

Research Article

State Estimation for Neural Networks with Leakage Delay and Time-Varying Delays

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Received 31 July 2013; Accepted 1 September 2013

Academic Editor: Jinde Cao

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The state estimation problem is investigated for neural networks with leakage delay and time-varying delay as well as for general activation functions. By constructing appropriate Lyapunov-Krasovskii functionals and employing matrix inequality techniques, a delay-dependent linear matrix inequalities (LMIs) condition is developed to estimate the neuron state with some observed output measurements such that the error-state system is globally asymptotically stable. An example is given to show the effectiveness of the proposed criterion.

1. Introduction

In the past few years, neural networks have been extensively studied and successfully applied in many areas such as combinatorial optimization, signal processing, associative memory, affine invariant matching, and pattern recognition [1]. In such applications, the stability analysis is a necessary step for the practical design of neural networks [2]. In hardware implementation, time delays occur due to finite switching speed of the amplifiers and communication time. The existence of time delay may lead to some complex dynamic behaviors such as oscillation, divergence, chaos, instability, or other poor performance of the neural networks [3]. Therefore, the issue of stability analysis of neural networks with time delay attracts many researchers, and a number of remarkable results have been built up in the open literature; for example, see [2–5] and references therein.

When a neural network is designed to handle complex nonlinear problems, a great number of neurons with tremendous connections are often required. In such relatively large-scale neural networks, it may be very difficult and expensive (or even impossible) to obtain the complete information of the neuron states. On the other hand, in many practical applications, one needs to know the neuron states and then use them to achieve certain objectives. For instance, a recurrent

neural network was presented in [6] to model an unknown nonlinear system, and the neuron states were utilized to implement a control law. Therefore, it is of great importance to study the state estimation problem of neural networks.

Recently, some results related to the state estimation problem for neural networks have been reported; for example, see [7–38] and references therein. In [7], authors initially studied the state estimation problem of delayed neural networks, where a delay-independent condition was obtained in terms of a linear matrix inequality (LMI). In [8], authors proposed a free-weighting matrix approach to discuss the state estimation problem for neural networks with time-varying delay. By using the Newton-Leibniz formula, some slack variables were introduced to derive a less conservative condition. In [11], attention was focused on the design of a state estimator to estimate the neuron states by using the delay-fractioning technique to reduce the possible conservatism. The authors in [13] first investigated the robust state estimator problem of delayed neural networks with parameter uncertainties. Delay-dependent conditions were presented to guarantee the global asymptotical stability of the error system. In [16], a further result on design problem of state estimator for a class of neural networks of neutral type was presented. A delay-dependent LMI criterion for existence of the estimator was derived. In [20], the state estimation problem for discrete

neural networks with partially unknown transition probabilities and time-varying delays was discussed. By utilizing a novel Lyapunov functional integrating both lower and upper time-delay bounds and some new techniques, some delay-range-dependent sufficient conditions under which the estimation error dynamics were stochastically stable are established. In [22], authors investigated the state estimation problem for neural networks with discrete time-varying delay and distributed time-varying delay; a delay-interval-dependent condition is developed to estimate the neuron state with some observed output measurements such that the error-state system was globally asymptotically stable. In [25], leakage delay in the leakage term was used to destabilize the neuron states. By constructing the Lyapunov-Krasovskii functional which contains a triple-integral term, an improved delay-dependent stability criterion was derived in terms of LMIs. In [27], the state estimation problem for a class of discrete-time stochastic neural networks with random delays was considered. By employing a Lyapunov-Krasovskii functional, sufficient delay distribution-dependent conditions were established in terms of LMIs that guarantee the existence of the state estimator. In [33], authors discussed the state estimation problem for Takagi-Sugeno (T-S) fuzzy Hopfield neural networks via strict output passivation of the error system. In [36–38], authors investigated the distributed state estimation problem for sensor networks and presented several new sufficient conditions to guarantee the convergence of the estimation error systems. To the best of the author's knowledge, there are no results on the problem of state estimation for neural networks with leakage delay and time-varying delays. As pointed out in [39], neural networks with leakage delay are a class of important neural networks; time delay in the leakage term also has great impact on the dynamics of neural networks since time delay in the stabilizing negative feedback term has a tendency to destabilize a system. Therefore, it is necessary to investigate the state estimation problem for neural networks with leakage delay [25].

Motivated by the previous discussions, the objective of this paper is to study the state estimation for neural networks with leakage delay and time-varying delays by employing new Lyapunov-Krasovskii functionals and using matrix inequality techniques. The obtained sufficient condition does not require the differentiability of time-varying delays and is expressed in terms of linear matrix inequalities, which can be checked numerically using the effective LMI toolbox in Matlab. An example is given to show the effectiveness of the proposed criterion.

