# ON GENERALIZED SUBHARMONIC FUNCTIONS

L. K. JACKSON

**1. Introduction.** In a previous paper [1], the notion of subharmonic functions was generalized in a manner corresponding to Beckenbach's [2] generalization of convex functions. This generalization was accomplished by replacing the dominating family of harmonic functions by a more general family of functions. In [1] the discussion was restricted to continuous subfunctions.

In the present paper we shall give some further properties of the dominating functions and extend the definition of subfunctions to permit upper semicontinuous subfunctions. We shall then show that results of J. W. Green [3] on approximately subharmonic functions extend to our subfunctions.

**2. $\{F\}$-functions and sub-$\{F\}$ functions.** Let $D$ be a given plane domain and let $\{\gamma\}$ be a given family of contours bounding subdomains $\Gamma$ of $D$ such that $\overline{\Gamma} = \gamma + \Gamma \subset D$ where $\overline{\Gamma}$ indicates the closure of $\Gamma$. We assume that $\{\gamma\}$ contains all circles of radii less than a fixed number which lie, together with their interiors, in $D$. We shall use the Greek letter $\kappa$ to represent a circle of $\{\gamma\}$ and $K$ its interior. We shall use single small Roman letters to represent points in the plane. Let there be given a family of functions $\{F(x)\}$ which we shall call $\{F\}$-functions satisfying the following postulates.

POSTULATE 1. For any $\gamma \in \{\gamma\}$ and any continuous boundary value function $h(x)$ on $\gamma$, there is a unique $F(x; h; \gamma) \in \{F(x)\}$ such that

(a) $F(x; h; \gamma) = h(x)$ on $\gamma$,

(b) $F(x; h; \gamma)$ is continuous in $\overline{\Gamma}$.

POSTULATE 2. If $h_1(x)$ and $h_2(x)$ are continuous on $\gamma$ and if $h_1(x) - h_2(x) \leq M$ on $\gamma$, $M \geq 0$, then

$$F(x; h_1; \gamma) - F(x; h_2; \gamma) \leq M$$

Received August 1, 1953. This research was supported in part by a Summer Research Fellowship granted by the Research Council of the University of Nebraska.

*Pacific J. Math.* 5 (1955), 215-228