PRE								
$\hat{\bar{Y}}_{\mathrm{R}}$	136.37	125.78	120.13	115.26	124.08	117.55	113.91	110.70	115.43	111.43	109.14	107.09	107.99	106.03	104.88	103.82
$\hat{\bar{Y}}_{\xi}$	136.57	125.91	120.23	115.34	121.85	114.89	111.64	108.92	109.30	104.43	103.14	102.36	81.85	77.44	80.08	83.97
$\hat{\bar{Y}}_{\mathrm{P}}$	38.52	44.92	49.94	55.79	78.21	82.35	85.08	87.82	85.53	88.51	90.41	92.27	92.30	93.97	95.02	96.02
$\hat{\bar{Y}}_{\nu}$	129.56	122.75	119.38	110.37	110.03	108.94	109.58	108.05	107.97	104.32	102.91	102.04	110.52	113.58	107.02	106.49
$\hat{\bar{Y}}_{\mathrm{MM1}}$	138.10	126.90	120.96	115.87	123.51	117.16	113.61	110.47	115.05	111.16	108.93	106.93	107.79	105.88	104.76	103.72
$\hat{\bar{Y}}_{\mathrm{MM2}}$	147.72	133.02	125.46	119.10	118.69	113.78	111.00	108.51	111.83	108.83	107.10	105.52	106.15	104.66	103.78	102.96
$\hat{\bar{Y}}_{\mathrm{MM3}}$	139.94	128.09	121.85	116.51	111.05	108.29	106.68	105.21	106.91	105.22	104.22	103.31	103.65	102.78	102.26	101.78
$\hat{\bar{Y}}_{\mathrm{MM4}}$	64.19	69.99	74.05	78.31	89.50	91.72	93.12	94.48	93.29	94.77	95.69	96.56	96.41	97.22	97.72	98.19
$\hat{\bar{Y}}_{\mathrm{MM5}}$	148.43	133.46	125.78	119.33	149.29	134.05	126.24	119.67	147.58	132.88	125.33	118.99	148.69	133.64	125.92	119.43
$\hat{\bar{Y}}_{\mathrm{Reg}}$	148.43	133.46	125.78	119.33	149.29	134.05	126.24	119.67	147.58	132.88	125.33	118.99	148.69	133.64	125.92	119.43
$\hat{\bar{Y}}_{\mathrm{Rao}}$	148.96	133.75	125.97	119.46	157.78	138.63	129.31	121.71	167.77	143.78	132.67	123.87	226.22	175.45	154.06	138.12

$\hat{\hat{Y}}_{\mathrm{MM1}} - \hat{\hat{Y}}_{\mathrm{MM4}}$. This is a clear indication that the available auxiliary information concerning variability and kurtosis is not used in a profitable way and an optimum estimator could be found following, for instance, the suggestions given in Diana and Perri (2007). Second, it is quite surprising to observe that $\hat{\hat{Y}}_{\xi}$ may not only be less efficient than $\hat{\hat{Y}}_{\mathrm{R}}$ but also less efficient than the sample mean $\bar{Y}_{[n]}$ which does not employ any auxiliary information at the estimation stage. Yet, it is not surprising that $\hat{\hat{Y}}_{\xi}$ never outperforms $\hat{\hat{Y}}_{\mathrm{Reg}}$ and, hence, $\hat{\hat{Y}}_{\mathrm{Rao}}$ (see, e.g., Diana and Perri, 2007). Furthermore, we observe that among the ratio-type estimators it is not possible to identify the most efficient estimator since their performance varies as the population parameters change. Finally, with regard to the product estimator $\hat{\hat{Y}}_{\mathrm{P}}$ and the product-type estimator $\hat{\hat{Y}}_{\mathrm{MM4}}$, we note that they are always less efficient than the sample mean $\bar{Y}_{[n]}$. This behaviour is, however, not surprising given that the correlation between X and Y is positive.

We have repeated the simulation study by considering increasing values for C_x. Results are shown in Table 2 only for $C_x = 1$. Compared with the results given for $C_x = 0.25$, the PRB seems to increase in many situations, particularly for $\hat{\hat{Y}}_{\mathrm{R}}$ and $\hat{\hat{Y}}_{\nu}$. In a few cases, the PRB of $\hat{\hat{Y}}_{\xi}$, $\hat{\hat{Y}}_{\nu}$ and $\hat{\hat{Y}}_{\mathrm{MM5}}$ appears irregular as it does not always decrease when k increases. For the Rao estimator, the behaviour of the PRB remains almost identical, rather we note a slight reduction for $C_y = 1.5, 3$.

With regard to the PRE, also for $C_x = 1$ we observe that the Rao estimator always outperforms all the other estimators and, in general, the increase of C_x does not significantly affect the efficiency of $\hat{\hat{Y}}_{\mathrm{Reg}}$, $\hat{\hat{Y}}_{\mathrm{MM5}}$ and $\hat{\hat{Y}}_{\mathrm{Rao}}$. On the contrary, the efficiency of the other estimators may drastically deteriorate for low values of C_y. The results show that, for $C_y = 0.25$, the estimators $\hat{\hat{Y}}_{\mathrm{R}}$, $\hat{\hat{Y}}_{\xi}$, $\hat{\hat{Y}}_{\nu}$, $\hat{\hat{Y}}_{\mathrm{MM1}} - \hat{\hat{Y}}_{\mathrm{MM3}}$ are even less efficient than $\bar{Y}_{[n]}$.

4.2 Simulation with estimated population parameters

Previous efficiency comparisons have been performed by assuming that all the population parameters concerning both the study and the auxiliary variable, and appearing in the expression of the estimators, of the bias and the mean squared error, were known. Indeed, population information mainly related to the study variable is generally unknown in real situations. Hence, it may be interesting to integrate previous comparisons, based on theoretical results, with some considerations stemming from practical issues.

When population parameters are unknown and cannot be properly guessed on the basis of previous data, expert opinion or a pilot survey, they have to be estimated. In such a circumstance, estimates of the target parameter are affected by an extra source of variability and their behaviour may differ from the case when the population parameters are assumed to be given. To shed light on the matter,

Table 2 *Results for the theoretical PRB and PRE for $C_x = 1$ and $\rho_{xy} = 0.7$. The PRB of $\bar{Y}_{[n]}$ and $\hat{\hat{Y}}_{\text{Reg}}$ is omitted since the estimators are unbiased*