Notations. The notations are quite standard. Throughout this paper, \mathbb{R}^n and $\mathbb{R}^{n \times m}$ denote, respectively, the n -dimensional Euclidean space and the set of all $n \times m$ real matrices. $\|\cdot\|$ refers to the Euclidean vector norm. A^T represents the transpose of matrix A , and the asterisk “*” in a matrix is used to represent the term which is induced by symmetry. I is the identity matrix with compatible dimension. $X > Y$ means that X and Y are symmetric matrices and that $X - Y$ is positive definite. Matrices, if not explicitly specified, are assumed to have compatible dimensions.

2. Model Description and Preliminaries

Consider the following neural networks with leakage delay and time-varying delays:

$$\dot{x}(t) = -Cx(t - \delta) + Af(x(t)) + Bf(x(t - \tau(t))) + J(t) \quad (1)$$

for $t \geq 0$, where $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T \in \mathbb{R}^n$ is the state vector of the network at time t , and n corresponds to the number of neurons; $C \in \mathbb{R}^{n \times n}$ is a positive diagonal matrix, and $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times n}$ are the connection weight matrix and the delayed connection weight matrix, respectively; $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \dots, f_n(x_n(t)))^T \in \mathbb{R}^n$ denotes the neuron activation at time t ; $J(t) \in \mathbb{R}^n$ is an external input vector; δ and $\tau(t)$ denote the leakage delay and time-varying delay, respectively.

Throughout this paper, we make the following assumptions.

(H1) (see [40]) For any $j \in \{1, 2, \dots, n\}$ there exist constants F_j^- and F_j^+ such that

$$F_j^- \leq \frac{f_j(\alpha_1) - f_j(\alpha_2)}{\alpha_1 - \alpha_2} \leq F_j^+ \quad (2)$$

for all $\alpha_1 \neq \alpha_2$.

(H2) (see [25]) The leakage delay δ and time-varying delays $\tau(t)$ satisfy the following conditions:

$$0 \leq \delta, \quad 0 \leq \tau(t) \leq \tau, \quad (3)$$

where δ and τ are constants.

As is well known, information about the neuron states is often incomplete from the network measurements (outputs) in applications, and the network measurements are subject to nonlinear disturbances. Similar to [7], we assume that the network measurements satisfy

$$y(t) = Dx(t) + g(t, x(t)), \quad (4)$$

where $y(t) \in \mathbb{R}^m$ is the measurement output, and $D \in \mathbb{R}^{m \times n}$ is a known constant matrix with appropriate dimension. $g: \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}^m$ is the neuron-dependent nonlinear disturbances on the network outputs and satisfies the following condition.

(H3) (see [22]) For any $j \in \{1, 2, \dots, m\}$ there exist constants G_j^- and G_j^+ such that

$$G_j^- \leq \frac{g_j(t, \alpha_1(t)) - g_j(t, \alpha_2(t))}{\alpha_1(t) - \alpha_2(t)} \leq G_j^+ \quad (5)$$

for $t \geq 0$ and all $\alpha_1(t) \neq \alpha_2(t)$.

Remark 1. As pointed out in [40], the constants F_j^- and F_j^+ in assumption (H1) of this paper are allowed to be positive, negative, or zero. Hence, assumption (H1), first proposed by Liu et al. in [40], is weaker than the Lipschitz condition.

For the delay neural networks (1), we construct the full-order state estimation as follows:

$$\begin{aligned} \dot{u}(t) = & -Cu(t - \delta) + Af(u(t)) \\ & + Bf(u(t - \tau(t))) + J(t) + K(y(t) - v(t)) \end{aligned} \tag{6}$$

$$v(t) = Du(t) + g(t, u(t)), \tag{7}$$

where $u(t) \in \mathbb{R}^n$ is an estimation of the state $x(t)$ of (1), and $K \in \mathbb{R}^{n \times m}$, to be determined, is a gain matrix of the state estimator.

Let the error state be $e(t) = x(t) - u(t)$ then it follows from (1), (4), (6), and (7) that

$$\begin{aligned} \dot{e}(t) = & -KDe(t) - Ce(t - \delta) + Ah(e(t)) \\ & + Bh(e(t - \tau(t))) - Kr(t, e(t)), \end{aligned} \tag{8}$$

where $h(e(t)) = f(x(t)) - f(u(t))$, $r(t, e(t)) = g(t, x(t)) - g(t, u(t))$.

The problem to be addressed in this study is to find out the gain matrix K such that the system (8) is globally asymptotically stable.

