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Research Article

A Novel Approach to Calculation of Reproducing Kernel on Infinite Interval and Applications to Boundary Value Problems

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A new analytical method for the computation of reproducing kernel is proposed and tested on some examples. The expression of reproducing kernel on infinite interval is obtained concisely in polynomial form for the first time. Furthermore, as a particular effective application of this method, we give an explicit representation formula for calculation of reproducing kernel in reproducing kernel space with boundary value conditions.

1. Introduction

It is well known that reproducing kernel theory has been used in many research fields such as complex analysis, dilation of linear operators, stochastic processes [1–5], and solution of various differential and integral equations [6–10]. Recently, there are a rather small number of methods for calculating the reproducing kernel expression. One is using Green's function for a differential operator to construct a reproducing kernel [11, 12]. Another very standard method involves boundary value conditions depending on the property of δ function [13, 14]. The disadvantages of the above two approaches are clear. In the first method, the expression of reproducing kernel including integral form is complicated as a result of Green's function. Because δ function is highly abstractive in the second approach, it is quite difficult to calculate the kernel in the procedure.

The purpose of this paper is to avoid the complex operation of Green's function and δ function and simply give representation of reproducing kernel in polynomial form. The principal step of the procedure consists of classifying and discussing the infinite interval to satisfy the reproducing property. Therefore, our approach has the advantages that no additional condition is required in order to solve the kernel and a much simpler formalism which is in contrast to the previous two methods. In effect, the universal formula can be obtained in the case of finite interval.

Numerically solving an initial and boundary value problem for a differential equation by the reproducing kernel method can be described as follows: construct reproducing kernel spaces which can absorb initial or boundary value conditions, and then, transfer the initial and boundary value problem into an operator equation in the reproducing kernel space where the exact solution to the initial and boundary value problem is expressed by the reproducing kernel, and at last solve the operator equation by approximation. It is obvious that constructing reproducing kernel space which satisfies the initial or boundary conditions and effectively solving for the reproducing kernel become the key to apply reproducing kernel method for initial and boundary value problems.

In this work, in order to apply the new approach to solving differential equations with multiform boundary value problems, the explicit formula for calculation of reproducing kernel in the appropriate reproducing kernel space is provided successfully by using the orthogonal decomposition property.

The rest of the paper is organized as follows. In Section 2, a new reproducing kernel space on infinite interval is presented. Section 3 shows for representation of class one how reproducing kernel can be expressed in polynomial form and gives some examples. Then, these basic ideas are shown to extend to cases involving reproducing kernel space with boundary value condition in Section 4. Finally, in Section 5, we give a brief conclusion and discuss extensions and generalizations of the present work.

2. A New Reproducing Kernel Space on Infinite Interval

Definition 1 (see [15]). Let H be a Hilbert function space on a set D. H is called a reproducing kernel space if and only if for any $x \in D$, there exists a unique function $R_x(y) \in H$, such that $\langle f, R_x \rangle = f(x)$ for any $f \in H$. Meanwhile, $R(x,y) \triangleq R_x(y)$ is called a reproducing kernel function.

Definition 2. $H_m(-\infty,+\infty) = \{f(x) \mid f^{(m-1)}(x)$ is an absolutely continuous real value function in $(-\infty,+\infty)$, $f^{(m)}(x) \in L^2(-\infty,+\infty)\}$. The inner product and norm are given, respectively, by

$$\begin{aligned}\langle f(x), g(x) \rangle_H &= \sum_{i=0}^{m-1} f^{(i)}(0)\, g^{(i)}(0) \\ &\quad + \int_{-\infty}^{+\infty} f^{(m)}(x)\, g^{(m)}(x)\, dx, \end{aligned} \tag{1}$$

$$\|f\|_H = \sqrt{\langle f(x), f(x) \rangle_H}.$$

Theorem 3. *$H_m(-\infty,+\infty)$ is a Hilbert reproducing kernel space; namely, it is complete, and for any fixed $x \in (-\infty,+\infty)$ and $f(x) \in H_m(-\infty,+\infty)$, there exists a $c_x > 0$, such that $|f(x)| \le c_x \|f\|_H$.*

