ON THE L^p THEORY OF HANKEL TRANSFORMS

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1. Introduction. Under suitable restrictions on $f(x)$ and ν, the Hankel transform $g(t)$ of $f(x)$ is defined by the relation

$$g(t) = \int_0^\infty (xt)^{1/2} J_\nu(xt) f(x)\, dx. \tag{1}$$

The inverse is then given formally by

$$f(x) = \int_0^\infty (xt)^{1/2} J_\nu(xt) g(t)\, dt. \tag{2}$$

These integrals represent generalizations of the Fourier sine and cosine transforms to which they reduce when $\nu = \pm 1/2$. The L^p theory for the Fourier case has been studied in considerable detail. In this note we present some results concerning the inversion formula (2) in the L^p_a case.

It is clear that if $f(x) \in L$ and $\Re(\nu) \geq -1/2$ then the integral in (1) exists. It has been shown [3,6] that if $f(x) \in L^p$, $1 < p \leq 2$, then

$$g_a(t) = \int_0^a (xt)^{1/2} J_\nu(xt) f(x)\, dx \tag{3}$$

converges strongly to a function $g(t)$ in $L^{p'}$. For this case Kober has obtained the inversion formula,

$$f(x) = x^{-1/2-\nu} \frac{d}{dx} \left\{ x^{\nu+1/2} \int_0^\infty \frac{(xt)^{1/2} J_{\nu+1}(xt)}{t} g(t)\, dt \right\},$$

which holds for almost all x. In her investigation of Watson transforms, Busbridge [1] has given analogous results for more general kernels. Except when p = 2 the question of the strong convergence of the inversion integral has apparently been considered only in the Fourier case [2]. We now investigate this problem

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