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Research Article

The Fractal Dimension of River Length Based on the Observed Data

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Although the phenomenon that strictly meets the constant dimension fractal form in the nature does not exist, fractal theory provides a new way and means for the study of complex natural phenomena. Therefore, we use some variable dimension fractal analysis methods to study river flow discharge. On the basis of the flood flow corresponding to the waterline length, the river of the overall and partial dimensions are calculated and the relationships between the overall and partial dimensions are discussed. The law of the length in section of Chongqing city of Yangtze River is calibrated by using of variable fractal dimension. The results conclude that it does express a second-order accumulated variable-dimensional fractal phenomenon, and the dimension can reflect the degree of the river; the greater dimension, the more the river bend. It has different dimensions at a different location in the same river. In the same river, the larger dimension, the worse flow discharge capacity of the river and the more obvious of the flood will be on the performance.

1. Introduction

A fractal dimension is a ratio providing a statistical index of complexity comparing how detail in a pattern changes with the scale at which it is measured. It has also been characterized as a measure of the space-filling capacity of a pattern that tells how a fractal scales differently than the space it is embedded in; a fractal dimension does not have to be an integer.

Fractals have been introduced in order to quantify the self-similarity observed in nature while at the same time to make the study of nondifferentiable processes possible. Given that this self-similar behavior has often a “local” character, the theory of fractals was generalized to multifractals, enabling the description of more complex phenomena with varying fractal properties. Examples of processes that have been thus treated are the energy dissipation in turbulence and the price increments in finance. In the field of geophysics and atmospheric physics, fractal and multifractal analyses have been extensively applied [1], since self-similarity is present in a wide variety of such phenomena, from the distribution of earthquake epicenters [2–4] and hypocenters [5] to climate change [6] and atmospheric turbulence [7, 8]. In the same field of research, fractal and multifractal methods have been used both to characterize the long-term behavior of related signals and to indicate possible precursors in experimental time series of their records, yielding very promising results [9–12].

Fractal processes have become one of the most widely used modeling tools in science and engineering, with diverse applications in finance, physics, network traffic, and recently geography sciences. In the study of geometric properties of dynamical systems or fractal measures, one is often interested in the asymptotic behaviour of local quantities associated with the underlying dynamical or geometric structure. For example, one is often interested in the ergodic average of a continuous function, the local entropy or the local Lyapunov exponent, or the local dimension of a measure. These quantities provide a description of various aspects of measures or dynamical systems, for example, chaoticity, sensitive dependence, and so forth. All these quantities provide important information about the underlying geometric or

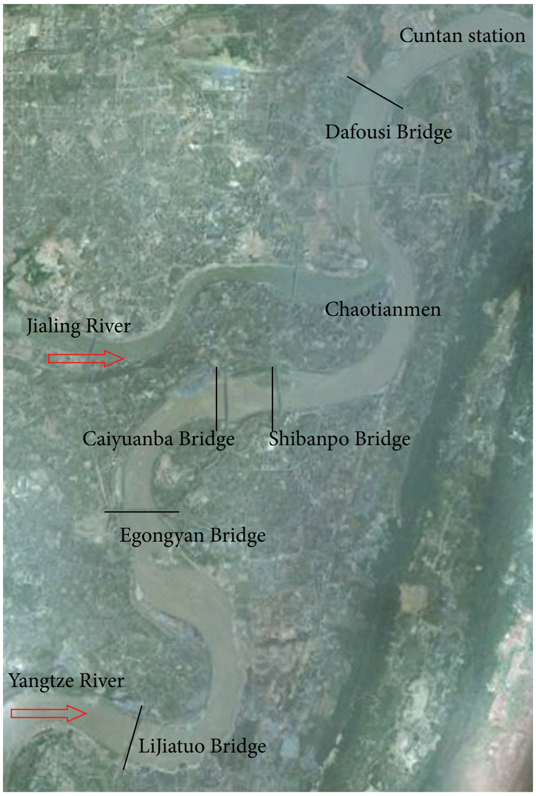


Figure 1: The plan of Chongqing city of Yangtze River.

dynamical structure [13]. The mixing length based on fractal theory has been calculated and analyzed [14]. Jou et al. [15] by assuming a self-similar structure for the Kelvin waves along vortex loops with successive smaller scale features model the fractal dimension of a super fluid vortex tangle in the zero temperature limits. Their model assumes that at each step the total energy of the vortices is conserved but the total length can change. They obtain a relation between the fractal dimension and the exponent describing how the vortex energy per unit length changes with the length scale. In addition, many scholars have also concerned about relationship between the fractal theory and river systems [16–22].