Proof. Suppose that $f_n(x)$ is a Cauchy sequence in $H_m(-\infty,+\infty)$. Then it holds that

$$\begin{aligned}\left\|f_{n+p} - f_n\right\|_H^2 &= \sum_{i=0}^{m-1} \left| f_{n+p}^{(i)}(0) - f_n^{(i)}(0) \right|^2 \\ &\quad + \int_{-\infty}^{+\infty} \left| f_{n+p}^{(m)}(x) - f_n^{(m)}(x) \right|^2 dx \longrightarrow 0. \end{aligned} \tag{2}$$

Therefore, we have

$$\begin{gathered} f_{n+p}^{(i)}(0) - f_n^{(i)}(0) \longrightarrow 0, \quad i = 0, 1, \ldots, m-1, \\ \int_{-\infty}^{+\infty} \left[f_{n+p}^{(m)}(x) - f_n^{(m)}(x) \right]^2 dx \longrightarrow 0, \end{gathered} \tag{3}$$

which indicate that the sequences $f_n^{(i)}(0)$ $(0 \le i \le m-1)$ and $f_n^{(m)}(x)$ are Cauchy sequences, respectively, in R and $L^2(-\infty,+\infty)$. So, there exist unique real number λ_i $(i = 0, 1, \ldots, m-1)$ and function $h(x) \in L^2(-\infty,+\infty)$, such that

$$\begin{gathered} \lim_{n\to\infty} f_n^{(i)}(0) = \lambda_i, \quad (i = 0, 1, \ldots, m-1), \\ \lim_{n\to\infty} \left[f_n^{(m)}(x) - h(x) \right]^2 \overset{L^2}{=} 0. \end{gathered} \tag{4}$$

Let

$$g(x) = \sum_{i=0}^{m-1} \frac{\lambda_i}{i!} x^i + \frac{1}{(m-1)!} \int_0^x (x-t)^{m-1} h(t)\, dt. \tag{5}$$

One easily sees that

$$\begin{gathered} g^{(i)}(0) = \lambda_i, \quad (i = 0, 1, \ldots, m-1), \\ g^{(m-1)}(x) = \lambda_m - 1 + \int_0^x h(t)\, dt. \end{gathered} \tag{6}$$

Because $h(t) \in L^2(-\infty,+\infty)$, we get $g(x) \in H_m(-\infty,+\infty)$. Moreover, it holds

$$\begin{aligned} \|f_n - g\|_H^2 &= \sum_{i=0}^{m-1} \left| f_n^{(i)}(0) - g^{(i)}(0) \right|^2 \\ &\quad + \int_{-\infty}^{+\infty} \left| f_n^{(m)}(x) - g^{(m)}(x) \right|^2 dx \\ &= \sum_{i=0}^{m-1} \left| f_n^{(i)}(0) - \lambda_i \right|^2 \\ &\quad + \int_{-\infty}^{+\infty} \left| f_n^{(m)}(x) - h(x) \right|^2 dx \longrightarrow 0, \end{aligned} \tag{7}$$

which means that $H_m(-\infty,+\infty)$ is a Hilbert space.

Meanwhile, we can introduce that

$$\begin{aligned} \left| f^{(m-1)}(x) \right| &= f^{(m-1)}(0) + \int_0^x f^{(m)}(t)\, dt \\ &\le \left| f^{(m-1)}(0) \right| + \int_0^x \left| f^{(m)}(t) \right| dt \\ &\le \|f\|_H + \sqrt{x \int_0^x \left| f^{(m)}(t) \right|^2 dt} \\ &\le \|f\|_H \\ &\quad + \sqrt{x} \left(\sum_{i=0}^{m-1} \left| f^{(i)}(0) \right|^2 + \int_{-\infty}^{+\infty} \left| f^{(m)}(x) \right|^2 dx \right)^{1/2} \\ &\le (1 + \sqrt{x}) \|f\|_H = a_x \|f\|_H, \\ \left| f^{(m-2)}(x) \right| &\le \left| f^{(m-2)}(0) \right| + \int_0^x \left| f^{(m-1)}(t) \right| dt \\ &\le \|f\|_H + \int_0^x a_t \|f\|_H dt \le b_x \|f\|_H, \end{aligned} \tag{8}$$

where a_x and b_x are positive numbers.