	$C_y = 0.25$				$C_y = 1$				$C_y = 1.5$				$C_y = 3$			
k	3	5	7	10	3	5	7	10	3	5	7	10	3	5	7	10
								PRB								
$\hat{\hat{Y}}_{\text{R}}$	47.26	19.59	10.78	5.66	17.65	7.32	4.03	2.11	−2.97	−1.23	−0.68	−0.36	−62.56	−25.94	−14.27	−7.49
$\hat{\hat{Y}}_{\xi}$	−5.39	−2.54	−1.56	−0.94	−3.39	−1.84	−1.25	−0.83	−0.90	−0.17	−0.01	0.05	11.37	10.29	8.25	6.16
$\hat{\hat{Y}}_{\text{P}}$	0.10	0.04	0.02	0.01	0.41	0.17	0.09	0.05	0.61	0.25	0.14	0.07	1.09	0.45	0.25	0.13
$\hat{\hat{Y}}_{\nu}$	−3.37	−0.38	−0.80	−0.93	6.16	−10.80	−4.91	1.19	−24.42	2.34	−12.46	−8.07	10.43	10.63	5.48	2.28
$\hat{\hat{Y}}_{\text{MM1}}$	3.82	1.58	0.87	0.46	1.12	0.47	0.26	0.13	−0.75	−0.31	−0.17	−0.09	−5.81	−2.41	−1.33	−0.70
$\hat{\hat{Y}}_{\text{MM2}}$	2.64	1.09	0.60	0.32	0.34	0.14	0.08	0.04	−1.25	−0.52	−0.29	−0.15	−5.55	−2.30	−1.27	−0.66
$\hat{\hat{Y}}_{\text{MM3}}$	2.63	1.09	0.60	0.31	0.34	0.14	0.08	0.04	−1.25	−0.52	−0.29	−0.15	−5.55	−2.30	−1.27	−0.66
$\hat{\hat{Y}}_{\text{MM4}}$	0.78	0.32	0.18	0.09	3.13	1.30	0.71	0.37	4.67	1.94	1.07	0.56	9.00	3.73	2.05	1.08
$\hat{\hat{Y}}_{\text{MM5}}$	1.09	0.23	0.01	−0.08	−3.52	−3.24	−2.73	−2.16	−10.91	−8.48	−6.77	−5.15	−48.69	−34.72	−26.82	−19.96
$\hat{\hat{Y}}_{\text{Rao}}$	−0.34	−0.21	−0.15	−0.10	−5.41	−3.32	−2.39	−1.69	−11.25	−7.07	−5.15	−3.67	−31.08	−21.29	−16.20	−11.92

Table 2 *Continued*

	$C_y = 0.25$				$C_y = 1$				$C_y = 1.5$				$C_y = 3$			
k	3	5	7	10	3	5	7	10	3	5	7	10	3	5	7	10
								PRE								
$\hat{\hat{Y}}_{\mathrm{R}}$	12.39	15.47	18.24	21.96	136.46	125.88	120.22	115.35	150.13	134.65	126.70	120.03	134.33	124.61	119.33	114.73
$\hat{\hat{Y}}_{\xi}$	12.37	15.45	18.23	21.96	139.65	128.11	121.92	116.59	153.86	135.46	126.90	119.99	119.94	106.02	103.51	102.53
$\hat{\hat{Y}}_{\mathrm{P}}$	6.32	8.03	9.62	11.84	38.00	44.33	49.33	55.17	51.58	58.02	62.79	68.07	70.21	75.31	78.81	82.42
$\hat{\hat{Y}}_{\nu}$	12.37	15.28	18.23	21.92	111.13	111.12	119.51	111.35	141.06	127.93	120.20	118.08	121.83	103.56	108.75	110.85
$\hat{\hat{Y}}_{\mathrm{MM1}}$	15.28	18.93	22.16	26.41	142.75	129.95	123.24	117.53	148.95	133.91	126.17	119.64	131.49	122.70	117.89	113.68
$\hat{\hat{Y}}_{\mathrm{MM2}}$	22.01	26.75	30.81	35.96	148.46	133.54	125.87	119.41	145.08	131.49	124.40	118.38	126.93	119.61	115.54	111.93
$\hat{\hat{Y}}_{\mathrm{MM3}}$	22.09	26.84	30.90	36.06	148.46	133.54	125.87	119.41	145.06	131.47	124.39	118.37	126.92	119.60	115.53	111.93
$\hat{\hat{Y}}_{\mathrm{MM4}}$	9.52	11.98	14.24	17.31	46.75	53.29	58.24	63.81	59.92	65.98	70.31	74.95	76.18	80.54	83.46	86.42
$\hat{\hat{Y}}_{\mathrm{MM5}}$	152.12	136.01	127.74	120.80	149.33	134.08	126.26	119.69	150.32	134.77	126.79	120.08	152.06	135.97	127.71	120.78
$\hat{\hat{Y}}_{\mathrm{Reg}}$	152.12	136.01	127.74	120.80	149.33	134.08	126.26	119.69	150.32	134.77	126.79	120.08	152.06	135.97	127.71	120.78
$\hat{\hat{Y}}_{\mathrm{Rao}}$	152.64	136.29	127.93	120.93	157.86	138.68	129.35	121.74	169.38	145.02	133.68	124.65	220.62	172.75	152.39	137.12

we therefore address the issue of evaluating the effect on the efficiency of the considered estimators when the population parameters μ_y, C_y, C_x, M_d, Q_d, $\beta_2(x)$, ρ_{xy}, $W_{yx[i]}$ and $W_{y[i]}$ are estimated from the final ranked sample. In order to keep the paper short, we do not repeat all the simulation experiments of Section 4.1. We focus instead only on the situation described in Table 2 and we investigate the performance of the estimators previously discussed by using a plug-in estimation approach. Specifically, from different bivariate Normal populations, we select a ranked set sample and obtain a plug-in estimate of $\mathrm{B}(\hat{\bar{Y}}_*)$ and $\mathrm{MSE}(\hat{\bar{Y}}_*)$ by replacing the aforementioned population parameters with their natural estimates. Hence, we independently repeat the procedure $L = 5,000$ times and evaluate the estimated PRB and PRE as

$$\widehat{\mathrm{PRB}}(\hat{\bar{Y}}_*) = \frac{\sum_{l=1}^{L} \widehat{\mathrm{B}}^{(l)}(\hat{\bar{Y}}_*)}{\sum_{l=1}^{L} \bar{Y}_{[n]}^{(l)}} \times 100$$

and

$$\widehat{\mathrm{PRE}}(\hat{\bar{Y}}_*) = \frac{\sum_{l=1}^{L} \widehat{\mathrm{Var}}^{(l)}(\bar{Y}_{[n]})}{\sum_{l=1}^{L} \widehat{\mathrm{MSE}}^{(l)}(\hat{\bar{Y}}_*)} \times 100,$$

where $\bar{Y}_{[n]}^{(l)}$ denotes the estimate of μ_y over the lth replication, and $\widehat{\mathrm{B}}^{(l)}(\hat{\bar{Y}}_*)$ and $\widehat{\mathrm{MSE}}^{(l)}(\hat{\bar{Y}}_*)$ the plug-in estimates of $\mathrm{B}(\hat{\bar{Y}}_*)$ and $\mathrm{MSE}(\hat{\bar{Y}}_*)$ over the lth replication. Accordingly, the meaning of $\widehat{\mathrm{Var}}^{(l)}(\bar{Y}_{[n]})$ easily follows. The findings of this simulation experiment are summarized in Table 3. All in all, the results confirm those discussed in Section 4.1. With the exception of a few cases, mostly referable to the estimator $\hat{\bar{Y}}_\nu$, the PRB seems to increase (in absolute value) in comparison to the previous simulation study. On the contrary, no general conclusion can be drawn about the behaviour of the PRE, although we can observe a major diversification in the efficiency of the estimators $\hat{\bar{Y}}_{\mathrm{MM5}}$, $\hat{\bar{Y}}_{\mathrm{Reg}}$ and $\hat{\bar{Y}}_{\mathrm{Rao}}$ for low values of C_y, and a very satisfactory performance of $\hat{\bar{Y}}_{\mathrm{Rao}}$ which notably improves as C_y increases.