The analysis of river flows has a long history; nevertheless some important issues have been lost. Many scholars use some fractal analysis methods to study river flow fluctuations. Sadegh Movahed and Hermanis [23] have studied one component of the climate system, the river flux, by using the novel approach in the fractal analysis like detrended fluctuation analysis, fourier-detrended fluctuation analysis and scaled windowed variance analysis methods. The statistical and fractal analysis of river flows should be an important issue in the geophysics and hydrological systems to recognize the influence of environmental conditions and to detect the relative effects. A set of most important results which can be given by using statistical tools are as follows: a concept of scale self-similarity for the topography of Earth's surface [24], the hydraulic-geometric similarity of river system and floods forced by the heavy rain [25], and so forth. Already more than half a century ago the engineer Hurst found that runoff records from various rivers exhibit "long-range statistical dependencies." Later, such long-term correlated fluctuation behavior has also been reported for many other geophysical records including precipitation data. These original approaches exclusively focused on the absolute values or the variances of the full distribution of the fluctuations, which can be regarded as the first and second moments of detrended fluctuation analysis [24, 26]. In the last decade it has been realized that a multifractal description is required for a full characterization of the runoff records [27]. This multifractal description of the records can be regarded as a "fingerprint" for each station or river, which, among other things, can serve as an efficient nontrivial test bed for the state-of-the-art precipitation-runoff models.

In this work, the fractal dimension of the river (hereinafter referred to as dimension) is generated by studying characteristics of fractal structure. The dimension of the river is divided into river length and river network, to explore the fractal dimension from the view of the entire river length, to be called the unitary dimensions of river length. However, it has different dimensions at a different location in the same river. Meandering is different, so the dimension of rivers length (hereinafter referred to as part dimension) is, even vary considerably.

Chongqing is an oversize industrial city and the water-land transport hub that developed relying on the Yangtze River and Jialing River. The main section of Chongqing city is from Daduko to Tongluoxia; tributary section is from Jingkou of Jialing River to Chaotianmen. The total length of it is about 60 km. The section of Chongqing city of Yangtze River is located in the southeastern edge of Sichuan basin; the section of river is the lotus root shape. Because the river is affected by geological structure and its lithology changes, the longitudinal profile along the higher ups and downs changes greatly, and the river boundary conditions are very complicated as the section of Yangtze River is located in fluctuating backwater area of Three Gorges Project. Based on statistic water surface profile when flow discharge is 58000 m^3/s of Cuntan station, the average river width of Jiulongpo and Caiyuanba is more than 900 m, the total length of the two sections is about 6 km, and the length of narrow reach which is below 600 m is 1.7 km. The reach in the section of Chongqing city performance is of continuous and irregular curve shape, which has 6 continuous bends. These curves are slowly bending 150° and 90° elbow. Chongqing section of the Yangtze River in 2007 is shown in Figure 1.

At present, the research on the fractal characteristics of river length in section of Chongqing city of Yangtze River is few, and it is still in the theoretical exploration and analysis phase. Chongqing is located in the upper reaches of the Yangtze River, is an important strategic position, and is the largest port city in the upper reaches of the Yangtze River. Therefore, measurement on fractal dimension of river length in section of Chongqing city of Yangtze River can give full play to the effect of golden waterway in Yangtze River. It will play a leading role in water transport of Chongqing,

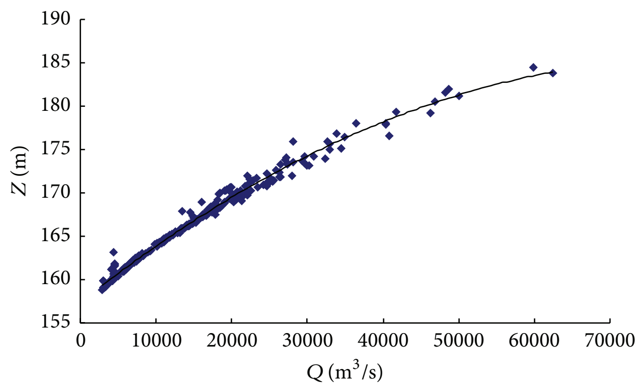


FIGURE 2: Discharge and water level relationship curve.

and provide better services for the upper reaches of Yangtze River and the western region. Thus, the study of the fractal dimension of river length in section of Chongqing city of Yangtze River is of great significance. In this paper, in the view of river length, the fractal dimension and its relationship with floods will to be studied.

2. Methods

2.1. Building of 1D Model Program. The former USSR, North America, Western Europe, and China have carried out research and application of hydrodynamic, sediment transport model since the 1950s. A one-dimensional mathematical model of water flow has been more mature after several decades' development and application.

This research is studied through one-dimensional flow mathematical model. In view of the constant flow calculation which has been more mature, a one-dimensional constant flow mathematical model is taken to the whole river.

2.1.1. Basic Equations. Here is a 1D water movement and continuity equation:

$$\begin{gathered} \frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0, \\ \frac{\partial Q}{\partial t} + \frac{\partial UQ}{\partial x} + gA\frac{\partial Z}{\partial x} = -\frac{B}{\rho}\tau_b, \end{gathered} \tag{1}$$

where A is cross-section area, Q is flow discharge, U is section average flow velocity, Z is water level, B is river width, x is horizontal direction, t is time, ρ is water density, and τ_b is shear stress of river bottom.