Then, we also introduce that $f(x) \le c_x \|f\|_H$. This completes the proof of Theorem 3. □

3. Calculation of Reproducing Kernel on Infinite Interval

In this section, we will give a novel method to calculate the reproducing kernel of $H_m(-\infty,+\infty)$. Suppose $R(x,y)$ is the reproducing kernel function. According to the definition of inner, for any fixed $f(y) \in H_m(-\infty,+\infty)$, we have

$$\begin{aligned}\langle f(y), R(x,y)\rangle_H &= \sum_{i=0}^{m-1} f^{(i)}(0)\,\partial_y^i R(x,0) \\ &\quad + \int_{-\infty}^{+\infty} f^{(m)}(y)\,\partial_y^m R(x,y)\,dy \\ &= f(0)R(x,0) + f'(0)\,\partial_y R(x,0) \\ &\quad + \cdots + f^{(m-1)}(0)\,\partial_y^{(m-1)} R(x,0) \\ &\quad + \begin{cases} \left(\int_{-\infty}^{0} + \int_{0}^{x} + \int_{x}^{+\infty}\right) f^{(m)} \\ \qquad \times (y)\,\partial_y^m R(x,y)\,dy, \\ \qquad\qquad x \ge 0, \\ \left(\int_{-\infty}^{x} + \int_{x}^{0} + \int_{0}^{+\infty}\right) f^{(m)} \\ \qquad \times (y)\,\partial_y^m R(x,y)\,dy, \\ \qquad\qquad x < 0. \end{cases}\end{aligned} \tag{9}$$

According to the formula

$$\begin{aligned} f(x) &= f(0) + f'(0)\,x + \frac{f''(0)}{2}x^2 \\ &\quad + \cdots + \frac{1}{m!}\int_0^x f^{(m+1)}(t)\,(x-t)^m dt, \end{aligned} \tag{10}$$

it follows that if

$$\begin{aligned} &R(x,0) = 1, \\ &\partial_y R(x,0) = x, \\ &\partial_y^2 R(x,0) = \frac{1}{2}x^2, \\ &\vdots \\ &\partial_y^m R(x,y) \\ &= \begin{cases} \dfrac{1}{(m-1)!}\operatorname{Sign}(x)\,(x-y)^{m-1}, & y \in [0,x] \text{ or } [x,0], \\ 0, & \text{otherwise}, \end{cases} \end{aligned} \tag{11}$$

then $\langle f(y), R(x,y)\rangle_H = f(x)$ which indicates that $R(x,y)$ is the reproducing kernel function.

It is now clear how to compute $R(x,y)$ by solving (11). Moreover, the resulting function is represented locally by polynomials.

Example 4. $H_1(-\infty,+\infty) = \{f(x) \mid f(x)$ is an absolutely continuous real value function in $(-\infty,+\infty)$, $f'(x) \in L^2(-\infty,+\infty)\}$ endowed inner product

$$\langle f(x), g(x)\rangle_{H_1} = f(0)\,g(0) + \int_{-\infty}^{+\infty} f'(x)\,g'(x)\,dx. \tag{12}$$

By applying the procedure of the previous statement, we can construct the equations:

$$\begin{aligned} &R(x,0) = 1, \\ &\partial_y R(x,y) = \begin{cases} \operatorname{Sign}(x), & y \in [0,x] \text{ or } [x,0], \\ 0, & \text{otherwise}. \end{cases} \end{aligned} \tag{13}$$

Then, the reproducing kernel of $H_1(-\infty,+\infty)$ is given by the following.

(1) When $x < 0$

$$R(x,y) = \begin{cases} 1, & 0 \le y, \\ -y+1, & x < y \le 0, \\ -x+1, & y \le x. \end{cases} \tag{14}$$

(2) When $x = 0$, $R(x,y) = 1$.

(3) When $x > 0$

$$R(x,y) = \begin{cases} 1, & y \le 0, \\ y+1, & 0 < y \le x, \\ x+1, & x \le y. \end{cases} \tag{15}$$

It can be obviously obtained that for any $x \in R$, we have

$$R(x,y) = \frac{1}{2}\left(|y| + |x| - |y-x|\right) + 1. \tag{16}$$

The graph of the $R(x,y)$ of $H_1(-\infty,+\infty)$ up to $y = 1, -1$ is presented in Figures 1 and 2, and it shows the curvilinear figure of $R(x,y)$.