5 Application to real data

In this section, the estimators discussed in Section 4 are compared on the basis of real data using the same framework of Section 4. Two datasets are considered. The first dataset, say Dataset 1, refers to the *total corn production* and to the *extension of agricultural land*. Similarly, the second dataset, say Dataset 2, considers the *total wine grapes production* and the *extension of agricultural land*. In the two datasets, the production (in quintals) represents the target variable (Y), while the

Table 3 *Results for the estimated PRB and PRE for* $C_x = 1$

	$C_y = 0.25$				$C_y = 1$				$C_y = 1.5$				$C_y = 3$			
k	3	5	7	10	3	5	7	10	3	5	7	10	3	5	7	10
								PRB								
$\hat{\hat{Y}}_{\mathrm{R}}$	−81.99	−44.46	−37.41	−26.60	−8.37	13.58	8.59	5.65	134.63	80.72	40.22	46.04	810.85	460.99	349.49	246.23
$\hat{\hat{Y}}_{\xi}$	−2.30	−1.93	−0.78	−0.52	−4.41	−4.45	−3.10	−2.18	−11.14	−7.36	−3.78	−4.28	−20.12	−15.88	−15.33	−12.19
$\hat{\hat{Y}}_{\mathrm{P}}$	0.11	0.04	0.02	0.01	0.48	0.18	0.10	0.05	0.85	0.32	0.16	0.08	1.83	0.29	0.37	0.17
$\hat{\hat{Y}}_{\nu}$	0.84	−0.45	0.24	−0.30	1.09	−1.84	−0.42	−0.52	−1.20	−0.62	1.27	−0.71	−3.53	1.45	−0.79	−0.23
$\hat{\hat{Y}}_{\mathrm{MM1}}$	21.13	16.51	6.51	4.12	−0.16	13.62	7.84	5.13	10.98	1.18	−14.42	4.20	24.42	−34.54	−3.55	−4.24
$\hat{\hat{Y}}_{\mathrm{MM2}}$	19.17	12.97	5.38	3.26	0.27	8.60	6.11	4.08	9.06	−0.78	−12.55	2.49	17.54	−33.10	−5.74	−4.42
$\hat{\hat{Y}}_{\mathrm{MM3}}$	21.47	13.77	5.72	3.41	2.26	9.42	6.44	4.22	10.99	−0.01	−12.18	2.64	19.71	−32.30	−5.31	−4.29
$\hat{\hat{Y}}_{\mathrm{MM4}}$	9.23	3.63	2.00	1.02	37.91	13.97	8.01	4.17	56.18	21.57	11.66	5.88	112.97	47.21	23.17	13.21
$\hat{\hat{Y}}_{\mathrm{MM5}}$	11.95	6.80	2.25	1.03	6.64	15.20	9.60	6.43	9.75	8.73	−5.27	10.17	−53.23	−52.93	18.03	4.69
$\hat{\hat{Y}}_{\mathrm{Reg}}$	−0.01	0.00	0.00	0.00	−0.07	−0.04	−0.01	0.00	0.02	−0.05	−0.08	−0.04	0.08	−0.22	−0.03	−0.04
$\hat{\hat{Y}}_{\mathrm{Rao}}$	−2.63	−2.77	−2.85	−2.89	−28.42	−30.71	−31.98	−32.51	−41.99	−46.91	−48.88	−50.41	−56.63	−66.18	−70.14	−73.23

Table 3 *Continued*

k	$C_y = 0.25$				$C_y = 1$				$C_y = 1.5$				$C_y = 3$			
	3	5	7	10	3	5	7	10	3	5	7	10	3	5	7	10
								PRE								
$\hat{\bar{Y}}_{\mathrm{R}}$	11.64	12.24	16.21	18.77	143.87	117.15	113.33	108.29	142.91	135.83	132.55	122.61	132.99	118.80	122.89	116.52
$\hat{\bar{Y}}_{\xi}$	11.68	12.25	16.21	18.78	162.82	123.08	117.53	111.09	205.16	167.70	155.77	133.49	364.46	261.14	217.26	175.25
$\hat{\bar{Y}}_{\mathrm{P}}$	6.36	6.81	9.17	10.83	38.82	35.96	40.67	44.39	44.76	53.06	82.34	56.05	56.99	79.56	70.98	78.13
$\hat{\bar{Y}}_{\nu}$	10.06	11.60	15.79	18.36	100.65	106.71	106.20	105.06	107.54	122.05	125.55	119.37	102.79	108.82	116.97	113.73
$\hat{\bar{Y}}_{\mathrm{MM1}}$	15.20	15.52	20.39	23.23	152.53	126.15	120.03	113.79	145.87	136.47	128.77	123.91	132.59	117.34	121.69	115.54
$\hat{\bar{Y}}_{\mathrm{MM2}}$	17.04	19.79	24.54	28.90	151.77	133.95	123.68	116.94	145.79	135.52	124.45	124.29	132.13	116.69	119.63	114.98
$\hat{\bar{Y}}_{\mathrm{MM3}}$	15.98	18.88	23.73	28.19	149.46	132.34	122.80	116.44	144.96	135.09	124.12	124.19	132.16	116.67	119.74	115.01
$\hat{\bar{Y}}_{\mathrm{MM4}}$	8.08	9.40	12.24	14.76	42.81	43.70	46.74	50.58	49.04	59.04	84.77	63.37	61.49	81.84	76.68	81.00
$\hat{\bar{Y}}_{\mathrm{MM5}}$	180.62	147.40	134.00	123.94	175.63	145.52	132.44	123.33	174.14	145.58	137.25	124.60	192.84	149.39	134.79	124.04
$\hat{\bar{Y}}_{\mathrm{Reg}}$	219.96	209.87	204.68	201.97	213.23	202.89	198.32	196.55	215.05	205.04	200.73	198.64	217.02	206.81	203.37	201.13
$\hat{\bar{Y}}_{\mathrm{Rao}}$	227.85	217.04	211.53	208.57	344.20	319.93	310.72	304.60	492.75	459.67	445.66	439.04	925.92	990.81	975.94	990.68

Table 4 *Population values for the transformed variables*

	N	μ_y	μ_x	C_y	C_x	$\beta_{2(x)}$	M_d	Q_d	ρ_{xy}	λ_y	λ_x
Dataset 1	101	96.48	171.93	0.46	0.58	3.38	157.52	135.76	0.91	0.33	0.52
Dataset 2	101	33.71	18.07	0.44	0.49	3.51	17.86	11.84	0.95	0.28	0.35

extension of the agricultural land (in hectares) is assumed to be the auxiliary variable (X). Data on both the variables refer to the Italian Provinces in 2011 (Source: Istat—Italian Statistical Institute). Original data are not normally distributed, thus, in order to eliminate non-normality and use results from normal order statistics (see Section 4), both the target and the auxiliary variables have been preliminarily transformed using the Box–Cox power transformation. The values of the population parameters for the transformed variables are summarized in Table 4, where we also report the λ power value used for the Box–Cox transformation to achieve normality.