Simplified equations is used in the calculation process, and water movement is changed to

$$\begin{gathered} \frac{\partial Q}{\partial x} = 0, \\ Z_2 + \frac{\alpha_2 V_2^2}{2g} = Z_1 + \frac{\alpha_1 V_1^2}{2g} + h_f + h_j, \end{gathered} \tag{2}$$

where Z_1 is downstream section level, Z_2 is upstream section level, V_1 is downstream section average velocity, V_2 is upstream section average velocity, α_1 is downstream section kinetic energy correction factor, α_2 is upstream section kinetic energy correction factor, h_f is frictional head loss, and h_j is local head loss.

2.1.2. Boundary Condition. Control conditions of this model are the flow discharge of upstream and water level of downstream. Stage-discharge data of Cuntan station is the boundary condition of downstream. According to the hydrological data in 2011 and 2012 of Cuntan station, stage-discharge relation of Cuntan station is shown in Figure 2 and Table 1.

2.1.3. Model Verification. Roughness is vital to the calculation of water surface profile. Roughness must adjust and revise repeatedly until to the deviation between calculated value and measured value. Finally, the ideal roughness is 0.036.

There is water level date from May 1, 2009, to October 17 of the section of Chongqing city. When water level of downstream is 161.66 m, flow discharge of Cuntan station is 6320 m^3/s. The water level of calculation and observation is shown in Table 2.

From Table 2 the following can be seen. The calculation water level of the section of Chongqing city is similar to the observe water level. The result of calculation is reasonable.

In this work, 1D model program mentioned above is used to calculate the length of waterline (L) corresponding to 25 class flow discharge (Q).

2.2. Building of Fractal Dimension. According to the definition of Mandelbrot, fractal refers to the body that the part is similar to the whole in some way. Mandelbrot (1967) put forward the formula of a statistical fractal dimension estimated for the self-similar fractal case:

$$L = AQ^{-D}, \tag{3}$$

where L is Euclidean length (waterline), Q is the measured size (flow discharge), D is the fractal dimension, and A is a proportion constant.

Take the natural logarithm to (3), then get it as follows:

$$\ln L = \ln A - D \ln Q. \tag{4}$$

And then paint $\ln L$ and $\ln Q$ on the coordinates of y-axis and x-axis, respectively, and last, use the least square method to fit the straight line and its slope is $-D$. We can derive its fractal dimension.

If the fractal dimension is calculated as a constant, it is simple fractal dimension, and if not, it needs to be described as variable fractal dimension [28]. In fact, the phenomenon of strictly meet the simple fractal dimension form does not exist in nature, a large number of complex phenomena need to use variable fractal dimension to describe.

In this thesis, fractal theory of cumulative sum sequence is used for calculation.

This specific method steps are as follows.

(1) Determine the raw data (L_i, Q_i), where Q_i orders from small to large, $i = 1, 2, \ldots, n$. There are some data

Table 1: The water level-discharge line in Chongqing Cuntan station (Yellow Sea Elevation).

Z (m)	158.784	159.125	159.464	159.801	160.136	160.469	160.8	161.129	161.456	161.781	162.104	162.425	162.744
Q (m^3/s)	2000	2500	3000	3500	4000	4500	5000	5500	6000	6500	7000	7500	8000
Z (m)	163.061	163.376	163.689	164	167	169.8	172.4	174.8	177	179	180.8	182.4	183.8
Q (m^3/s)	8500	9000	9500	10000	15000	20000	25000	30000	35000	40000	45000	50000	55000

Table 2: The water level of calculate and observation.

Section	Observation	Calculation	Deviation
Dafousi	162.11	162.204	0.094
Hudie Dam	164.86	164.80	−0.060

points in log-log coordinates. And then calculate the slope $l_{i,i+1}$ of two adjacent points by the use of (5); the sub variable fractal dimension is $D_{i,i+1} = -l_{i,i+1}$. In general, the fractal dimensions change much and there is no law:

$$D_{i,i+1} = \frac{\ln(L_i/L_{i+1})}{\ln(Q_{i+1}/Q_i)}. \quad (5)$$

(2) Construct the accumulated sum of a total order, and $(L_1, L_2, L_3, \ldots)$ is the basic sequence. Then it is constructed by following the rules:

$$\{S1_i\} = \{L_1, L_1 + L_2, L_1 + L_2 + L_3, \ldots\}, \quad i = 1, 2, \ldots, n, \quad (6)$$

$$\{S2_i\} = \{S1_1, S1_1 + S1_2, S1_1 + S1_2 + S1_3, \ldots\}, \quad i = 1, 2, \ldots, n, \quad (7)$$

$$\{S3_i\} = \{S2_1, S2_1 + S2_2, S2_1 + S2_2 + S2_3, \ldots\}, \quad i = 1, 2, \ldots, n, \quad (8)$$

where $S1$, $S2$, $S3, \ldots$ are the accumulated sum of first order, second order, and third order, $N = 1, 2, 3, \ldots$

