Ark. Mat., **45** (2007), 269–271
DOI: 10.1007/s11512-007-0042-0

Erratum to "A new example of a uniformly Levi degenerate hypersurface in $\mathbf{C}^3$"

Hervé Gaussier and Joël Merker

Abstract. The purpose of this erratum is to point out a mistake in a previous article by the authors and to explain how this affects the results of that article.

The paper [4], "A new example of a uniformly Levi degenerate hypersurface in $\mathbf{C}^3$", was devoted to the study of the hypersurface

$$M_0 := \left\{ (z_1, z_2, w) : w + \overline{w} = \frac{2z_1\bar{z}_1 + z_1^2\bar{z}_2 + \bar{z}_1^2 z_2}{1 - z_2\bar{z}_2} \text{ and } |z_2| < 1 \right\}.$$

This hypersurface belongs to the family $\mathcal{M}$ of homogeneous, two-nondegenerate, uniformly Levi degenerate hypersurfaces in $\mathbf{C}^3$, with rank one.

The aim of the paper was to compare the CR geometry of M_0 and of the tube $\Gamma_C := C + i\mathbf{R}^3 \subset \mathbf{C}^3$ over the real two-dimensional cone $C := \{(x_1, x_2, x_3) : x_1^2 + x_2^2 - x_3^2 = 0\} \subset \mathbf{R}^3$. The hypersurface Γ_C was characterized by P. Ebenfelt in [1], among the elements of the family $\mathcal{M}$, by the vanishing of some curvature form.

The main statement in [4] was the CR inequivalence between M_0 and of Γ_C. The approach was based on the Lie theory of prolongation of vector fields and on the observation that the Segre varieties attached to M_0 are solutions of a nonlinear system of partial differential equations. More precisely, let us write $(x_1, x_2, u, \bar{x}_1, \bar{x}_2, \bar{u})$ instead of $(z_1, z_2, w, \bar{z}_1, \bar{z}_2, \overline{w})$ and let us consider $\bar{u}$, $\bar{x}_1$ and $\bar{x}_2$ as complex parameters, x_1 and x_2 as two independent variables and u as a dependent variable. Then, as mentioned on p. 90 of [4], every partial derivative $u_{x_1^k x_2^l} = \partial^{k+l} u / \partial x_1^k \partial x_2^l$ may be expressed in terms of u_{x_1} and $u_{x_1^2}$. The exact expressions are provided by system (3.4) in [4]. The scheme of the proof of the nonequivalence between M_0 and Γ_C in [4] was the following one.

The online version of the original article can be found at http://dx.doi.org/10.1007/BF02384568

Let M be a homogeneous, two-nondegenerate, uniformly Levi degenerate hypersurface in $\mathbf{C}^3$, with rank one and let $p \in M$. We denote by $\mathrm{Aut}_{\mathrm{CR}}(M)$ the real Lie algebra of infinitesimal CR automorphisms of M and by $\mathrm{Aut}_{\mathrm{CR}}(M,p)$ the isotropy subalgebra at point p.

The Lie theory of prolongation of vector fields enables one to reduce the resolution of the nonlinear system (3.4) to the resolution of the linear system (3.12) of partial differential equations. Solving system (3.12) we exhibited seven independent CR automorphisms for M_0. Using the following estimates stated in [1]:

$$\dim_{\mathbf{R}} \mathrm{Aut}_{\mathrm{CR}}(M) \leq 7, \quad \dim_{\mathbf{R}} \mathrm{Aut}_{\mathrm{CR}}(M,p) \leq 2, \tag{0.1}$$

we obtained the equality $\dim_{\mathbf{R}} \mathrm{Aut}_{\mathrm{CR}}(M_0) = 7$. Moreover, the Lie algebra generated by the two corresponding vector fields in the isotropy subalgebra $\mathrm{Aut}_{\mathrm{CR}}(M_0,0)$ at the origin is commutative. Finding two noncommuting vector fields in $\mathrm{Aut}_{\mathrm{CR}}(\Gamma_C, p)$, where $p=(1,0,1)$, we concluded the nonequivalence between M_0 and Γ_C.

The aim of this erratum is to correct the computation lead in [4]. As a consequence, we exhibit ten vector fields generating $\mathrm{Aut}_{\mathrm{CR}}(M_0)$. We point out that, by a careful inspection of the structures of $\mathrm{Aut}_{\mathrm{CR}}(M_0)$ and of $\mathrm{Aut}_{\mathrm{CR}}(\Gamma_C)$, G. Fels–W. Kaup [3] proved that M_0 and Γ_C are actually CR equivalent. Finally the estimates (0.1) were revised in [2].