The bias and the mean squared error of the considered estimators have been computed on the transformed populations under the two scenarios depicted in Sections 4.1 and 4.2. Accordingly, Tables 5 and 6 show the results for the theoretical PRB and PRE computed using population data, and the results for the estimated PRB and PRE computed over $L = 5,000$ replications. The result for the bias and the mean squared error are instead reported in the Appendix (see Tables 10 and 11).

The two illustrative examples further confirm the outperforming behaviour of the Rao estimator. We observe that a negligible bias marks $\hat{\bar{Y}}_{\text{Rao}}$ in all the considered situations. Moreover, due to the high value of the correlation coefficient, $\hat{\bar{Y}}_{\text{Rao}}$ slightly outperforms $\hat{\bar{Y}}_{\text{Reg}}$ and $\hat{\bar{Y}}_{\text{MM5}}$ in terms of efficiency, even if the differences appear more evident in the case of estimated parameters.

6 Conclusions

In this article, following part of the current literature on RSS from finite population, we have introduced an alternative estimator of the population mean. The Rao regression-type estimator, originally proposed in the context of SRS, has been adapted to RSS, and its efficiency investigated by means of a comparative study based on simulated and real data.

The results of the study would be of interest for survey practitioners for at least two reasons: (i) the proposed estimator always outperforms certain competitive estimators previously introduced in the literature and mentioned in this article; (ii) some serious inefficiencies of competitive estimators are disclosed.

Table 5 *Results for the theoretical and estimated PRB and PRE of the corn production estimates*

	True population parameters				Estimated population parameters			
k	3	5	7	10	3	5	7	10
				PRB				
$\hat{\bar{Y}}_{\mathrm{R}}$	51.22	21.23	11.68	6.13	−296.35	−39.37	−117.86	−45.76
$\hat{\bar{Y}}_{\xi}$	−0.89	−0.51	−0.35	−0.24	1.53	−0.50	0.54	−0.01
$\hat{\bar{Y}}_{\mathrm{P}}$	0.01	0.01	0.00	0.00	0.02	0.01	0.00	0.00
$\hat{\bar{Y}}_{\nu}$	3.93	3.81	3.75	−5.50	1.97	−0.27	0.80	0.07
$\hat{\bar{Y}}_{\mathrm{MM1}}$	0.52	0.22	0.12	0.06	−143.60	42.47	−59.62	−5.64
$\hat{\bar{Y}}_{\mathrm{MM2}}$	0.48	0.20	0.11	0.06	−141.97	41.74	−58.02	−5.76
$\hat{\bar{Y}}_{\mathrm{MM3}}$	0.45	0.19	0.10	0.05	−138.83	41.11	−56.33	−5.64
$\hat{\bar{Y}}_{\mathrm{MM4}}$	1.35	0.56	0.31	0.16	141.04	57.50	30.57	15.73
$\hat{\bar{Y}}_{\mathrm{MM5}}$	−1.24	−1.23	−1.05	−0.84	−109.39	42.20	−45.09	−1.49
$\hat{\bar{Y}}_{\mathrm{Reg}}$	0.00	0.00	0.00	0.00	0.56	0.28	0.20	0.15
$\hat{\bar{Y}}_{\mathrm{Rao}}$	−0.42	−0.26	−0.18	−0.13	−2.94	−3.19	−3.33	−3.42
				PRE				
$\hat{\bar{Y}}_{\mathrm{R}}$	251.73	213.80	191.70	171.51	170.48	185.71	238.46	215.04
$\hat{\bar{Y}}_{\xi}$	251.85	214.17	192.08	171.85	170.39	185.90	237.91	215.29
$\hat{\bar{Y}}_{\mathrm{P}}$	23.36	25.66	27.75	30.59	32.67	21.59	48.83	37.68
$\hat{\bar{Y}}_{\nu}$	272.43	243.94	201.43	180.34	150.50	154.43	252.50	198.15
$\hat{\bar{Y}}_{\mathrm{MM1}}$	253.29	214.79	192.41	172.01	173.80	187.39	241.25	215.93
$\hat{\bar{Y}}_{\mathrm{MM2}}$	260.73	219.48	195.78	174.35	179.81	189.35	262.07	219.70
$\hat{\bar{Y}}_{\mathrm{MM3}}$	267.08	223.43	198.59	176.29	185.58	191.32	277.21	219.99
$\hat{\bar{Y}}_{\mathrm{MM4}}$	24.21	26.57	28.70	31.59	33.43	21.95	51.16	39.56
$\hat{\bar{Y}}_{\mathrm{MM5}}$	337.81	264.31	226.59	194.92	525.98	333.24	355.69	222.63
$\hat{\bar{Y}}_{\mathrm{Reg}}$	337.81	264.31	226.59	194.92	691.26	639.88	609.48	594.65
$\hat{\bar{Y}}_{\mathrm{Rao}}$	339.25	264.99	227.00	195.17	717.60	663.89	632.52	617.19

In conclusion, the Rao regression-type estimator appears to be a valuable alternative to other estimators currently available in the literature. We recommend, therefore, its use since it can substantially improve the estimates of the population mean in the RSS.