Example 5. $H_3[0,+\infty) = \{f(x) \mid f''(x)$ is an absolutely continuous real value function in $[0,+\infty)$, $f'''(x) \in L^2[0,+\infty)\}$ endowed inner product

$$\begin{aligned}\langle f(x), g(x)\rangle_{H_3} &= f(0)\,g(0) + f'(0)\,g'(0) \\ &\quad + f''(0)\,g''(0) + \int_0^{+\infty} f'''(x)\,g'''(x)\,dx.\end{aligned} \tag{17}$$

Similarly, the $R(x,y)$ of $H_3[0,+\infty)$ satisfying the following:

$$\begin{aligned} &R(x,0) = 1, \\ &\partial_y R(x,0) = x, \\ &\partial_y^2 R(x,0) = \frac{1}{2}x^2, \\ &\partial_y^3 R(x,y) = \begin{cases} \dfrac{1}{2}(x-y)^2, & y \le x, \\ 0, & y > x \end{cases} \end{aligned} \tag{18}$$

can be obtained; namely,

$$R(x,y) = \begin{cases} 1 + xy + \dfrac{1}{4}x^2y^2 + \dfrac{1}{12}x^2y^3 - \dfrac{1}{24}xy^4 + \dfrac{1}{120}y^5, \\ \qquad y \le x, \\ 1 + xy + \dfrac{1}{4}x^2y^2 + \dfrac{1}{12}x^3y^2 - \dfrac{1}{24}x^4y + \dfrac{1}{120}x^5, \\ \qquad y > x. \end{cases} \tag{19}$$

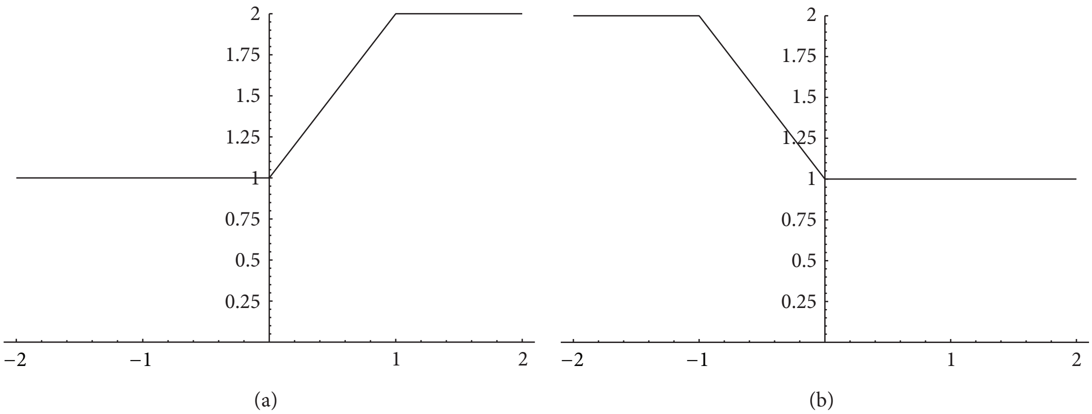

FIGURE 1: The curves are, respectively, $R(x, 1)$ and $R(x, -1)$ for Example 4, where $x \in [-2, 2]$.

4. A Concrete Application to Boundary Value Problems

4.1. Application to Initial Value Problems. Actually, when we apply reproducing kernel theory to solve problems with boundary value conditions [16, 17], it is important to find the representation of reproducing kernel in the appropriate reproducing kernel space. In this section, we show how to express reproducing kernel function in terms of reproducing kernel space with boundary value condition on infinite interval.

Set $D = (-\infty, +\infty)$. Let $H(D)$ be a reproducing kernel Hilbert space and $R(x, y)$ its reproducing kernel function. As is well known,

$$H_0(D) = \{f(x) \mid f(x_0) = 0,\ x_0 \in D,\ f \in H(D)\} \quad (20)$$

is the closed subspace of $H(D)$.