To prove our results, the following lemmas that can be found in [22] are necessary.

Lemma 2 (see [22]). *For any constant matrix $W \in \mathbb{R}^{m \times m}$, $W > 0$, scalar $0 < h(t) < h$, and vector function $\omega(\cdot) : [0, h] \rightarrow \mathbb{R}^m$ such that the integrations concerned are well defined, then*

$$\begin{aligned} & \left(\int_0^{h(t)} \omega(s) ds \right)^T W \left(\int_0^{h(t)} \omega(s) ds \right) \\ & \leq h(t) \int_0^{h(t)} \omega^T(s) W \omega(s) ds. \end{aligned} \tag{9}$$

Lemma 3 (see [22]). *Given constant matrices P, Q , and R , where $P^T = P, Q^T = Q$, then*

$$\begin{bmatrix} P & R \\ R^T & -Q \end{bmatrix} < 0 \tag{10}$$

is equivalent to the following conditions:

$$Q > 0, \quad P + RQ^{-1}R^T < 0. \tag{11}$$

3. Main Results

For presentation convenience, in the following, we denote

$$\begin{aligned} F_1 &= \text{diag}(F_1^-, F_2^-, \dots, F_n^-), \\ F_2 &= \text{diag}(F_1^+, F_2^+, \dots, F_n^+), \\ F_3 &= \text{diag}(F_1^- F_1^+, F_2^- F_2^+, \dots, F_n^- F_n^+), \\ F_4 &= \text{diag}\left(\frac{F_1^- + F_1^+}{2}, \frac{F_2^- + F_2^+}{2}, \dots, \frac{F_n^- + F_n^+}{2}\right), \\ G_1 &= \text{diag}(G_1^- G_1^+, G_2^- G_2^+, \dots, G_n^- G_n^+), \\ G_2 &= \text{diag}\left(\frac{G_1^- + G_1^+}{2}, \frac{G_2^- + G_2^+}{2}, \dots, \frac{G_n^- + G_n^+}{2}\right). \end{aligned} \tag{12}$$

Theorem 4. *Assume that the assumptions (H1)–(H3) hold. If there exist four symmetric positive definite matrices P_i ($i = 1, 2, 3, 4$), five positive diagonal matrices W_1, W_2, R_1, R_2 , and R_3 , and seven matrices $Q_1, Q_2, Q_3, X_{11}, X_{12}, X_{22}$, and Z , such that the following LMIs hold:*

$$X = \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} > 0, \tag{13}$$

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & Q_3 & 0 & \Omega_{15} & X_{22} & \Omega_{17} & Q_1 B & \Omega_{19} & Q_3 & 0 \\ * & \Omega_{22} & 0 & 0 & -Q_1 C & X_{12} & \Omega_{27} & Q_1 B & -Z & 0 & 0 \\ * & * & \Omega_{33} & Q_2 & 0 & 0 & F_4 R_2 & 0 & 0 & 0 & Q_2 \\ * & * & * & -P_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -P_1 & -X_{22} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -P_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -R_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -R_2 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -R_3 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\frac{1}{\tau} P_4 & 0 \\ * & * & * & * & * & * & * & * & * & 0 & -\frac{1}{\tau} P_4 \end{bmatrix} < 0, \tag{14}$$

where $\Omega_{11} = X_{12} + X_{12}^T + P_1 + \delta^2 P_2 + P_3 - ZD - D^T Z^T - Q_3 - Q_3^T - F_3 R_1 - G_1 R_3$, $\Omega_{15} = -X_{12} - Q_1 C$, $\Omega_{17} = Q_1 A + F_4 R_1$, $\Omega_{19} = -Z + G_2 R_3$, $\Omega_{22} = -Q_1 - Q_1^T + \tau P_4$, $\Omega_{27} = W_1 - W_2 + Q_1 A$, $\Omega_{33} = -Q_2 - Q_2^T -$

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$F_3 R_2$, then error-state system (8) of the delayed neural network described by (1) and (6) is globally asymptotically stable, and the estimator gain matrix K can be designed as $K = Q_1^{-1} Z$.