Table 6 *Results for the theoretical and estimated PRB and PRE of the wine grapes production estimates*

	True population parameters				Estimated population parameters			
k	3	5	7	10	3	5	7	10
				PRB				
$\hat{\hat{Y}}_{\mathrm{R}}$	6.30	2.61	1.44	0.75	−9.05	−24.63	−28.98	−11.94
$\hat{\hat{Y}}_{\xi}$	−0.34	−0.20	−0.14	−0.10	−0.28	0.37	0.60	0.17
$\hat{\hat{Y}}_{\mathrm{P}}$	0.04	0.01	0.01	0.00	0.04	0.02	0.01	0.00
$\hat{\hat{Y}}_{\nu}$	−3.38	−1.93	−1.23	−1.01	0.12	0.59	0.63	0.31
$\hat{\hat{Y}}_{\mathrm{MM1}}$	0.15	0.06	0.03	0.02	8.22	−13.53	−20.58	−6.78
$\hat{\hat{Y}}_{\mathrm{MM2}}$	−0.03	−0.01	−0.01	0.00	5.16	−11.23	−15.53	−5.41
$\hat{\hat{Y}}_{\mathrm{MM3}}$	−0.15	−0.06	−0.03	−0.02	3.07	−8.53	−11.42	−4.17
$\hat{\hat{Y}}_{\mathrm{MM4}}$	0.85	0.35	0.19	0.10	35.66	11.07	6.42	3.36
$\hat{\hat{Y}}_{\mathrm{MM5}}$	−1.52	−1.25	−1.01	−0.78	12.43	3.77	−4.14	1.46
$\hat{\hat{Y}}_{\mathrm{Reg}}$	0.00	0.00	0.00	0.00	0.29	0.21	0.14	0.09
$\hat{\hat{Y}}_{\mathrm{Rao}}$	−0.19	−0.12	−0.08	−0.06	−1.35	−1.50	−1.55	−1.59
				PRE				
$\hat{\hat{Y}}_{\mathrm{R}}$	553.51	424.94	355.18	294.51	500.82	419.59	491.65	423.90
$\hat{\hat{Y}}_{\xi}$	553.74	425.33	355.54	294.78	505.21	420.35	490.78	423.53
$\hat{\hat{Y}}_{\mathrm{P}}$	24.57	25.87	27.08	28.77	22.91	35.69	44.91	46.42
$\hat{\hat{Y}}_{\nu}$	549.52	424.32	355.19	293.88	273.10	302.27	429.96	379.51
$\hat{\hat{Y}}_{\mathrm{MM1}}$	576.67	437.53	363.40	299.67	555.62	476.99	537.74	451.37
$\hat{\hat{Y}}_{\mathrm{MM2}}$	622.65	461.66	378.85	309.23	621.22	465.15	537.08	348.14
$\hat{\hat{Y}}_{\mathrm{MM3}}$	540.82	417.91	350.56	291.57	602.36	284.30	266.91	225.64
$\hat{\hat{Y}}_{\mathrm{MM4}}$	33.73	35.29	36.73	38.70	27.10	51.04	75.61	67.53
$\hat{\hat{Y}}_{\mathrm{MM5}}$	625.73	463.24	379.85	309.84	948.13	719.94	524.83	467.30
$\hat{\hat{Y}}_{\mathrm{Reg}}$	625.73	463.24	379.85	309.84	1419.60	1279.50	1219.90	1184.20
$\hat{\hat{Y}}_{\mathrm{Rao}}$	626.95	463.79	380.17	310.02	1446.70	1303.40	1242.40	1205.80

Appendix: Results for the bias and the MSE of the compared estimators

Table 7 *Results for the theoretical bias and MSE for $C_x = 0.25$ and $\rho_{xy} = 0.7$. The bias of $\bar{Y}_{[n]}$ and $\hat{\bar{Y}}_{\text{Reg}}$ is omitted since the estimators are unbiased*

	$C_y = 0.25$				$C_y = 1$				$C_y = 1.5$				$C_y = 3$			
k	3	5	7	10	3	5	7	10	3	5	7	10	3	5	7	10
								Bias								
$\hat{\bar{Y}}_{\text{R}}$	0.11	0.04	0.02	0.01	−0.64	−0.27	−0.15	−0.08	−1.11	−0.46	−0.25	−0.13	−2.64	−1.10	−0.60	−0.32
$\hat{\bar{Y}}_{\xi}$	−0.02	−0.01	−0.01	0.00	0.15	0.13	0.11	0.08	0.37	0.35	0.29	0.23	1.71	1.74	1.44	1.10
$\hat{\bar{Y}}_{\text{P}}$	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.02	0.01	0.00	0.00	0.03	0.01	0.01	0.00
$\hat{\bar{Y}}_{\nu}$	−0.10	−0.07	−0.07	−0.14	0.29	0.17	0.07	0.04	0.20	0.20	0.18	0.14	−0.05	−0.35	−0.13	−0.16
$\hat{\bar{Y}}_{\text{MM1}}$	0.01	0.00	0.00	0.00	−0.06	−0.03	−0.01	−0.01	−0.11	−0.05	−0.03	−0.01	−0.26	−0.11	−0.06	−0.03
$\hat{\bar{Y}}_{\text{MM2}}$	0.00	0.00	0.00	0.00	−0.06	−0.02	−0.01	−0.01	−0.09	−0.04	−0.02	−0.01	−0.21	−0.09	−0.05	−0.03
$\hat{\bar{Y}}_{\text{MM3}}$	0.00	0.00	0.00	0.00	−0.04	−0.02	−0.01	0.00	−0.06	−0.02	−0.01	−0.01	−0.13	−0.05	−0.03	−0.02
$\hat{\bar{Y}}_{\text{MM4}}$	0.01	0.00	0.00	0.00	0.05	0.02	0.01	0.01	0.07	0.03	0.02	0.01	0.14	0.06	0.03	0.02
$\hat{\bar{Y}}_{\text{MM5}}$	−0.02	−0.02	−0.01	−0.01	−0.46	−0.33	−0.26	−0.19	−1.06	−0.76	−0.59	−0.44	−4.38	−3.11	−2.40	−1.78
$\hat{\bar{Y}}_{\text{Rao}}$	−0.04	−0.02	−0.02	−0.01	−0.54	−0.33	−0.24	−0.17	−1.17	−0.74	−0.54	−0.38	−3.40	−2.36	−1.81	−1.34