(3) Establish variable fractal dimension model of the accumulated sum of a total order, taking the first order as an example, and the variable fractal dimension is the opposite slope of data points calculated by (6) in the log-log coordinates.
According to the data of n, fractal dimensions of $n - 1$ are obtained, known as the fractal dimension sequence. $DN_{i,i+1}$ is variable fractal dimension sequence of the accumulated sum of a total order, $N = 1, 2, \ldots$; $i = 1, 2, \ldots, n - 1$.

(4) Determine the better order of the accumulated sum, and identify the corresponding fractal dimension.

3. Data and Results

Based on the field observations of topographic map in the Chongqing section of the Yangtze River in 2007 (Figure 1), a valuable data and analysis results by (4) are given. In this paper, the results are shown in Figure 3.

From the double logarithmic coordinates it can be seen that (Figure 3) the data point is clearly not a straight line. From the analysis of it, the relationship that the river length of Chongqing section of the Yangtze River was able to meet the variable fractal dimension should be applicable to subdimensional variable fractal model. Therefore, we have adopted variable fractal model to calculate the fractal dimension. The calculation result of the fractal dimension during median flood period on the left bank $D2$ is −0.3321, the correlation coefficient R is 0.9855; the fractal dimension on the right bank $D2$ is −0.3323, the correlation coefficient R is 0.9854. The calculation result of the fractal dimension during flood period on the left bank $D2$ is −1.6844, and the correlation coefficient R is 0.9834; the fractal dimension on the right bank $D2$ is −1.6859, and the correlation coefficient R is 0.9834 (Tables 3 and 4).

From Figure 4 it can be seen that, after the transformation of second-order accumulated and variable dimensional fractal, the data points fit better to a straight line, which means that the river length in section of Chongqing city of Yangtze River has the characteristics of second-order accumulated variable dimensional fractal. Thus, the river length in the section of Chongqing city of Yangtze River has characteristics of second-order fractal dimension.

4. Analysis and Discussion

The fractal dimension of the river length reflects the degree of bending of the river. It is shown that the greater the fractal dimension of the river length, the more tortuous of the river. On the contrary, the river is straighter. It has different dimensions of the river length and the degree of bending at a different location in the same river. In terms of the flood, the possibility and intensity of flooding in a different reach are different. Thus, calculating the fractal dimension values of the whole river length has little significance. The river segmentation being carried out, which calculated the value of the fractal dimension in different sections, found out the correlation between the fractal dimension of each reach and flood.

From the qualitative analysis of the possibility of flood and its fractal dimension on various river reaches, it is shown that there are some relationships between them. The greater fractal dimensions of the river length, the more tortuous of the river. The worse flood carrying capacity of the rivers, the more obvious the flood will be on the performance. However, from quantitative analysis, what kind of relationship exists between the fractal dimension of river length and the flood? Based on Chongqing city segments of Yangtze River, the

Table 3: Result of subdimensional fractal dimension of Chongqing city of Yangtze River left bank.

	Q (m^3/s)	L_i/m	$D_{i,i+1}$	$S1_i$	$D1_{i,i+1}$	$S2_i$	$D2_{i,i+1}$
Median flood	2000	29743.12	—	29743.12	—	29743.12	—
	2500	29725.56	504.65	59468.68	−0.44	89211.80	−0.29
	3000	29708.00	401.50	89176.68	−0.64	178388.48	−0.39
	3500	29712.25	−1371.56	118888.93	−0.77	297277.41	−0.46
	4000	29716.51	−1167.50	148605.44	−0.86	445882.85	−0.51
	4500	29720.76	−1014.47	178326.21	−0.93	624209.06	−0.55
	5000	29725.02	−895.65	208051.22	−0.98	832260.28	−0.58
	5500	29728.44	−995.69	237779.66	−1.03	1070039.95	−0.61
	6000	29731.86	−899.53	267511.53	−1.06	1337551.47	−0.63
	6500	29735.28	−819.68	297246.81	−1.09	1634798.28	−0.65
	7000	29738.71	−752.37	326985.52	−1.12	1961783.80	−0.66
	7500	29678.17	39.23	356663.68	−1.14	2318447.48	−0.68
	8000	29617.63	36.34	386281.31	−1.16	2704728.79	−0.69
	8500	29557.09	33.83	415838.40	−1.18	3120567.19	−0.70
	9000	29496.55	31.62	445334.95	−1.19	3565902.15	−0.71
	9500	29456.19	44.50	474791.14	−1.21	4040693.29	−0.72
Flood	10000	29415.83	—	474750.79	—	4040652.93	—
	15000	29274.54	92.04	504025.33	−9.43	4544678.26	−5.60
	20000	29133.25	62.65	533158.58	−6.90	5077836.83	−4.09
	25000	29105.34	238.94	562263.91	−5.54	5640100.75	−3.29
	30000	29077.42	191.15	591341.34	−4.69	6231442.09	−2.79
	35000	29079.26	−2412.46	620420.60	−4.12	6851862.69	−2.45
	40000	29081.10	−2061.36	649501.70	−3.70	7501364.39	−2.21
	45000	29602.35	−6.40	679104.05	−3.32	8180468.44	−2.02
	50000	29123.60	6.17	708227.65	−3.13	8888696.09	−1.88
	55000	29144.84	−123.62	737372.49	−2.93	9626068.58	−1.77