Proof. From assumption (H1), we know that

$$\int_0^{e_i(t)} (h_i(s) - F_i^- s) ds \geq 0, \quad (15)$$

$$\int_0^{e_i(t)} (F_i^+ s - h_i(s)) ds \geq 0, \quad i = 1, 2, \dots, n. \quad (16)$$

Let $W_1 = \text{diag}(w_{11}, w_{12}, \dots, w_{1n})$, $W_2 = \text{diag}(w_{21}, w_{22}, \dots, w_{2n})$, and consider the following Lyapunov-Krasovskii functional as

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \quad (17)$$

where

$$V_1(t) = \begin{bmatrix} e(t) \\ \int_{t-\delta}^t e(s) ds \end{bmatrix}^T \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} \begin{bmatrix} e(t) \\ \int_{t-\delta}^t e(s) ds \end{bmatrix},$$

$$V_2(t) = 2 \sum_{i=1}^n w_{1i} \int_0^{e_i(t)} (h_i(s) - F_i^- s) ds \\ + 2 \sum_{i=1}^n w_{2i} \int_0^{e_i(t)} (F_i^+ s - h_i(s)) ds,$$

$$V_3(t) = \int_{t-\delta}^t e^T(s) P_1 e(s) ds + \delta \int_{-\delta}^0 \int_{t+\xi}^t e^T(s) P_2 e(s) ds d\xi,$$

$$V_4(t) = \int_{t-\tau}^t e^T(s) P_3 e(s) ds + \int_{-\tau}^0 \int_{t+\xi}^t \dot{e}^T(s) P_4 \dot{e}(s) ds d\xi. \quad (18)$$

Calculating the time derivative of $V_i(t)$ ($i = 1, 2, 3, 4$), we obtain

$$\dot{V}_1(t) = 2 \begin{bmatrix} e(t) \\ \int_{t-\delta}^t e(s) ds \end{bmatrix}^T \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} \begin{bmatrix} \dot{e}(t) \\ e(t) - e(t-\delta) \end{bmatrix} \\ = e^T(t) (X_{12} + X_{12}^T) e(t) + 2e^T(t) X_{11} \dot{e}(t) \\ - 2e^T(t) X_{12} e(t-\delta) + 2e^T(t) X_{22} \int_{t-\delta}^t e(s) ds \\ + 2\dot{e}^T(t) X_{12} \int_{t-\delta}^t e(s) ds \\ - 2e^T(t-\delta) X_{22} \int_{t-\delta}^t e(s) ds, \quad (19)$$

$$\dot{V}_2(t) = 2\dot{e}^T(t) W_1 (h(e(t)) - F_1 e(t)) \\ + 2\dot{e}^T(t) W_2 (F_2 e(t) - h(e(t))) \quad (20)$$

$$= 2e^T(t) (-F_1 W_1 + F_2 W_2) \dot{e}(t)$$

$$+ 2\dot{e}^T(t) (W_1 - W_2) h(e(t)),$$

$$\dot{V}_3(t) = e^T(t) (P_1 + \delta^2 P_2) e(t)$$

$$- e^T(t-\delta) P_1 e(t-\delta) - \delta \int_{t-\delta}^t e^T(s) P_2 e(s) ds$$

$$\leq e^T(t) (P_1 + \delta^2 P_2) e(t) - e^T(t-\delta) P_1 e(t-\delta)$$

$$- \left(\int_{t-\delta}^t e(s) ds \right)^T P_2 \left(\int_{t-\delta}^t e(s) ds \right), \quad (21)$$

$$\dot{V}_4(t) = e^T(t) P_3 e(t) - e^T(t-\tau) P_3 e(t-\tau)$$

$$+ \tau \dot{e}^T(t) P_4 \dot{e}(t) - \int_{t-\tau}^t \dot{e}^T(s) P_4 \dot{e}(s) ds. \quad (22)$$

In deriving inequality (21), we have made use of Lemma 2. It follows from inequalities (19)–(22) that

$$\dot{V}(t) \leq e^T(t) (X_{12} + X_{12}^T + P_1 + \delta^2 P_2 + P_3) e(t) \\ + 2e^T(t) (X_{11} - F_1 W_1 + F_2 W_2) \dot{e}(t) \\ - 2e^T(t) X_{12} e(t-\delta) \\ + 2e^T(t) X_{22} \int_{t-\delta}^t e(s) ds \\ + \tau \dot{e}^T(t) P_4 \dot{e}(t) + 2\dot{e}^T(t) X_{12} \int_{t-\delta}^t e(s) ds \\ + 2\dot{e}^T(t) (W_1 - W_2) h(e(t)) \\ - e^T(t-\delta) P_1 e(t-\delta) \\ - 2e^T(t-\delta) X_{22} \int_{t-\delta}^t e(s) ds \\ - e^T(t-\tau) P_3 e(t-\tau) \\ - \left(\int_{t-\delta}^t e(s) ds \right)^T P_2 \left(\int_{t-\delta}^t e(s) ds \right) \\ - \int_{t-\tau}^t \dot{e}^T(s) P_4 \dot{e}(s) ds. \quad (23)$$