Table 7 *Continued*

	$C_y = 0.25$				$C_y = 1$				$C_y = 1.5$				$C_y = 3$			
k	3	5	7	10	3	5	7	10	3	5	7	10	3	5	7	10
								MSE								
$\bar{Y}_{[n]}$	0.53	0.29	0.19	0.13	8.50	4.58	3.08	2.04	19.21	10.38	6.99	4.65	76.35	41.18	27.71	18.40
$\hat{\bar{Y}}_{\mathrm{R}}$	0.39	0.23	0.16	0.11	6.85	3.89	2.70	1.85	16.64	9.31	6.41	4.34	70.71	38.83	26.42	17.72
$\hat{\bar{Y}}_{\xi}$	0.39	0.23	0.16	0.11	6.97	3.98	2.76	1.88	17.58	9.94	6.78	4.54	93.29	53.17	34.60	21.91
$\hat{\bar{Y}}_{\mathrm{P}}$	1.39	0.64	0.39	0.23	10.86	5.56	3.62	2.33	22.46	11.73	7.73	5.04	82.72	43.82	29.17	19.16
$\hat{\bar{Y}}_{\nu}$	0.41	0.24	0.16	0.12	7.72	4.20	2.81	1.89	17.79	9.95	6.79	4.55	69.09	36.25	25.89	17.28
$\hat{\bar{Y}}_{\mathrm{MM1}}$	0.39	0.23	0.16	0.11	6.88	3.91	2.71	1.85	16.70	9.34	6.42	4.35	70.84	38.89	26.45	17.74
$\hat{\bar{Y}}_{\mathrm{MM2}}$	0.36	0.22	0.15	0.11	7.16	4.02	2.77	1.88	17.18	9.54	6.53	4.40	71.93	39.34	26.70	17.87
$\hat{\bar{Y}}_{\mathrm{MM3}}$	0.38	0.23	0.16	0.11	7.65	4.23	2.89	1.94	17.97	9.86	6.71	4.50	73.67	40.06	27.10	18.08
$\hat{\bar{Y}}_{\mathrm{MM4}}$	0.83	0.41	0.26	0.16	9.49	4.99	3.31	2.16	20.59	10.95	7.31	4.81	79.19	42.35	28.36	18.74
$\hat{\bar{Y}}_{\mathrm{MM5}}$	0.36	0.22	0.15	0.11	5.69	3.41	2.44	1.71	13.02	7.81	5.58	3.90	51.35	30.81	22.01	15.41
$\hat{\bar{Y}}_{\mathrm{Reg}}$	0.36	0.22	0.15	0.11	5.69	3.41	2.44	1.71	13.02	7.81	5.58	3.90	51.35	30.81	22.01	15.41
$\hat{\bar{Y}}_{\mathrm{Rao}}$	0.36	0.22	0.15	0.11	5.39	3.30	2.38	1.68	11.45	7.22	5.27	3.75	33.75	23.47	17.99	13.32

Table 8 *Theoretical results for the bias and the MSE for $C_x = 1$ and $\rho_{xy} = 0.7$. The bias of $\bar{Y}_{[n]}$ and $\hat{\hat{Y}}_{\text{Reg}}$ is omitted since the estimators are unbiased*

	$C_y = 0.25$				$C_y = 1$				$C_y = 1.5$				$C_y = 3$			
k	3	5	7	10	3	5	7	10	3	5	7	10	3	5	7	10
								Bias								
$\hat{\hat{Y}}_{\text{R}}$	4.74	1.97	1.08	0.57	1.76	0.73	0.40	0.21	−0.29	−0.12	−0.07	−0.04	−6.67	−2.76	−1.52	−0.80
$\hat{\hat{Y}}_{\xi}$	−0.54	−0.26	−0.16	−0.09	−0.34	−0.18	−0.12	−0.08	−0.09	−0.02	0.00	0.01	1.21	1.10	0.88	0.66
$\hat{\hat{Y}}_{\text{P}}$	0.01	0.00	0.00	0.00	0.04	0.02	0.01	0.00	0.06	0.03	0.01	0.01	0.12	0.05	0.03	0.01
$\hat{\hat{Y}}_{\nu}$	−0.34	−0.04	−0.08	−0.09	0.61	−1.08	−0.49	0.12	−2.42	0.23	−1.23	−0.80	1.11	1.13	0.58	0.24
$\hat{\hat{Y}}_{\text{MM1}}$	0.38	0.16	0.09	0.05	0.11	0.05	0.03	0.01	−0.07	−0.03	−0.02	−0.01	−0.62	−0.26	−0.14	−0.07
$\hat{\hat{Y}}_{\text{MM2}}$	0.26	0.11	0.06	0.03	0.03	0.01	0.01	0.00	−0.12	−0.05	−0.03	−0.01	−0.59	−0.25	−0.13	−0.07
$\hat{\hat{Y}}_{\text{MM3}}$	0.26	0.11	0.06	0.03	0.03	0.01	0.01	0.00	−0.12	−0.05	−0.03	−0.01	−0.59	−0.25	−0.13	−0.07
$\hat{\hat{Y}}_{\text{MM4}}$	0.08	0.03	0.02	0.01	0.31	0.13	0.07	0.04	0.46	0.19	0.11	0.06	0.96	0.40	0.22	0.11
$\hat{\hat{Y}}_{\text{MM5}}$	0.11	0.02	0.00	−0.01	−0.35	−0.32	−0.27	−0.21	−1.08	−0.84	−0.67	−0.51	−5.19	−3.70	−2.86	−2.13
$\hat{\hat{Y}}_{\text{Rao}}$	−0.03	−0.02	−0.01	−0.01	−0.54	−0.33	−0.24	−0.17	−1.11	−0.70	−0.51	−0.36	−3.31	−2.27	−1.73	−1.27

Table 8 *Continued*

	$C_y = 0.25$				$C_y = 1$				$C_y = 1.5$				$C_y = 3$			
k	3	5	7	10	3	5	7	10	3	5	7	10	3	5	7	10
								MSE								
$\bar{Y}_{[n]}$	0.53	0.28	0.19	0.13	8.48	4.57	3.07	2.04	18.68	10.05	6.75	4.48	77.93	41.81	28.05	18.57
$\hat{\bar{Y}}_{\mathrm{R}}$	4.25	1.83	1.04	0.57	6.21	3.63	2.55	1.77	12.44	7.46	5.33	3.73	58.02	33.56	23.51	16.19
$\hat{\bar{Y}}_{\xi}$	4.26	1.83	1.04	0.57	6.07	3.56	2.52	1.75	12.14	7.42	5.32	3.73	64.97	39.44	27.10	18.11
$\hat{\bar{Y}}_{\mathrm{P}}$	8.33	3.52	1.97	1.06	22.31	10.30	6.23	3.69	36.21	17.32	10.75	6.58	111.00	55.52	35.60	22.53
$\hat{\bar{Y}}_{\nu}$	4.26	1.85	1.04	0.57	7.63	4.11	2.57	1.83	13.24	7.85	5.62	3.79	63.97	40.38	25.80	16.75
$\hat{\bar{Y}}_{\mathrm{MM1}}$	3.44	1.49	0.86	0.47	5.94	3.51	2.49	1.73	12.54	7.50	5.35	3.74	59.27	34.08	23.79	16.34
$\hat{\bar{Y}}_{\mathrm{MM2}}$	2.39	1.06	0.61	0.35	5.71	3.42	2.44	1.71	12.87	7.64	5.43	3.78	61.40	34.96	24.28	16.59
$\hat{\bar{Y}}_{\mathrm{MM3}}$	2.38	1.05	0.61	0.35	5.71	3.42	2.44	1.71	12.88	7.64	5.43	3.78	61.40	34.96	24.28	16.59
$\hat{\bar{Y}}_{\mathrm{MM4}}$	5.53	2.36	1.33	0.72	18.13	8.57	5.27	3.19	31.17	15.23	9.60	5.97	102.30	51.91	33.61	21.49
$\hat{\bar{Y}}_{\mathrm{MM5}}$	0.35	0.21	0.15	0.10	5.68	3.41	2.43	1.70	12.42	7.45	5.32	3.73	51.25	30.75	21.97	15.38
$\hat{\bar{Y}}_{\mathrm{Reg}}$	0.35	0.21	0.15	0.10	5.68	3.41	2.43	1.70	12.42	7.45	5.32	3.73	51.25	30.75	21.97	15.38
$\hat{\bar{Y}}_{\mathrm{Rao}}$	0.34	0.21	0.15	0.10	5.37	3.29	2.37	1.67	11.03	6.93	5.05	3.59	35.32	24.20	18.41	13.54