Lemma 6. *If $H_0(D)$ is the proper subspace of $H(D)$, then for any $y \in D$, $R(x, y) \notin H_0(D)$; namely, $R(x_0, y) \neq 0$.*

Proof. The proof is by contradiction. Suppose that $R(x, y) \in H_0(D)$. Then for a fixed x_0 it holds $R(x_0, y) = 0$. Choosing $f(x) \in H(D)$, we have

$$f(x_0) = \langle f(y), R(x_0, y)\rangle_H = \langle f(y), 0\rangle_H = 0, \quad (21)$$

which infers $f(x) \in H_0(D)$. It is conflict about proper subspace. □

Lemma 7. *The orthogonal complement space of $H_0(D)$ about $H(D)$ is*

$$H_0^{\perp}(D) = \{g(x) \mid g(x) = \lambda R(x, x_0),\ \lambda \in C\}. \quad (22)$$

Proof. Due to the only restriction of $H_0(D)$, it means that $H_0^{\perp}$ is one dimensional space. Moreover, for any $f(x) \in H_0(D)$ we get

$$\langle f(x), R(x, x_0)\rangle_{H_0} = f(x_0) = 0; \quad (23)$$

Then, it indicates $R(x, x_0) \perp H_0(D)$. Therefore, we have $R(x, x_0) \in H_0^{\perp}(D)$, and $H_0^{\perp}(D)$ satisfies (22). □

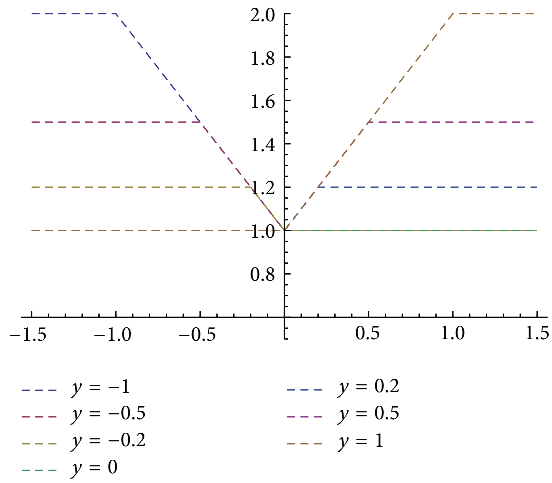

FIGURE 2: Image of $R(x, y)$ for Example 4.

By virtue of the orthogonal decomposition property of reproducing kernel, the reproducing kernel function $K(x, y)$ of $H_0(D)$ can be represented as

$$R(x, y) = K(x, y) + \lambda R(x, x_0). \quad (24)$$

So under the assumption that $K(x_0, y) = 0$, we can introduce

$$\lambda = \frac{R(x_0, y)}{R(x_0, x_0)}. \quad (25)$$

Here, according to Lemma 6, we know that $R(x_0, x_0) \neq 0$.

Now, the formula of reproducing kernel function of $H_0(D)$ can be obtained by

$$K(x, y) = R(x, y) - \frac{R(x_0, y)\, R(x, x_0)}{R(x_0, x_0)}. \quad (26)$$

Example 8. This problem corresponds to the closed subspace of $H_3[0, +\infty)$ in Example 5.

$H_3^0[0, +\infty) = \{f(x) \mid f''(x)$ is an absolutely continuous real value function in $[0, +\infty)$, $f'''(x) \in L^2[0, +\infty)$ and $f(10) = 0\}$.

Here, we simply use formula (26) to find the reproducing kernel function $K(x, y)$ of $H_3^0[0, +\infty)$

$$K(x, y) = \frac{R(10, 10) R(x, y) - R(10, y) R(x, 10)}{R(10, 10)}, \quad (27)$$

where $R(x, y)$ is determined by (19).

4.2. Application to Multipoint Boundary Value Problem. To see that the previous results can be generalized, let $H(D)$ be a reproducing kernel space with kernel $R(x, y)$. For any finite point set $\{x_1, x_2, \ldots, x_m\} \subset D$, we get

$$H_0(D) = \{f(x) \mid f(x_i) = 0,\ i = 1, 2, \ldots, m,\ f \in H(D)\}, \quad (28)$$

which is the closed subspace of $H(D)$. Because, for any $f \in H_0(D)$,

$$\langle f(\cdot), R(x_i, \cdot)\rangle_{H_0} = f(x_i) = 0 \quad i = 1, 2, \ldots m, \quad (29)$$

we know that $R(x_i, x) \in H_0^{\perp}(D)$, where $H_0^{\perp}(D)$ is m dimensional orthogonal complement space of $H(D)$.