From model (8), we have

$$\begin{aligned}
 0 &= 2 \left(e^T(t) + \dot{e}^T(t) \right) Q_1 \\
 &\quad \times \left[-\dot{e}(t) - KDe(t) - Ce(t - \delta) + Ah(e(t)) \right. \\
 &\quad \quad \left. + Bh(e(t - \tau(t))) - Kr(t, e(t)) \right] \\
 &= -e^T(t) \left(Q_1KD + D^TK^TQ_1^T \right) e(t) \\
 &\quad - 2e^T(t) Q_1\dot{e}(t) - 2e^T(t) Q_1Ce(t - \delta) \\
 &\quad + 2e^T(t) Q_1Ah(e(t)) + 2e^T(t) Q_1Bh \\
 &\quad \times (e(t - \tau(t))) - 2e^T(t) Q_1Kr(t, e(t)) \\
 &\quad - \dot{e}^T(t) \left(Q_1 + Q_1^T \right) \dot{e}(t) - 2e^T(t) D^TK^TQ_1^T\dot{e}(t) \\
 &\quad - 2\dot{e}^T(t) Q_1Ce(t - \delta) + 2\dot{e}^T(t) Q_1Ah(e(t)) \\
 &\quad + 2\dot{e}^T(t) Q_1Bh(e(t - \tau(t))) \\
 &\quad - 2\dot{e}^T(t) Q_1Kr(t, e(t)).
 \end{aligned} \tag{24}$$

By Newton-Leibniz formulation and assumption (H2), we have

$$\begin{aligned}
 0 &= -2e^T(t - \tau(t)) Q_2 \\
 &\quad \times \left(e(t - \tau(t)) - e(t - \tau) - \int_{t-\tau}^{t-\tau(t)} \dot{e}(s) ds \right) \\
 &\leq -2e^T(t - \tau(t)) Q_2e(t - \tau(t)) \\
 &\quad + 2e^T(t - \tau(t)) Q_2e(t - \tau) \\
 &\quad + \tau e^T(t - \tau(t)) Q_2P_4^{-1}Q_2^Te(t - \tau(t)) \\
 &\quad + \int_{t-\tau}^{t-\tau(t)} \dot{e}^T(s) P_4\dot{e}(s) ds, \\
 0 &= -2e^T(t) Q_3 \\
 &\quad \times \left(e(t) - e(t - \tau(t)) - \int_{t-\tau(t)}^t \dot{e}(s) ds \right) \\
 &\leq -2e^T(t) Q_3e(t) + 2e^T(t) Q_3e(t - \tau(t)) \\
 &\quad + \tau e^T(t) Q_3P_4^{-1}Q_3^Te(t) \\
 &\quad + \int_{t-\tau(t)}^t \dot{e}^T(s) P_4\dot{e}(s) ds.
 \end{aligned} \tag{25}$$

In addition, for positive diagonal matrices $R_i > 0$ ($i = 1, 2, 3$), we can get from assumption (H1) and assumption (H3) that [15]

$$\begin{aligned}
 \begin{bmatrix} e(t) \\ h(e(t)) \end{bmatrix}^T \begin{bmatrix} F_3R_1 & -F_4R_1 \\ -F_4R_1 & R_1 \end{bmatrix} \begin{bmatrix} e(t) \\ h(e(t)) \end{bmatrix} &\leq 0, \\
 \begin{bmatrix} e(t - \tau(t)) \\ h(e(t - \tau(t))) \end{bmatrix}^T \begin{bmatrix} F_3R_2 & -F_4R_2 \\ -F_4R_2 & R_2 \end{bmatrix} \\
 \times \begin{bmatrix} e(t - \tau(t)) \\ h(e(t - \tau(t))) \end{bmatrix} &\leq 0, \\
 \begin{bmatrix} e(t) \\ r(t, e(t)) \end{bmatrix}^T \begin{bmatrix} G_1R_3 & -G_2R_3 \\ -G_2R_3 & R_3 \end{bmatrix} \begin{bmatrix} e(t) \\ r(t, e(t)) \end{bmatrix} &\leq 0.
 \end{aligned} \tag{26}$$