Table 9 *Results for the estimated bias and MSE for $C_x = 1$*

	$C_y = 0.25$				$C_y = 1$				$C_y = 1.5$				$C_y = 3$			
k	3	5	7	10	3	5	7	10	3	5	7	10	3	5	7	10
								Bias								
$\hat{\bar{Y}}_{\mathrm{R}}$	−8.23	−4.46	−3.75	−2.67	−0.84	1.36	0.85	0.56	13.37	7.98	3.99	4.53	87.28	48.33	37.44	26.21
$\hat{\bar{Y}}_{\xi}$	−0.23	−0.19	−0.08	−0.05	−0.44	−0.45	−0.31	−0.22	−1.11	−0.73	−0.37	−0.42	−2.17	−1.66	−1.64	−1.30
$\hat{\bar{Y}}_{\mathrm{P}}$	0.01	0.00	0.00	0.00	0.05	0.02	0.01	0.01	0.08	0.03	0.02	0.01	0.20	0.03	0.04	0.02
$\hat{\bar{Y}}_{\nu}$	0.08	−0.05	0.02	−0.03	0.11	−0.18	−0.04	−0.05	−0.12	−0.06	0.13	−0.07	−0.38	0.15	−0.08	−0.02
$\hat{\bar{Y}}_{\mathrm{MM1}}$	2.12	1.66	0.65	0.41	−0.02	1.36	0.78	0.51	1.09	0.12	−1.43	0.41	2.63	−3.62	−0.38	−0.45
$\hat{\bar{Y}}_{\mathrm{MM2}}$	1.93	1.30	0.54	0.33	0.03	0.86	0.61	0.41	0.90	−0.08	−1.24	0.24	1.89	−3.47	−0.61	−0.47
$\hat{\bar{Y}}_{\mathrm{MM3}}$	2.16	1.38	0.57	0.34	0.23	0.94	0.64	0.42	1.09	0.00	−1.21	0.26	2.12	−3.39	−0.57	−0.46
$\hat{\bar{Y}}_{\mathrm{MM4}}$	0.93	0.36	0.20	0.10	3.79	1.40	0.80	0.41	5.58	2.13	1.16	0.58	12.16	4.95	2.48	1.41
$\hat{\bar{Y}}_{\mathrm{MM5}}$	1.20	0.68	0.23	0.10	0.66	1.52	0.95	0.64	0.97	0.86	−0.52	1.00	−5.73	−5.55	1.93	0.50
$\hat{\bar{Y}}_{\mathrm{Reg}}$	0.00	0.00	0.00	0.00	−0.01	0.00	0.00	0.00	0.00	−0.01	−0.01	0.00	0.01	−0.02	0.00	0.00
$\hat{\bar{Y}}_{\mathrm{Rao}}$	−0.26	−0.28	−0.29	−0.29	−2.84	−3.08	−3.18	−3.23	−4.17	−4.64	−4.85	−4.96	−6.10	−6.94	−7.51	−7.80

Table 9 *Continued*

	$C_y = 0.25$				$C_y = 1$				$C_y = 1.5$				$C_y = 3$			
k	3	5	7	10	3	5	7	10	3	5	7	10	3	5	7	10
								MSE								
$\bar{Y}_{[n]}$	0.52	0.28	0.19	0.12	8.29	4.49	3.01	2.00	18.35	9.82	6.64	4.37	76.30	40.84	27.48	18.25
$\hat{\bar{Y}}_{\mathrm{R}}$	4.45	2.26	1.14	0.66	5.76	3.83	2.66	1.84	12.84	7.23	5.01	3.57	57.37	34.38	22.36	15.66
$\hat{\bar{Y}}_{\xi}$	4.44	2.26	1.14	0.66	5.09	3.65	2.56	1.80	8.95	5.86	4.26	3.28	20.94	15.64	12.65	10.41
$\hat{\bar{Y}}_{\mathrm{P}}$	8.15	4.07	2.02	1.14	21.34	12.49	7.40	4.50	41.01	18.51	8.06	7.80	133.88	51.33	38.72	23.36
$\hat{\bar{Y}}_{\nu}$	5.15	2.39	1.17	0.67	8.23	4.21	2.83	1.90	17.07	8.05	5.29	3.66	74.23	37.53	23.49	16.05
$\hat{\bar{Y}}_{\mathrm{MM1}}$	3.41	1.78	0.91	0.53	5.43	3.56	2.51	1.76	12.58	7.20	5.16	3.53	57.55	34.80	22.58	15.80
$\hat{\bar{Y}}_{\mathrm{MM2}}$	3.04	1.40	0.76	0.43	5.46	3.35	2.43	1.71	12.59	7.25	5.33	3.52	57.74	35.00	22.97	15.87
$\hat{\bar{Y}}_{\mathrm{MM3}}$	3.24	1.47	0.78	0.44	5.54	3.39	2.45	1.72	12.66	7.27	5.35	3.52	57.73	35.00	22.95	15.87
$\hat{\bar{Y}}_{\mathrm{MM4}}$	6.41	2.94	1.52	0.83	19.36	10.28	6.44	3.95	37.42	16.64	7.83	6.90	124.09	49.90	35.84	22.53
$\hat{\bar{Y}}_{\mathrm{MM5}}$	0.29	0.19	0.14	0.10	4.72	3.09	2.27	1.62	10.54	6.75	4.84	3.51	39.57	27.34	20.39	14.71
$\hat{\bar{Y}}_{\mathrm{Reg}}$	0.24	0.13	0.09	0.06	3.89	2.21	1.52	1.02	8.53	4.79	3.31	2.20	35.16	19.75	13.51	9.07
$\hat{\bar{Y}}_{\mathrm{Rao}}$	0.23	0.13	0.09	0.06	2.41	1.40	0.97	0.66	3.72	2.14	1.49	1.00	8.24	4.12	2.82	1.84

Table 10 *Results for the theoretical and estimated bias and MSE of the corn production estimates*