Table 4: The second-order accumulated fractal dimension of the measured.

Station	Linear correlation equation	Dimension $D2$	Relative number R^2
The leftt bank (median flood)	$y = 0.3321x + 4.0393$	−0.3321	0.9855
The leftt bank (flood)	$y = 1.6884x - 16.166$	−1.6884	0.9834
The right bank (median flood)	$y = 0.3323x + 4.0518$	−0.3323	0.9854
The right bank (flood)	$y = 1.6859x - 16.047$	−1.6859	0.9834

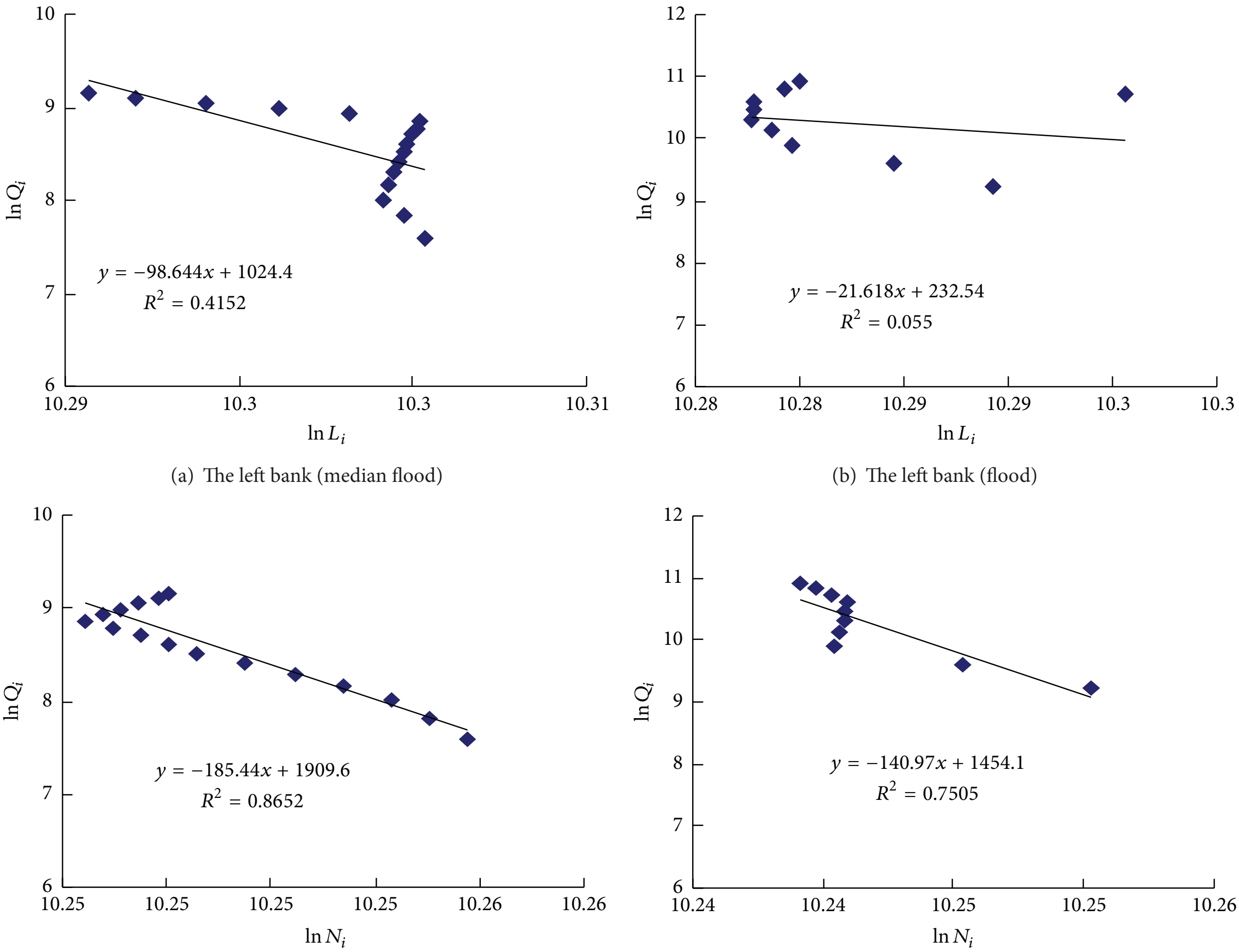
relationship between them is explored by using quantitative calculation and measured data.