Lemma 9. *If, for any $n \in N$ and differential points $x_1, x_2, \ldots, x_n$, the valuation functional $x_1^*, x_2^*, \ldots, x_n^*$ of reproducing kernel space $H(D)$ are linearly independent, then the reproducing kernel function $R(x, y)$ is positive definite.*

Proof. Suppose that $R(x, y)$ is not positive definite. Then there exist differential points $x_1, x_2, \ldots, x_n$ and nonzero vector $u = (u_1, u_2, \ldots, u_n) \in C^n$ such that

$$\begin{aligned}&\sum_{i,j} \overline{u_i} u_j R(x_i, x_j)\\ &\quad = \left\langle \sum_i \overline{u_i} R(x_i, \cdot), \sum_j \overline{u_j} R(x_j, \cdot) \right\rangle_H = 0.\end{aligned} \quad (30)$$

It follows $\sum_i \overline{u_i} R(x_i, \cdot) \equiv 0$. Therefore, for any $f \in H(D)$, we have

$$\begin{aligned}\left(\sum_i u_i x_j^*\right)(f) &= \sum_i u_i f(x_i)\\ &= \sum_i u_i \langle f, R(x_i, \cdot)\rangle_H\\ &= \left\langle f, \sum_i \overline{u_i} R(x_i, \cdot) \right\rangle_H = 0.\end{aligned} \quad (31)$$

It indicates that $x_1^*, x_2^*, \ldots, x_n^*$ are linearly dependent, which can be a contradiction. □

Theorem 10. *The reproducing kernel function $K(x, y)$ of $H_0(D)$ has the form*

$$K(x, y) = R(x, y) + \sum_{i=1}^{m} \lambda_i R(x, x_i). \quad (32)$$

Proof. It needs to be proven that $R(x, x_i)$ $(i = 1, 2, \ldots, m)$ is the basis of $H_0^{\perp}(D)$; namely, $R(x, x_i)$ are linearly independent.

Let $\sum_{i=1}^{m} u_i R(x, x_i) = 0$. Then $\sum_{i=1}^{m} u_i R(x_i, x_j) = 0$ $(j = 1, 2, \ldots, m)$. Applying Lemma 9, we get that matrix $(R(x_i, x_j))_{m\times m}$ is positive definite. So, $u_i = 0$ $(i = 1, 2, \ldots, m)$, and formula (32) holds. Meanwhile, according to $K(x, y) \in H_0(D)$, we can obtain the uniquely defined λ_i. □

Example 11. Here we apply the proposed method to solve the following simplified model of propagation of nonadiabatic flames inside long tubes [18–22]:

$$\begin{gathered}-x''(t) + cx'(t) + \lambda x(t) = f(t, x(t), x'(t)),\\ t \in (0, \infty),\\ x(0) - \alpha x(\eta) = a_0, \qquad \lim_{t\to+\infty} \frac{x'(t)}{e^{rt}} = b_0.\end{gathered} \quad (33)$$

Firstly, due to the complex boundary conditions of (33), the following reproducing kernel space is constructed.

$H = \{x(t) \mid x''(t)$ is absolutely continuous in $[0, +\infty)$ and $\lim_{t\to\infty}(x'(t)/e^{rt}) = 0$, $\int_0^{+\infty} e^{-rt}[x^{(3)}(t)]^2 dt < +\infty\}$. The inner product is defined, respectively, by

$$\begin{aligned}\langle x(t), y(t)\rangle_H &= \sum_{i=0}^{2} x^{(i)}(0)\, y^{(i)}(0)\\ &\quad + \int_0^{+\infty} e^{-rt} x^{(3)}(t)\, y^{(3)}(t)\, dt.\end{aligned} \quad (34)$$

Suppose that $R(t, s)$ is the reproducing kernel of H. According to the previous method for computation of reproducing kernel, we can get

$$R(t, s) = \begin{cases} r_1(t, s) = e^{rt}\sum_{i=1}^{3} a_{1i}(s)\, t^{i-1} + \sum_{i=4}^{6} a_{1i}(s)\, t^{i-4}, & t \le s,\\ r_2(t, s) = e^{rt}\sum_{i=1}^{3} a_{2i}(s)\, t^{i-1} + \sum_{i=4}^{6} a_{2i}(s)\, t^{i-4} & s \le t.\end{cases} \quad (35)$$

For $r = 1$, the concrete expression of $R(t, s)$ is given as follows:

$$\begin{aligned}R(t, s) = \frac{1}{2}\Big[&e^s\left(12 + 6t + t^2 - 2(3 + t)s\right)\\ &+ e^t\left(12 - 6t + t^2 - 2(t - 3)s + s^2\right)\\ &- 10 - 6t - t^2 - 6s\\ &- 2ts - t^2 s - s^2 - ts^2\\ &+ e^{(1/2)(t+s+|t-s|)}\left(6(t - s) - 12 - (t - s)^2\right)\Big].\end{aligned} \quad (36)$$