	True population parameters				Estimated population parameters			
k	3	5	7	10	3	5	7	10
				Bias				
$\hat{\hat{Y}}_{\mathrm{R}}$	49.41	20.49	11.27	5.92	−285.99	−38.04	−113.77	−44.16
$\hat{\hat{Y}}_{\xi}$	−0.86	−0.49	−0.34	−0.23	1.48	−0.48	0.52	−0.01
$\hat{\hat{Y}}_{\mathrm{P}}$	0.01	0.01	0.00	0.00	0.01	0.01	0.00	0.00
$\hat{\hat{Y}}_{\nu}$	3.80	3.68	3.62	−5.31	1.90	−0.26	0.77	0.07
$\hat{\hat{Y}}_{\mathrm{MM1}}$	0.50	0.21	0.12	0.06	−138.58	41.04	−57.55	−5.45
$\hat{\hat{Y}}_{\mathrm{MM2}}$	0.47	0.19	0.11	0.06	−137.01	40.33	−56.01	−5.56
$\hat{\hat{Y}}_{\mathrm{MM3}}$	0.44	0.18	0.10	0.05	−133.97	39.72	−54.37	−5.44
$\hat{\hat{Y}}_{\mathrm{MM4}}$	1.31	0.54	0.30	0.16	136.11	55.55	29.51	15.18
$\hat{\hat{Y}}_{\mathrm{MM5}}$	−1.20	−1.18	−1.01	−0.81	−105.56	40.77	−43.52	−1.44
$\hat{\hat{Y}}_{\mathrm{Reg}}$	0.00	0.00	0.00	0.00	0.54	0.28	0.20	0.14
$\hat{\hat{Y}}_{\mathrm{Rao}}$	−0.41	−0.25	−0.18	−0.12	−2.84	−3.08	−3.22	−3.30
				MSE				
$\bar{Y}_{[n]}$	134.22	63.01	38.58	23.23	135.30	62.58	37.94	22.74
$\hat{\hat{Y}}_{\mathrm{R}}$	53.32	29.47	20.13	13.55	79.37	33.69	15.91	10.58
$\hat{\hat{Y}}_{\xi}$	53.29	29.42	20.09	13.52	79.41	33.66	15.95	10.56
$\hat{\hat{Y}}_{\mathrm{P}}$	574.57	245.56	139.04	75.96	414.15	289.82	77.70	60.36
$\hat{\hat{Y}}_{\nu}$	49.27	25.83	19.15	12.88	89.90	40.52	15.03	11.48
$\hat{\hat{Y}}_{\mathrm{MM1}}$	52.99	29.33	20.05	13.51	77.85	33.39	15.73	10.53
$\hat{\hat{Y}}_{\mathrm{MM2}}$	51.48	28.71	19.71	13.33	75.25	33.05	14.48	10.35
$\hat{\hat{Y}}_{\mathrm{MM3}}$	50.25	28.20	19.43	13.18	72.91	32.71	13.69	10.34
$\hat{\hat{Y}}_{\mathrm{MM4}}$	554.31	237.16	134.42	73.54	404.72	285.05	74.15	57.49
$\hat{\hat{Y}}_{\mathrm{MM5}}$	39.73	23.84	17.03	11.92	25.72	18.78	10.67	10.22
$\hat{\hat{Y}}_{\mathrm{Reg}}$	39.73	23.84	17.03	11.92	19.57	9.78	6.22	3.82
$\hat{\hat{Y}}_{\mathrm{Rao}}$	39.56	23.78	17.00	11.90	18.86	9.43	6.00	3.68

Table 11 *Results for the theoretical and estimated bias and MSE of the wine grapes production estimates*

	True population parameters				Estimated population parameters			
k	3	5	7	10	3	5	7	10
				Bias				
$\hat{\bar{Y}}_{\mathrm{R}}$	2.12	0.88	0.48	0.25	−3.05	−8.29	−9.78	−4.02
$\hat{\bar{Y}}_{\xi}$	−0.11	−0.07	−0.05	−0.03	−0.10	0.13	0.20	0.06
$\hat{\bar{Y}}_{\mathrm{P}}$	0.01	0.00	0.00	0.00	0.01	0.01	0.00	0.00
$\hat{\bar{Y}}_{\nu}$	−1.14	−0.65	−0.42	−0.34	0.04	0.20	0.21	0.10
$\hat{\bar{Y}}_{\mathrm{MM1}}$	0.05	0.02	0.01	0.01	2.77	−4.56	−6.94	−2.28
$\hat{\bar{Y}}_{\mathrm{MM2}}$	−0.01	0.00	0.00	0.00	1.74	−3.78	−5.24	−1.82
$\hat{\bar{Y}}_{\mathrm{MM3}}$	−0.05	−0.02	−0.01	−0.01	1.03	−2.87	−3.85	−1.40
$\hat{\bar{Y}}_{\mathrm{MM4}}$	0.29	0.12	0.07	0.03	12.00	3.73	2.17	1.13
$\hat{\bar{Y}}_{\mathrm{MM5}}$	−0.51	−0.42	−0.34	−0.26	4.18	1.27	−1.40	0.49
$\hat{\bar{Y}}_{\mathrm{Reg}}$	0.00	0.00	0.00	0.00	0.10	0.07	0.05	0.03
$\hat{\bar{Y}}_{\mathrm{Rao}}$	−0.07	−0.04	−0.03	−0.02	−0.46	−0.50	−0.52	−0.54
				MSE				
$\bar{Y}_{[n]}$	13.86	6.16	3.61	2.06	14.25	6.24	3.61	2.03
$\hat{\bar{Y}}_{\mathrm{R}}$	2.50	1.45	1.02	0.70	2.84	1.49	0.73	0.48
$\hat{\bar{Y}}_{\xi}$	2.50	1.45	1.01	0.70	2.82	1.48	0.74	0.48
$\hat{\bar{Y}}_{\mathrm{P}}$	56.41	23.80	13.31	7.15	62.19	17.47	8.04	4.38
$\hat{\bar{Y}}_{\nu}$	2.52	1.45	1.02	0.70	5.22	2.06	0.84	0.54
$\hat{\bar{Y}}_{\mathrm{MM1}}$	2.40	1.41	0.99	0.69	2.56	1.31	0.67	0.45
$\hat{\bar{Y}}_{\mathrm{MM2}}$	2.23	1.33	0.95	0.67	2.29	1.34	0.67	0.58
$\hat{\bar{Y}}_{\mathrm{MM3}}$	2.56	1.47	1.03	0.71	2.37	2.19	1.35	0.90
$\hat{\bar{Y}}_{\mathrm{MM4}}$	41.09	17.44	9.82	5.32	52.57	12.22	4.78	3.01
$\hat{\bar{Y}}_{\mathrm{MM5}}$	2.21	1.33	0.95	0.66	1.50	0.87	0.69	0.44
$\hat{\bar{Y}}_{\mathrm{Reg}}$	2.21	1.33	0.95	0.66	1.00	0.49	0.30	0.17
$\hat{\bar{Y}}_{\mathrm{Rao}}$	2.21	1.33	0.95	0.66	0.98	0.48	0.29	0.17

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Department of Economics, Business,
Mathematics and Statistics
University of Trieste
P.le Europa 1
Trieste 34127
Italy
E-mail: elvira.pelle@deams.units.it

Department of Economics
Statistics and Finance
University of Calabria
Via Pietro Bucci
Arcavacata di Rende 87036
Italy
E-mail: pierfrancesco.perri@unical.it