4.1. Calculation of Local Fractal Dimension. The section of Chongqing city of Yangtze River is divided into six sections. So we can calculate the fractal dimension of each reach and the correlation coefficient by the left and right sides separately, and the concrete results were shown in Tables 5 and 6.

From Tables 5 and 6 it can be seen that the correlation coefficient on the fractal dimension of the length for each reach in section of Chongqing city of the Yangtze River is more than 98%. It is shown that the length has good characteristics of fractal dimension, and it can reflect the characteristics of it.

In Table 5, the fractal dimensions during flood period on the left bank of river length for the three reaches of Lijiatuo Bridge to Egongyan Bridge, Egongyan Bridge to Caiyuanba Bridge, and Caiyuanba Bridge to Shibanpo Bridge are −1.7203, −1.6844, and −1.6776, respectively. In Table 6, the fractal dimensions on the right bank of river length are −1.7071, −1.6805, and −1.6736, respectively. And the dimension can reflect the degree of bending of the river; the greater dimension, the more tortuous of the river. So we can get that the bending degree of the four reaches of Caiyuanba Bridge to Shibanpo Bridge, Egongyan Bridge to Caiyuanba Bridge, and Lijiatuo Bridge to Egongyan Bridge is more and more big.

4.2. The Local Fractal Dimension and Overall Fractal Dimension. The arithmetic average of river length does not mean the overall fractal dimension values (see Table 7). In order to further reveal the existence of the law, the situation of the level with the left bank in section of Chongqing city of the Yangtze



(a) The left bank (median flood)

(b) The left bank (flood)

(c) The right bank (median flood)

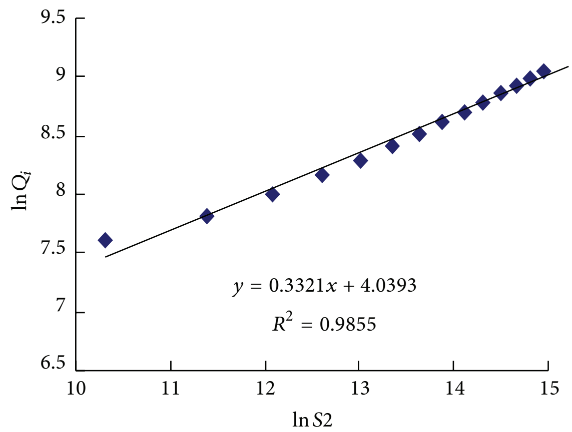
(d) The right bank (flood)

Figure 3: The original subdimensional fractal sequence of Chongqing city of Yangtze River.

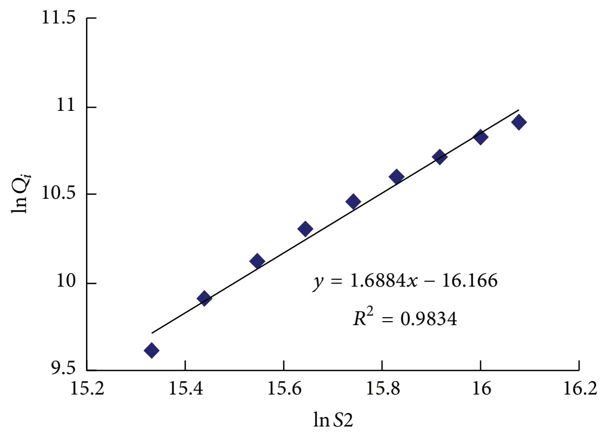
Table 5: The result of second-order accumulated variable-dimensional fractal sequence of Chongqing city of Yangtze River left bank.

	Station	Linear correlation equation	Dimension $D2$	Relative number R^2
Median flood	The upper reaches of the Yangtze River to Lijiatuo bridge	$y = 0.3318x + 4.8736$	−0.3318	0.9855
	Lijiatuo bridge to Ergongyan Yangtze River bridge	$y = 0.3327x + 4.4858$	−0.3327	0.9853
	Ergongyan bridge to Caiyuanba Yangtze River bridge	$y = 0.3328x + 4.6929$	−0.3328	0.9853
	Caiyuanba bridge to Shibanpo Yangtze River bridge	$y = 0.3316x + 5.0926$	−0.3216	0.9859
	Shibanpo bridge to the Big Temple Yangtze River bridge	$y = 0.3315x + 4.4454$	−0.3315	0.9856
	The Big Temple bridge to the lower reaches of the Yangtze River	$y = 0.3318x + 4.6043$	−0.3318	0.9855
Flood	The upper reaches of the Yangtze River to Lijiatuo bridge	$y = 1.6705x - 11.711$	−1.6705	0.9831
	Lijiatuo bridge to Ergongyan Yangtze River bridge	$y = 1.7203x - 14.294$	−1.7203	0.9839
	Ergongyan bridge to Caiyuanba Yangtze River bridge	$y = 1.6844x - 12.606$	−1.6844	0.9831
	Caiyuanba bridge to Shibanpo Yangtze River bridge	$y = 1.6776x - 10.838$	−1.6776	0.9836
	Shibanpo bridge to the Big Temple Yangtze River bridge	$y = 1.6822x - 14.053$	−1.6822	0.9833
	The Big Temple bridge to the lower reaches of the Yangtze River	$y = 1.6766x - 13.155$	−1.6766	0.9832