Secondly, we need to establish the subspace of H:

$$H_0 = \{x(t) \mid x(t) \in H,\ x(0) - \alpha x(\eta) = 0\}. \tag{37}$$

Now, we use the formula (32) to get the reproducing kernel of H_0:

$$K(t,s) = R(t,s) - \frac{f(t) f(s)}{f(0) - \alpha f(\eta)}, \tag{38}$$

where $f(t) = R(t,0) - \alpha R(t,\eta)$ and $R(t,s)$ is the reproducing kernel function of H.

Thirdly, we will prove that $K(t,s)$ in (38) is the reproducing kernel of H_0. For any $x(t) \in H_0$, there holds that

$$\begin{aligned}
&\langle x(t), K(t,s)\rangle_{H_0} \\
&\quad = \left\langle x(t), R(t,s) - \frac{f(t) f(s)}{f(0) - \alpha f(\eta)} \right\rangle_{H_0} \\
&\quad = x(s) - \frac{f(s)}{f(0) - \alpha f(\eta)} \langle x(t), R(t,0) - \alpha R(t,\eta)\rangle_{H_0} \\
&\quad = x(s) - \frac{f(s)}{f(0) - \alpha f(\eta)} [x(0) - \alpha x(\eta)] = x(s).
\end{aligned} \tag{39}$$

For $r = 1$, $\alpha = 2$, and $\eta = 1$, the concrete expression of $K(t,s)$ is given as follows

$$K(t,s) = \begin{cases} k_1(t,s) & s \le t \le \eta, \\ k_2(t,s) & s \le \eta \le t, \\ k_3(t,s) & \eta \le s \le t, \\ k_4(t,s) & \eta \le t \le s, \\ k_5(t,s) & t \le \eta \le s, \\ k_6(t,s) & t \le s \le \eta; \end{cases}$$

$$\begin{aligned}
k_1(t,s) &= \frac{1}{10}\left(-52 + 5s^2(-1+t) - 5e^s\left(12 - 6s + s^2 - t\right)\right. \\
&\quad \times (-1+t) + 61t + 60est - 5t^2 \\
&\quad \left. -5e^t s\left(19 - 8t + t^2\right) + s\left(61 - 68t + 5t^2\right)\right); \\
k_2(t,s) &= \frac{1}{10}\left(-52 + 5s^2(-1+t) - 5e^s\left(12 - 6s + s^2 - t\right)\right. \\
&\quad \times (-1+t) + 61t - 5t^2 - 5es\left(7 - 8t + t^2\right) \\
&\quad \left. +s\left(61 - 68t + 5t^2\right)\right); \\
k_3(t,s) &= \frac{1}{10}\left(-52 + 5s^2(-1+t)\right. \\
&\quad + 61t - 5t^2 + s\left(61 - 68t + 5t^2\right) \\
&\quad + 5e^s\left(12 + s^2 + 6t + t^2 - 2s(3+t)\right) \\
&\quad \left. - 5e\left(7t + s^2 t + s\left(7 - 4t + t^2\right)\right)\right); \\
k_4(t,s) &= k_3(s,t); \qquad k_5(t,s) = k_2(s,t); \\
k_6(t,s) &= k_1(s,t).
\end{aligned} \tag{40}$$

Compared with the procedure for computation of the reproducing kernel in [22], we can see that our method is easier to implement, and it avoids the complexity of δ function.

5. Conclusions and Future Work

To summarize, in this paper, a new method for the calculation of reproducing kernel on infinite interval was introduced, and the representation in polynomial form was obtained for the first time. The scheme was then used to generate the formula for the reproducing kernel in reproducing kernel space with boundary value conditions. We end this paper by mentioning the following applications. On one hand, the approach detailed here can be readily adapted to the case of reproducing kernel on the finite interval. According to the former method in [15], which cannot represent reproducing kernel on the infinite interval in polynomial form, the advantages of the present approach are that we use theory of elementary to avoid the complex operation and numerical algorithm will be much more timesaving. On the other hand, the formula in Section 4 can be used to solve multipoint boundary value problems on the positive half-line, such as in [23–25].

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