It follows from (23)–(26) that

$$\begin{aligned}
 \dot{V}(t) &\leq e^T(t) \left(X_{12} + X_{12}^T + P_1 + \delta^2P_2 + P_3 \right. \\
 &\quad \left. - Q_1KD - D^TK^TQ_1^T - Q_3 - Q_3^T \right. \\
 &\quad \left. + \tau Q_3P_4^{-1}Q_3^T - F_3R_1 - G_1R_3 \right) e(t) \\
 &\quad + 2e^T(t) \left(X_{11} - F_1W_1 + F_2W_2 \right. \\
 &\quad \quad \left. - Q_1 - D^TK^TQ_1^T \right) \dot{e}(t) \\
 &\quad + 2e^T(t) Q_3e(t - \tau(t)) \\
 &\quad - 2e^T(t) \left(X_{12} + Q_1C \right) e(t - \delta) \\
 &\quad + 2e^T(t) X_{22} \int_{t-\delta}^t e(s) ds \\
 &\quad + 2e^T(t) \left(Q_1A + F_4R_1 \right) h(e(t)) \\
 &\quad + 2e^T(t) Q_1Bh(e(t - \tau(t))) \\
 &\quad + 2e^T(t) \left(-Q_1K + G_2R_3 \right) r(t, e(t)) \\
 &\quad + \dot{e}^T(t) \left(-Q_1 - Q_1^T + \tau P_4 \right) \dot{e}(t) \\
 &\quad - 2\dot{e}^T(t) Q_1Ce(t - \delta) \\
 &\quad + 2\dot{e}^T(t) X_{12} \int_{t-\delta}^t e(s) ds \\
 &\quad + 2\dot{e}^T(t) \left(W_1 - W_2 + Q_1A \right) h(e(t)) \\
 &\quad + 2\dot{e}^T(t) Q_1Bh(e(t - \tau(t))) \\
 &\quad - 2\dot{e}^T(t) Q_1Kr(t, e(t)) + e^T(t - \tau(t)) \\
 &\quad \times \left(-Q_2 - Q_2^T + \tau Q_2P_4^{-1}Q_2^T \right. \\
 &\quad \quad \left. - F_3R_2 \right) e(t - \tau(t)) \\
 &\quad + 2e^T(t - \tau(t)) Q_2e(t - \tau) \\
 &\quad + 2e^T(t - \tau(t)) F_4R_2h(e(t - \tau(t)))
 \end{aligned}$$

$$\begin{aligned}
 & -e^T(t-\tau)P_3e(t-\tau) - e^T(t-\delta)P_1e \\
 & \times (t-\delta) - 2e^T(t-\delta)X_{22} \int_{t-\delta}^t e(s) ds \\
 & - \left(\int_{t-\delta}^t e(s) ds \right)^T P_2 \left(\int_{t-\delta}^t e(s) ds \right) \\
 & - h^T(e(t))R_1h(e(t)) \\
 & - h^T(e(t-\tau(t)))R_2h(e(t-\tau(t))) \\
 & - r^T(t, e(t))R_3r(t, e(t)) = \xi^T(t)\Pi\xi(t),
 \end{aligned} \tag{27}$$

where $\xi(t) = (e^T(t), \dot{e}^T(t), e^T(t-\tau(t)), e^T(t-\tau), e^T(t-\delta), \int_{t-\delta}^t e^T(s)ds, h^T(e(t)), h^T(e(t-\tau(t))), r^T(t, e(t)))^T$,

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & Q_3 & 0 & -X_{12} - Q_1C & X_{22} & Q_1A + F_4R_1 & Q_1B & -Q_1K + G_2R_3 \\ * & \Pi_{22} & 0 & 0 & -Q_1C & X_{12} & W_1 - W_2 + Q_1A & Q_1B & -Q_1K \\ * & * & \Pi_{33} & Q_2 & 0 & 0 & F_4R_2 & 0 & 0 \\ * & * & * & -P_3 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -P_1 & -X_{22} & 0 & 0 & 0 \\ * & * & * & * & * & -P_2 & 0 & 0 & 0 \\ * & * & * & * & * & * & -R_1 & 0 & 0 \\ * & * & * & * & * & * & * & -R_2 & 0 \\ * & * & * & * & * & * & * & * & -R_3 \end{bmatrix} \tag{28}$$

with $\Pi_{11} = X_{12} + X_{12}^T + P_1 + \delta^2P_2 + P_3 - Q_1KD - D^TK^TQ_1^T - Q_3 - Q_3^T + \tau Q_3P_4^{-1}Q_3^T - F_3R_1 - G_1R_3$, $\Pi_{12} = X_{11} - F_1W_1 + F_2W_2 - Q_1 - D^TK^TQ_1^T$, $\Pi_{22} = -Q_1 - Q_1^T + \tau P_4$, $\Pi_{33} = -Q_2 - Q_2^T + \tau Q_2P_4^{-1}Q_2^T - F_3R_2$.

