A GENERALIZATION OF THE CENTRAL ELEMENTS OF A GROUP

EUGENE SCHENKMAN

1. Introduction. If a and g are elements of a group G, we shall denote by $a^{(1)}(g)$ or $a(g)$ the element $g^{-1}ag$, and then for $n = 2, 3, 4, \cdots$ define $a^{(n)}(g) = a(a^{(n-1)}(g))$.

If for some n and all $g \in G$, $a^{(n)}(g) = a$ then a will be called *weakly central of order n* or simply *weakly central*. Thus the center elements of G are weakly central of order 1.

As usual, let

$$[g, a] = a^{-1}\, g^{-1}\, ag = a^{-1} \cdot a(g);$$

then it can readily be verified by induction on n that

$$a^{-1} \cdot a^{(n)}(g) = a^{-1} \cdot [\overbrace{a \cdots [a}^{n-1 \text{ times}}, g] \cdots]^{-1} \cdot a \cdot [\overbrace{a \cdots [a}^{n-1 \text{ times}}, g] \cdots]$$

$$= [\overbrace{a \cdots [a}^{n \text{ times}}, g] \cdots].$$

Thus $a^{(n)}(g) = a$ is equivalent to

$$[\overbrace{a \cdots [a}^{n \text{ times}}, g] \cdots] = e,$$

where e is the identity of G. It follows that if a is an element of a normal nilpotent finite subgroup of G then a is weakly central. Another easy consequence of the definition is that if a is weakly central in G then a is its own normalizer in G if and only if $\{a\} = G$; here $\{a\}$ denotes the subgroup generated by a. It should also be noted that if a is weakly central in G, then $\overline{a}$ is weakly central in $\overline{G}$, where $\overline{a}$ is the image of a under a homomorphism which takes G onto $\overline{G}$.

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