SUBFUNCTIONS AND THE DIRICHLET PROBLEM

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1. Introduction. In previous papers [1 ; 6] the notion of subharmonic functions was generalized by replacing the dominating family of harmonic functions by a more general family of functions. The object was to require of the dominating functions the minimum properties necessary to study the boundary value problem by subfunction techniques. In a natural way these properties were separated into two parts: first, those properties sufficient to obtain functions which are solutions in the interior of a domain and, second, those properties sufficient to obtain agreement of the solution with the prescribed boundary values on the boundary of the domain. In particular the aim was to choose properties which would be sufficient to insure that a solution would take on prescribed boundary values at any boundary point p at which an exterior circle could be drawn intersecting the closed domain only in the point p. In a recent paper Inoue [5] points out an error in this second aspect of [1]. Inoue then lists properties of the dominating functions which are sufficient to insure the regularity of boundary points at which exterior triangles can be drawn. In his paper these properties are embodied in six postulates the first four of which are essentially the same as the first four postulates of [1]. Postulates 5 and 6 given by Inoue are used in studying the behavior at the boundary and are naturally more restrictive but they are such that the theory can be applied to elliptic partial differential equations which have the property that the difference between two solutions is subharmonic when positive.

In the present paper we use only the portion of the theory of subfunctions which is based on the first four postulates of [1] to obtain some results concerning the Dirichlet problem for certain types of elliptic equations. We shall give some results concerning the linear equation

$$\Delta z+a(x, y)z_x+b(x, y)z_y+c(x, y)z=f(x, y) \ , \tag{1}$$

where $\Delta z=\dfrac{\partial^2 z}{\partial x^2}+\dfrac{\partial^2 z}{\partial y^2}$, and the quasi-linear equation

$$a(p, q)r+2b(p, q)s+c(p, q)t=0 \ , \tag{2}$$

where $p=\dfrac{\partial z}{\partial x}$, $q=\dfrac{\partial z}{\partial y}$, $r=\dfrac{\partial^2 z}{\partial x^2}$, $s=\dfrac{\partial^2 z}{\partial x \partial y}$, and $t=\dfrac{\partial^2 z}{\partial x^2}$. In particular we

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