By using Lemma 3, and noting $K = Q_1^{-1}Z$, it is easy to verify the equivalence of $\Pi < 0$ and $\Omega < 0$. Thus, one can derive from (14) and (27) that

$$\dot{V}(t) \leq 0, \tag{29}$$

which implies that the error-state system (8) of the delayed neural networks described by (1) and (6) is globally asymptotically stable. The proof is completed. \square

4. Numerical Example

To verify the effectiveness of the theoretical result of this paper, consider the following example.

Example 1. Consider a two-neuron neural network (1), where

$$\begin{aligned}
 C &= \begin{bmatrix} 1.3 & 0 \\ 0 & 0.9 \end{bmatrix}, & A &= \begin{bmatrix} 0.5 & -0.2 \\ 0.7 & 0.5 \end{bmatrix}, \\
 B &= \begin{bmatrix} 0.6 & -0.1 \\ -1.2 & -0.8 \end{bmatrix}, & J(t) &= \begin{bmatrix} 1.6 \cos(21t) \\ -1.3 \sin(1.1t) \end{bmatrix}, \\
 f_1(x) &= f_2(x) = \tanh(x), \\
 \delta &= 0.1, & \tau(t) &= 0.27 |\sin(7t)|.
 \end{aligned} \tag{30}$$

Figure 1 shows that the considered neural network has a chaotic attractor, where the initial condition is $x_1(t) = 0.5 \cos(0.5t)$, $x_2(t) = -0.2 \sin(18t)$, and $t \in [-0.27, 0]$.

It can be verified that assumptions (H1) and (H2) are satisfied, and $F_1 = 0$, $F_2 = I$, $F_3 = 0$, $F_4 = \text{diag}\{0.5, 0.5\}$, $\tau = 0.27$.

Choose network measurement (4), where

$$D = \begin{bmatrix} 0.7 & -0.2 \\ 0.1 & 0.5 \end{bmatrix}, \tag{31}$$

$$g(t, x(t)) = 0.1 \cos x(t) + 0.3.$$

It is obvious that assumption (H3) is satisfied with $G_1 = -0.01I$ and $G_2 = 0$. By the Matlab LMI Control Toolbox, we find a solution to the LMIs in (13) and (14) as follows:

$$\begin{aligned}
 P_1 &= \begin{bmatrix} 0.5042 & 0.0068 \\ 0.0068 & 0.1895 \end{bmatrix}, & P_2 &= \begin{bmatrix} 2.7464 & 0.0896 \\ 0.0896 & 2.0468 \end{bmatrix}, \\
 P_3 &= \begin{bmatrix} 0.3288 & -0.0027 \\ -0.0027 & 0.1564 \end{bmatrix}, & P_4 &= \begin{bmatrix} 0.1907 & 0.0138 \\ 0.0138 & 0.1027 \end{bmatrix}, \\
 W_1 &= \begin{bmatrix} 0.1184 & 0 \\ 0 & 0.0779 \end{bmatrix}, & W_2 &= \begin{bmatrix} 0.2313 & 0 \\ 0 & 0.1183 \end{bmatrix}, \\
 R_1 &= \begin{bmatrix} 0.9466 & 0 \\ 0 & 0.4360 \end{bmatrix}, & R_2 &= \begin{bmatrix} 0.4704 & 0 \\ 0 & 0.2050 \end{bmatrix}, \\
 R_3 &= \begin{bmatrix} 2.9274 & 0 \\ 0 & 3.4376 \end{bmatrix}, & Q_1 &= \begin{bmatrix} 0.2168 & 0.0134 \\ 0.0144 & 0.0812 \end{bmatrix}, \\
 Q_2 &= \begin{bmatrix} 0.2025 & 0.0032 \\ 0.0032 & 0.1054 \end{bmatrix}, & Q_3 &= \begin{bmatrix} -0.0134 & 0.0159 \\ -0.0162 & 0.0155 \end{bmatrix}, \\
 X_{11} &= \begin{bmatrix} 0.4753 & 0.0123 \\ 0.0123 & 0.1554 \end{bmatrix}, & X_{12} &= \begin{bmatrix} -0.5667 & -0.0378 \\ -0.0362 & -0.2141 \end{bmatrix}, \\
 X_{22} &= \begin{bmatrix} 0.8376 & 0.0508 \\ 0.0508 & 0.4503 \end{bmatrix}, & Z &= \begin{bmatrix} 0.5858 & 0.1452 \\ -0.0249 & 0.4239 \end{bmatrix}.
 \end{aligned} \tag{32}$$

Subsequently, we can obtain from $K = Q_1^{-1}Z$ that

$$K = \begin{bmatrix} 2.7511 & 0.3508 \\ -0.7949 & 5.1560 \end{bmatrix}. \tag{33}$$