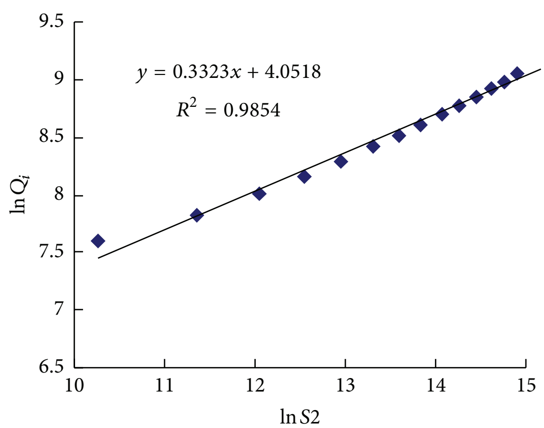
Where $y = \ln(Q_i)$, $x = \ln(S2)$.



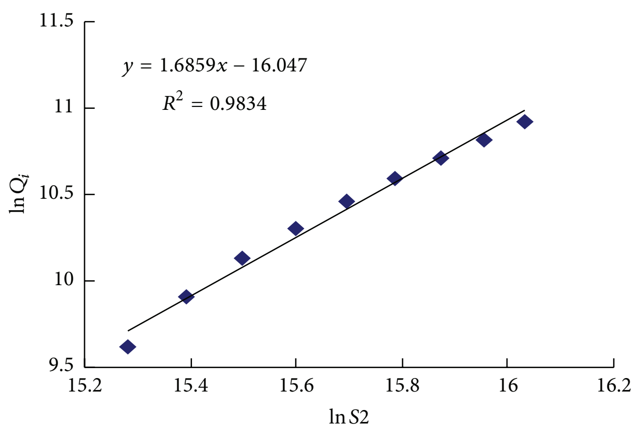
(a) The left bank (median flood)



(b) The left bank (flood)

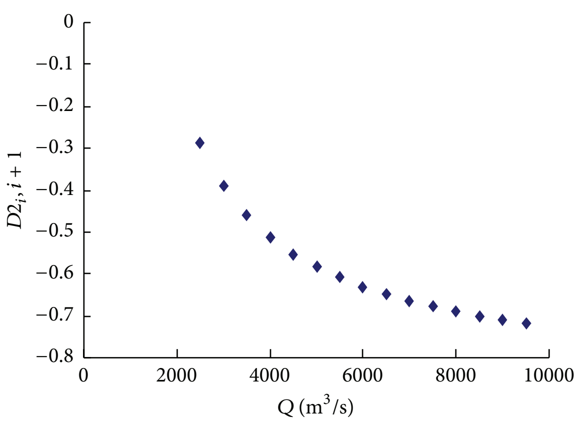


(c) The right bank (median flood)

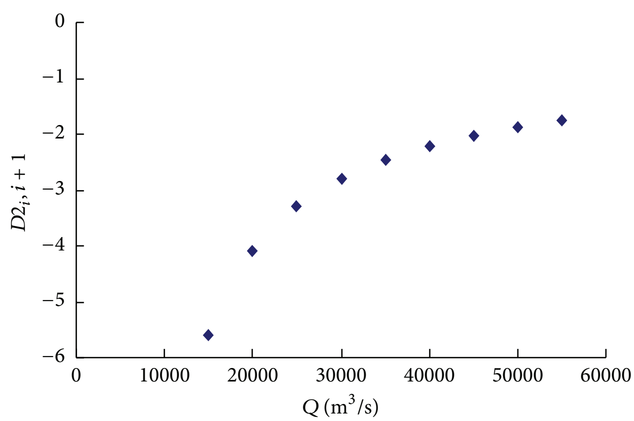


(d) The right bank (flood)

FIGURE 4: Second-order accumulated variable-dimensional fractal sequence of Chongqing city of Yangtze River.



(a) Median flood



(b) Flood

FIGURE 5: Second-order accumulated variable-dimensional fractal-flow discharge of Chongqing city of Yangtze River left bank.