Computation of generators of $\mathrm{Aut}_{\mathrm{CR}}(M_0)$

The mistake in the computation occurs in equation (3.10) p. 92, where the coefficient of $U^3_{1,1,2}$ in the expansion of $R^3_{1,1,1}$ is $-3Q^2_{x_1}$ instead of $-5Q^2_{x_1}$.

Consequently the fourth and the ninth equations in system (3.12) are identical. The resolution of the eight first equations in system (3.12) provides the following general form for a generator of the symmetry group of system (3.4):

$$\begin{aligned} Y = {} & [a_{10}+a_{11}x_1+a_{12}x_1^2+(a_{20}+a_{21}x_1)x_2+(a_{30}+a_{31}x_1)u-a_{12}ux_2]\frac{\partial}{\partial x_1} \\ & +[b_{10}-2a_{30}x_1-a_{31}x_1^2+2a_{12}x_1x_2+b_{20}x_2+a_{21}x_2^2]\frac{\partial}{\partial x_2} \\ & +[c_{10}+(2a_{11}-b_{20})u+a_{31}u^2+(-2a_{20}+2a_{12}u)x_1-a_{21}x_1^2]\frac{\partial}{\partial u}, \end{aligned}$$

where a_{10}, a_{11}, a_{12}, a_{20}, a_{21}, a_{30}, a_{31}, b_{10}, b_{20} and c_{10} are ten complex constants.

A basis of $\mathrm{Aut}_{\mathrm{CR}}(M_0,0)$ is then given by the real parts of the ten following holomorphic vector fields:

$$X^1 = i\frac{\partial}{\partial w},$$

$$
\begin{aligned}
X^2 &= z_1\frac{\partial}{\partial z_1}+2w\frac{\partial}{\partial w},\\
X^3 &= i\left(z_1\frac{\partial}{\partial z_1}+2z_2\frac{\partial}{\partial z_2}\right),\\
X^4 &= (z_2-1)\frac{\partial}{\partial z_1}-2z_1\frac{\partial}{\partial w},\\
X^5 &= i\left((1+z_2)\frac{\partial}{\partial z_1}-2z_1\frac{\partial}{\partial w}\right),\\
X^6 &= z_1z_2\frac{\partial}{\partial z_1}+(z_2^2-1)\frac{\partial}{\partial z_2}-z_1^2\frac{\partial}{\partial w},\\
X^7 &= i\left(z_1z_2\frac{\partial}{\partial z_1}+(z_2^2+1)\frac{\partial}{\partial z_2}-z_1^2\frac{\partial}{\partial w}\right),\\
X^8 &= i\left(wz_1\frac{\partial}{\partial z_1}-z_1^2\frac{\partial}{\partial z_2}+w^2\frac{\partial}{\partial w}\right),\\
X^9 &= (z_1^2-wz_2-w)\frac{\partial}{\partial z_1}+(2z_1z_2+2z_1)\frac{\partial}{\partial z_2}+2wz_1\frac{\partial}{\partial w},\\
X^{10} &= i\left((-z_1^2+wz_2-w)\frac{\partial}{\partial z_1}+(-2z_1z_2+2z_1)\frac{\partial}{\partial z_2}-2wz_1\frac{\partial}{\partial w}\right).
\end{aligned}
$$

References

1. Ebenfelt, P., Uniformly Levi degenerate CR manifolds: the 5-dimensional case, *Duke Math. J.* **110** (2001), 37–80.
2. Ebenfelt, P., Correction to: "Uniformly Levi degenerate CR manifolds: the 5-dimensional case", *Duke Math. J.* **131** (2006), 589–591.
3. Fels, G. and Kaup, W., CR-manifolds of dimension 5: A Lie algebra approach, to appear in *J. Reine Angew. Math.*
4. Gaussier, H. and Merker, J., A new example of a uniformly Levi degenerate hypersurface in $\mathbf{C}^3$, *Ark. Mat.* **41** (2003), 85–94.

Hervé Gaussier
Université de Provence
Centre de Mathématiques et Informatique
39 rue Joliot-Curie
FR-13453 Marseille Cedex 13
France
gaussier@cmi.univ-mrs.fr

Joël Merker
Université de Provence
Centre de Mathématiques et Informatique
39 rue Joliot-Curie
FR-13453 Marseille Cedex 13
France
merker@cmi.univ-mrs.fr

Received March 13, 2006
published online May 15, 2007