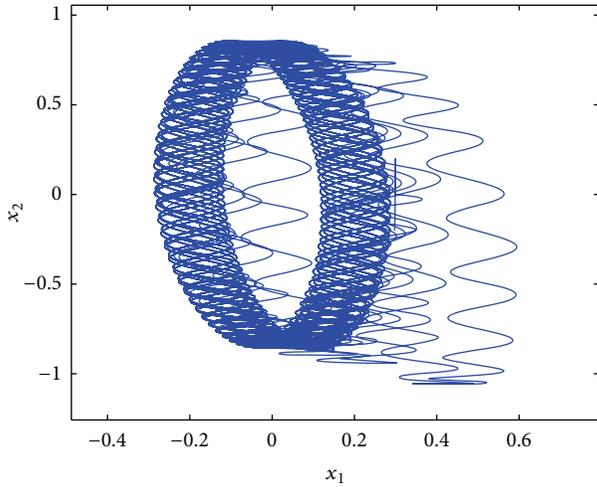


FIGURE 1: Phase trajectory of model (1).

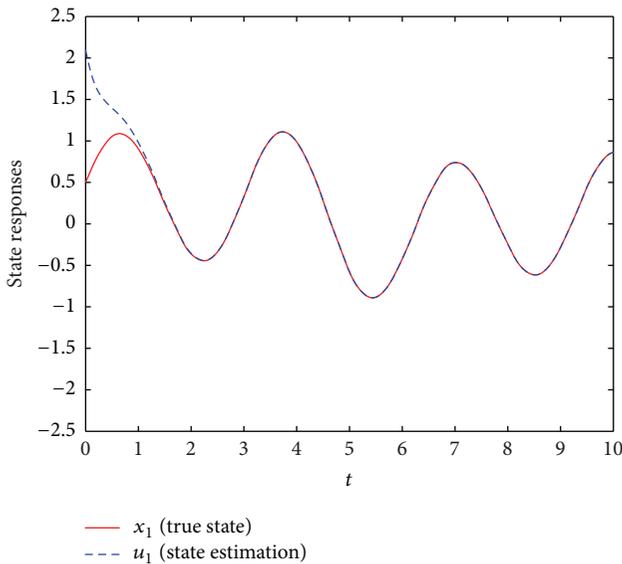


FIGURE 2: The responses of the true state $x_1(t)$ (solid line) and its estimation $u_1(t)$ (dash line).

Therefore, we know from Theorem 4 that error-state system (8) of the delayed neural network described by (1) and (6) is globally asymptotically stable. The simulation results are shown in Figures 2 and 3, which demonstrate the effectiveness of the developed approach for the design of the state estimator for neural networks with leakage delay and time-varying delay.

5. Conclusions

In this paper, the state estimation problem has been investigated for neural networks with leakage delay and time-varying delay as well as general activation functions. By employing Lyapunov functional method and the matrix inequality techniques, a delay-dependent LMIs condition has been established to estimate the neuron state with some

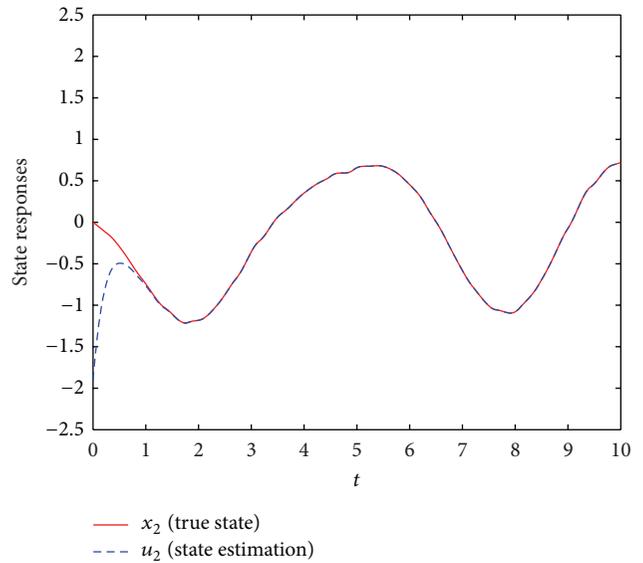


FIGURE 3: The responses of the true state $x_2(t)$ (solid line) and its estimation $u_2(t)$ (dash line).

observed output measurements such that the error-state system is globally asymptotically stable. An example has been provided to show the effectiveness of the proposed criterion.

Acknowledgments

The authors would like to thank the reviewers and the editor for their valuable suggestions and comments which have led to a much improved paper. This work was supported by the National Natural Science Foundation of China under Grants 61273021, 60974132, 11172247, and 51208538 and in part by the Natural Science Foundation Project of CQ cstc2013jjB40008.

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