River is calculated. The fractal dimensions with the upper reaches of the Yangtze River to Lijiatuo bridge and Lijiatuo bridge to Egongyan bridge are −0.3318, −0.3327, respectively. From the upper reaches of the Yangtse River to Egongyan bridge, the arithmetic mean is −0.3322, and it is not equal to fractal dimension −0.3325 (Table 5). The results are also consistent with other reaches of the river and the right bank. So, it can be found that the fractal dimension of length in section of Chongqing city of the Yangtze River is not equal to its part fractal dimension of the arithmetic mean.

TABLE 6: The result of second-order accumulated variable-dimensional fractal sequence of Chongqing city of Yangtze River right bank.

	Station	Linear correlation equation	Dimension $D2$	Relative number R^2
Median flood	The upper reaches of the Yangtze River to Lijiatuo bridge	$y = 0.3326x + 4.873$	−0.3326	0.9853
	LijJiatuo bridge to Ergongyan Yangtze River bridge	$y = 0.332x + 4.4746$	−0.3320	0.9855
	Ergongyan bridge to Caiyuanba Yangtze River bridge	$y = 0.3286x + 4.7992$	−0.3286	0.9852
	Caiyuanba bridge to Shibanpo Yangtze River bridge	$y = 0.332x + 5.0899$	−0.3320	0.9855
	Shibanpo bridge to the Big Temple Yangtze River bridge	$y = 0.3319x + 4.4296$	−0.3319	0.9855
	The Big Temple bridge to the lower reaches of the Yangtze River	$y = 0.3328x + 4.6864$	−0.3328	0.9853
Flood	The upper reaches of the Yangtze River to Lijiatuo bridge	$y = 1.7111x - 12.181$	−1.7111	0.9839
	Lijiatuo bridge to Ergongyan Yangtze River bridge	$y = 1.7071x - 13.804$	−1.7071	0.9832
	Ergongyan bridge to Caiyuanba Yangtze River bridge	$y = 1.6805x - 12.625$	−1.6805	0.9835
	Caiyuanba bridge to Shibanpo Yangtze River bridge	$y = 1.6736x - 10.772$	−1.6736	0.9832
	Shibanpo bridge to the Big Temple Yangtze River bridge	$y = 1.6802x - 14.081$	−1.6802	0.9833
	The Big Temple bridge to the lower reaches of the Yangtze River	$y = 1.6931x - 12.889$	−1.6931	0.9834

Where $y = \ln(Q_i)$, $x = \ln(S2)$.

TABLE 7: The result of average arithmetic dimension to overall and partial fractal of Chongqing city of Yangtze River.

Fractal dimension	Left bank (median flood/flood)	Right bank (median flood/flood)
The average arithmetic partial dimension	−0.3304/−1.6853	−0.3317/−1.6909
Overall dimension	−0.3321/−1.6884	−0.3323/−1.6859

4.3. The Relationship between Fractal Dimension and the Flow Discharge. In general, from Tables 5 and 6 and Figure 5, the dimension during median flood period is smaller than the dimension during flood period in the same observation station. And the absolute value of dimension during median flood period is inversely proportional to the flow discharge.

Some theory can be derived from the relationship between stage and discharge at the customs of Chongqing Cuntan hydrological station (Table 1); the larger flow, the higher level in the same observation station. Conversely, the greater dimension, the higher flow and the more obvious the flood will be on the performance. This just confirms the relationship between the possibility of flood and fractal dimension: the greater dimension, the more the river bend, and the larger dimension, the worse flow discharge capacity of the river and the more obvious the flood will be on the performance.

5. Conclusions

In this paper, taking the measured data in section of Chongqing city of Yangtze River as an example explored the fractal characteristics from the perspective of fractal scale. Through analysis, comparison, and discussion in this paper, it draws the following conclusions.

(1) The phenomenon of variable dimension fractal, with second-order fractal dimension, exists on the main reaches in section of Chongqing city of the Yangtze River. The fractal dimension value during median flood period of the left bank is −0.3321, the right bank is −0.3323, the fractal dimension value during flood period of the left bank is −1.6884, and the right bank is −1.6859.

(2) The dimension can reflect the degree of bending of the river; the greater dimension, the more tortuous of the river. It can be got that the bending degree of the three reaches of Caiyuanba Bridge to Shibanpo Bridge, Egongyan Bridge to Caiyuanba Bridge, and Lijiatuo Bridge to Egongyan Bridge is more and more big.

(3) The fractal dimension of length in section of Chongqing city of the Yangtze River is not equal to its part fractal dimension of the arithmetic mean.

(4) In the same river, the larger dimension, the more obvious the flood will be on the performance. Therefore, the fractal dimension of the river can be used as a quantitative indicator of flood forecasting. The larger fractal dimensions, the worse capacity of flood carrying. However, due to the impact of floods produced by many factors, such as water level and sediment, the fractal dimension of the river can only be one of the indicators as forecasting floods. Considered, we should identify more predictors of faster, more accurate prediction of flood.

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